Homework
Due Friday, 14 November

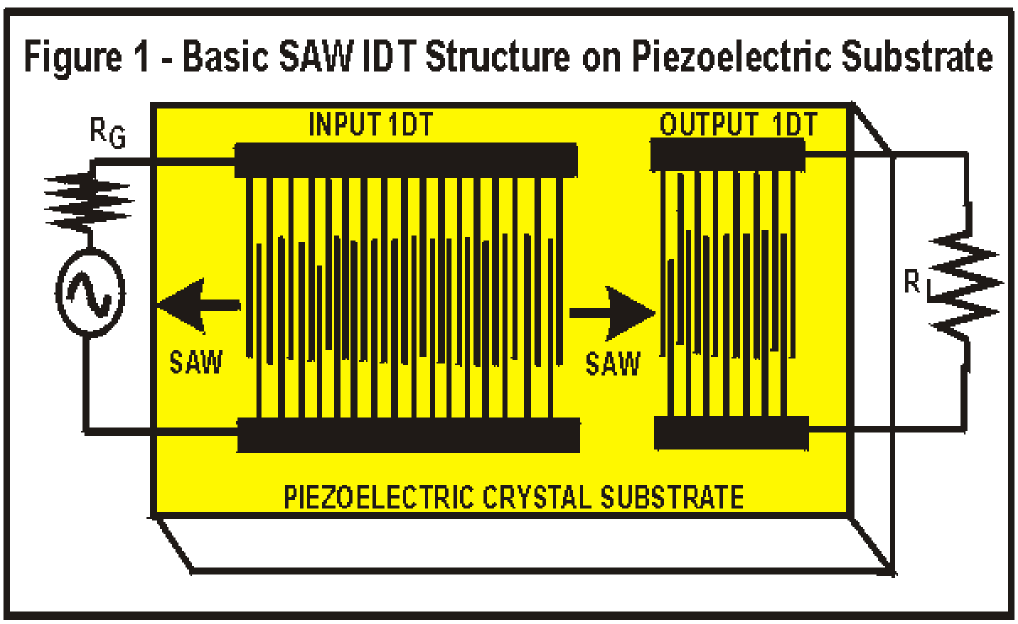
34. Surface Acoustic Wave (SAW) device

The Surface Acoustic Wave (SAW) device was invented by R.M. White and F.W. Voltmer (Direct piezoelectric coupling to surface elastic waves, Applied Physics Letters, vol. 17, pp. 314-316, 1965). The device consists of two sets of metal thin-film interdigital transducers on a piezoelectric substrate. When an alternating voltage is applied to one transducer, the substrate underneath the transducer vibrates, launching an acoustic wave on the surface of the substrate. When the acoustic wave reaches the other transducer, the acoustic vibration is converted back to an alternating voltage. The device transforms an electrical signal into an acoustic one and, after some processing, transforms it back to an electrical signal. The short acoustic wavelength, as compared to the electromagnetic wavelength, allows that the device to be made compact. Each device is only about a few millimeters in size, and costs less than $1. The SAW devices are used in TV and wireless phones. To appreciate the size, consider the following situation.

An acoustic wave is generated on the surface of a piezoelectric crystal by applying an alternating voltage to an interdigital transducer at a frequency of 1 GHz. The velocity of the acoustic wave in the crystal is 3488 m/s.

(a) Determine the wavelength of the acoustic wave.

(b) Compare the value of this wavelength with that of an electromagnetic wave propagating in free space at the same frequency.



http://www3.sympatico.ca/colin.kydd.campbell/

35. Approximate a rod as a 2DOF system

In the lecture notes on vibration (http://www.imechanica.org/node/339), we used a single linear-displacement element to represent a rod, and then used the PVW to determine the natural frequency. Now divide the rod into two linear displacement element of equal length. Use the PVW to obtain a pair of ODEs for the two degrees of freedom. Determine the natural frequencies. Compare your results with the exact solutions, as well as with the approximate solution obtained by representing the rod as a 1DOF system.

36. Soft tissues: large difference in the velocities of longitudinal and transverse waves.

Two kinds of plane waves exist in an isotropic elastic medium

a) A *longitudinal wave* has the displacement in the direction of the wave propagation. That is, if we call the wave propagation direction *x*, the only nonzero displacement component is $u(x,t)$. What stress components are nonzero? Reduce the governing equations to the equations for the longitudinal wave. Determine the longitudinal wave speed.

b) A *transverse wave* has the displacement normal to the direction of wave propagation. Reduce the governing equations to the equations for the transverse wave. Determine the transverse wave speed.

c) A soft tissue has a low shear modulus $G \sim 1\text{MPa}$, and is nearly incompressible $\nu \sim 0.49$. Take the mass density to be $\rho = 1500\text{kg/m}^3$. Calculate the speeds of the longitudinal and transverse waves. For a given frequency 100 Hz, calculate the wavelengths of the two waves. The large differences in speeds and wavelengths of the two waves have been exploited in acoustic imaging of human bodies (Professor Mathias Fink, http://www.loa.espci.fr/~mathias/en/).

37. A general approach to determine body waves.

In class we have described a general method to determine body waves in anisotropic media. This problem guides you through the method, using isotropic media as an example.

a) Start with momentum balance

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}$$

and stress-strain relations

$$\sigma_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda \frac{\partial u_k}{\partial x_k}\delta_{ij}.$$

Derive the equation of motion that governs the displacement field:

$$\mu \frac{\partial^2 u_i}{\partial x_k \partial x_k} + (\mu + \lambda)\frac{\partial^2 u_k}{\partial x_k \partial x_i} = \rho \frac{\partial^2 u_i}{\partial t^2}.$$

This equation is known as Navier's equation.

b) Because the equation contains no length scale, the plane waves must be nondispersive. Let **s** be the unit vector pointing in the direction of propagation, **a** be the unit vector pointing in the direction of the displacement field, and *c* be the wave velocity. The displacement field of a plane wave takes the form

$$\mathbf{u}(\mathbf{x},t) = \mathbf{a} f\left(\frac{\mathbf{x}\cdot\mathbf{s}}{c} - t\right),$$

where $f(\xi)$ is an arbitrary function. Insert this field into Navier's equation, and show that the wave speed *c* and the vector **a** are determined by an eigenvalue problem.

c) Solve the eigenvalue problem. Interpret the solution in terms of the longitudinal and transverse waves.

38. Reflection and refraction of a transverse wave

Two half spaces of dissimilar materials are bonded at a plane interface. A transverse wave is incident upon the interface, with the displacement normal to the plane of incidence. Knowing the displacement field of the incident wave, determine the displacement field of the reflected and refracted waves.