Due in class, Thursday, 15 April 2010

33. Use the J integral to determine the order of singularity of the HRR field

In class we determined the order of singularity by using the Ilyushin Theorem. In the original HRR papers, however, the order of singularity was determined by invoking the J integral. Read the original papers and reproduce their method of determining the order of singularity.

J.W. Hutchinson, Singular behavior at the end of a tensile crack in a hardening material. Journal of the Mechanics and Physics of Solids **16**, 13-31 (1968). http://www.seas.harvard.edu/hutchinson/papers/312.pdf

J.R. Rice and G.F. Rosengren, Plane-strain deformation near a crack tip in a power-law hardening material. Journal of the Mechanics and Physics of Solids **16**, 1-12 (1968). http://esag.harvard.edu/rice/016_RiceRosengren_CrackSing_JMPS68.pdf

34. The stress ahead of the crack tip

When a metal is loaded by a uniaxial tensile stress far beyond the elastic limit, the stress-strain curve fits the power law:

$$\varepsilon = \left(\frac{\sigma}{B}\right)^n .$$

This experimental curve is generalized to predict the multiaxial stress-strain behavior by a procedure described in class.

According to HRR, the tensile stress σ_{22} a distance r ahead of the crack tip takes the form:

$$\sigma_{22} = B\left(\frac{G}{Br}\right)^{\frac{1}{n+1}} f(n),$$

where G is the energy release rate, and $f(n)$ is a dimensionless number dependent on n only.

(a) Determine $f(1)$ from the linear elastic solution.

(b) Find the function $f(n)$ in the literature. List your references.

35. The Rice-Johnson paper (1970)

R. Rice and M. A. Johnson, The Role of Large Crack Tip Geometry Changes in Plane Strain Fracture, in Inelastic Behavior of Solids (eds. M. F. Kanninen, et al.), McGraw-Hill, N.Y., 1970, pp. 641-672. http://esag.harvard.edu/rice/025_RiceJohnson_CrackTip_InBeSo70.pdf

Read this paper and carefully describe Fig. 10.

36. The RKR model (1973)

The HRR field is the basis for a model of toughness, known as the RKR model (R.O. Ritchie, J.F. Knott, and Rice, On the relationship between the critical tensile stress and fracture toughness in the mild steel. Journal of the Mechanics and Physics of Solids 21, 395-410, 1973). http://esag.harvard.edu/rice/045_RitchieKnottRi_cleavage_JMPS73.pdf

Read the RKR paper and describe how the HRR solution is used to estimate toughness. Describe the main conclusion of the paper.