

## ES 242r Homework Problems 26-31

Due by submission to TFs on 15 May 2007

### 26. Subcritical cracking and delayed fracture of optical fibers

Optical fibers are susceptible to stress-corrosion. The fracture toughness can be measured by fast fracture test. Let's say the experimental value is  $K_C = 0.4 \text{ MPa}\sqrt{\text{m}}$ . In subcritical crack growth, the crack velocity  $v$  depends on the applied stress intensity factor  $K$ . The experimental data usually fit a power law  $v = BK^n$ . Say  $n = 22$ , and the crack velocity is  $v = 1 \text{ nm/s}$  when  $K = 0.1 \text{ MPa}\sqrt{\text{m}}$ . Use  $K = 1.12\sigma\sqrt{\pi a}$ .

- (a) If a fiber is designed to carry 50 MPa stress for 50 years, what initial flaw size can be tolerated?
- (b) In practice, the initial flaw size is estimated by a proof test. Describe the principle of the proof test. What stress level is needed in the proof test to warrant that the fiber will carry 50 MPa stress over 50 years?

### 27. The Dugdale model

Read D.S. Dugdale, Yielding of steel sheets containing slits. *Journal of the Mechanics and Physics of Solids*, **8**, 100-104 (1960).

- (a) Use linear superposition to show that

$$K_{\text{tip}} = \sigma_{\text{appl}}\sqrt{\pi(a+L)} - \sigma_0\sqrt{\pi(a+L)}\frac{2}{\pi}\cos^{-1}\left(\frac{a}{a+L}\right)$$
$$\delta_{\text{tail}} = \frac{4\sigma_{\text{appl}}}{E}\sqrt{(a+L)^2 - a^2} - \frac{8\sigma_0}{\pi E}\left[\cos^{-1}\left(\frac{a}{a+L}\right)\sqrt{(a+L)^2 - a^2} - a\ln\left(\frac{a+L}{a}\right)\right]$$

The notation is different from that used by Dugdale.

- (b) Derive the relation between the plastic zone size and the applied stress.
- (c) Derive the relation between the plastic zone size and the opening displacement at the tail of the plastic zone.
- (d) How does the model bridge the strength-limited design to the toughness-limited design?

### 28. Adhesive toughness

Read M.D. Thouless, J.L. Adams, M.S. Kafkalidis, S.M. Ward, R.A. Dickie, G.L. Westerbeek, Determining the toughness of plastically deforming joints. *Journal of Materials Science*, **33**, 189-197 (1998).

- (a) Describe the experimental procedure.
- (b) Derive Equation (10) from first principles.

29. A test specimen under a prescribed loading has the following relation among the quasi-static stress intensity factor, the prescribed overall stress,  $\sigma_0$ , and crack length,  $a$ :

$$K_s = \frac{\sigma_0 B}{\sqrt{a}} \quad (B = 0.1m)$$

The material has the following dynamic toughness:

$$K_{IC}(V) = \frac{K_{IC}^0}{(1 - (V/V_A)^2)}, \quad K_{IC}^0 = 50 \text{ MPa m}^{1/2}, \quad V_A = 1500 \text{ ms}^{-1}$$

In what follows assume no waves are reflected back onto the crack tip such that the dynamic stress intensity factor is given by  $K = k(m)K_S$ . In addition, as a simple approximation use

$$k(m) = (1 - m), \quad m = V/c, \quad c = 3000 \text{ ms}^{-1}$$

Suppose the crack has initial length  $a = 0.01 \text{ m}$  and that is initially blunt such that the overall stress reaches  $\sigma_0 = 100 \text{ MPa}$  before crack advance is initiated. Thereafter,  $\sigma_0$  remains fixed.

- Determine and plot the history of crack tip velocity as a function of time and the history of crack length  $a$  as a function of time.
- Verify that the crack length at which the crack arrests is given by the quasi-static condition:  $K_S = K_{IC}^0$ .

**30.** Reconsider the above problem which is unchanged except that the dynamic toughness of the material has reduced levels in an intermediate range of crack tip velocities between quasi-static and very high crack speeds according to

$$K_{IC}(V) = \frac{(3/4)K_{IC}^0}{(1 - ((V - V_A/2)/V_A)^2)}, \quad K_{IC}^0 = 50 \text{ MPa m}^{1/2}, \quad V_A = 1000 \text{ ms}^{-1}$$

(It would be good to plot the two curves for  $K_{IC}(V)$  on the same plot.)

Carry out parts a) and b) of the previous problem for this case. Now, however, in the case of b) you should find that the crack does not arrest quasi-statically but instead abruptly arrests at a finite velocity. What velocity is that? And does the quasi-static condition,  $K_S = K_{IC}^0$ , under or over-predict the crack length at arrest?

**31.** With reference to the figure below, consider the wall of a vessel designed with a brittle high temperature material (#1) on the inside well bonded to a tough material (#2) on the outside. For simplicity, assume the elastic properties of the two materials are the same.

Suppose each material has a dynamic toughness,  $K_{IC}^{(1)}(V)$  and  $K_{IC}^{(2)}(V)$ , respectively, which increases monotonically with  $V$ .

Denote the respective quasi-static toughnesses by  $K_{IC}^{(1)}(0)$  and  $K_{IC}^{(2)}(0)$ .

Given the design stress,  $\sigma_0$ , determine a simple, conservative condition on the choice of the toughness of material #2 based on quasi-static fracture analysis that guarantees that any crack that propagates through the brittle layer will arrest when it hits the interface with the tougher layer. Using dynamic fracture mechanics, give a convincing argument that your condition is conservative. You need not do any calculations on this problem. For the sake of discussion assume that the static stress intensity factor is given by  $K_S = \sigma_0 \sqrt{h_1} f(a)$  where  $f(a)$  is a monotonically increasing function of crack length  $a$ .

