Due in class, Thursday, 11 March 2010

17. The resistance curve of a brittle material fits the expression $K_R(\Delta a) = K_0 + \beta\Delta a$, where K_0 and β are material parameters. Consider the effects of small cracks in a body designed to carry a fixed stress level σ. Assume that the stress intensity factor is given by $K = \sigma\sqrt{\pi a}$.
(a) Obtain the critical crack size a_c for initiation.
(b) Determine the relationship between K_0 and β to ensure that this critical crack is stable.
(c) Discuss the implication of increasing K_0.
(d) Discuss the implication of increasing β.
(e) Discuss the implication of increasing the design stress σ.

18. The resistance curve of a material fits the expression

$$K_R(\Delta a) = K_{ss} - (K_{ss} - K_0)\exp(-\Delta a / L_{ss}).$$

Regard K_0, K_{ss} and L_{ss} as material parameters. The material has crack like flaws of length $2a_0$. When the material is under stress σ, the cracks may grow stably to size $2a$ and. Assume that the stress intensity factor is given by $K = \sigma\sqrt{\pi a}$.
(a) Sketch this curve and explain the meanings of the parameters K_0, K_{ss} and L_{ss}.
(b) Describe a procedure to determine the maximum stress, σ^*, which the material can carry without breaking, and the stable crack extension, Δa^*, before the material breaks under the maximum stress.
(c) A material has $K_0 = 1\text{MPa}\sqrt{\text{m}}$, $K_{ss} = 10\text{MPa}\sqrt{\text{m}}$, and $L_{ss} = 10\mu\text{m}$. Plot the strength as a function of the flaw size in the range $a_0 = 5 - 50\mu\text{m}$. Make a similar plot for a material having $L_{ss} = 20\mu\text{m}$. Discuss the effect of increasing L_{ss}.
(d) Plot your results in a dimensionless form. For example, plot $\sigma^*\sqrt{L_{ss}} / K_0$ as a function of a_0 / L_{ss}, at several levels of K_{ss} / K_0. Discuss the implications of increasing the initial flaw size or K_{ss} / K_0.
(e) Plot $\Delta a^* / L_{ss}$ as a function of a_0 / L_{ss}, at several levels of K_{ss} / K_0. Discuss this plot.

19. Fatigue crack growth

An aluminum has a fracture toughness of K_c = 30 MPa√m. Under a cyclic load, a crack in the aluminum extends per cycle by

$$\frac{da}{dN} = \beta\left(\frac{\Delta K}{K_c}\right)^4$$

where β = 1 μm/cycle. A large plate of the aluminum is subject to a cyclic tensile stress varying between 0 and 200 MPa. A surface crack has initial depth a_0 = 500 μm.
a) How many cycles does it take for the crack to double its depth?
b) How many cycles does it take for the aluminum plate to undergo fast fracture?
c) The threshold stress intensify factor has been determined to be K_{th} = 3 MPa √m. Determine the critical stress range, below which the crack will not grow.

20. Fatigue of a pressure vessel

A cylindrical steel pressure vessel of 7.5 m diameter and 40 mm wall thickness is to operate at working pressure of 5.1 MPa. The fracture toughness for the steel is 200 $\mathrm{MPa}\sqrt{\mathrm{m}}$. The growth of the crack by fatigue may be represented approximately by the equation $da/dN = A(\Delta K)^4$, where $A = 2.44 \times 10^{-14} (\mathrm{MPa})^{-4} \mathrm{m}^{-1}$. The design assumes that failure will take place by fast fracture from a crack which has extended gradually along the length of the vessel by fatigue. To prevent fast fracture, the total number of loading cycles from zero to full load and back to zero again must not exceed 3000. Find the minimum pressure to which the vessel must be tested before use to guarantee against failure in under the 3000 load cycles.