Homework
Due Friday, 12 December

43. Energy loss
A rectangular polymer bar of dimension $4\,\text{mm}\times 10\,\text{mm}\times 100\,\text{mm}$ is subject to an axial force oscillating sinusoidally between – 150 and 150 N at a frequency of 50 Hz. The storage and loss modulus of the polymer at 50 Hz are $E' = 3\times 10^9\,\text{Pa}$ and $E'' = 10^9\,\text{Pa}$. Calculate the power input required to sustain the oscillation.

44. Zener model and stress relaxation test
Use the Zener model to analyze the stress relaxation test (i.e., the step-strain test). From physical considerations, the compliance and the modulus at the two equilibrium configurations are related: $E_U = 1/D_U$, $E_R = 1/D_R$. Show that the stress relaxation modulus has the time dependence

$$E(t) = E_R + (E_U - E_R)\exp\left(-\frac{t}{\beta}\right)$$

and that the relaxation time of this test (β) relates to the relaxation time of the creep test (τ) as

$$\tau/\beta = E_U/E_R.$$

45. Zener model and cyclic test
Use the Zener model to analyze the stress-strain behavior under cyclic loading. In particular, demonstrate the relations

$$D'(\omega) = D_U + \frac{D_R - D_U}{1+(\omega\tau)^2}, \quad D''(\omega) = \frac{(D_R - D_U)\omega\tau}{1+(\omega\tau)^2}$$

Sketch the two moduli as functions of the loading frequency. Interpret the trends for various regimes of the frequency.

46. Vibration of a viscoelastic rod. A viscoelastic rod is made of a Kelvin material. That is, the stress and strain obeys

$$\sigma = E\varepsilon + H\frac{\partial\varepsilon}{\partial t}.$$

The rod vibrates in the axial direction under no external force. The two ends are free from constraint. Formulate the boundary value problem and determine all modes of vibration. Determine the time scale for each mode to decay. Discuss the implications of your results.