Due in class, Tuesday, 4 May 2010

41 Four-point bend specimen

Show that the energy release rate for the four-point bend specimen is

$$G = 6\frac{M^2}{EH^3}\left[1-\frac{1}{(1+h/H)^3}\right].$$

where M is the applied moment per unit thickness, h and H are the thickness of the two layers. Assume that both layers have the same Young's modulus E.

The following two papers developed the technique.

P.G. Charalambides, J. Lund, A.G. Evans, and R.M. McMeeking. A test specimen for determining the fracture resistance of bimaterial interfaces. Journal of Applied Mechanics. 56, 77-82 (1989).

R.H. Dauskardt, M. Lane, Q. Ma, N. Krishna. Adhesion and debonding of multi-layer thin film structures. Engineering Fracture mechanics 61, 141-162 (1998).

42. Mode mixity of the four-point bend specimen

Charalambides et al (1989) used the finite element method to determine the mode angle. When the two layers have identical elastic constants and identical thickness, one can determine the mode angle analytically. Show that in this special case

$$\tan\psi = \frac{K_{II}}{K_I} = \frac{\sqrt{3}}{2}$$

That is, the specimen has slightly more mode I than mode II, or $\psi \approx 41°$.

Hint: Look at Figure 1 of Z. Suo, G. Bao, and B. Fan, Delamination R-curve phenomena due to damage. Journal of the Mechanics and Physics of Solids **40**, 1-16 (1992).
http://www.seas.harvard.edu/suo/papers/015.pdf

43. Mixed mode loading. Crack kinking

A straight crack is cut in a material with a diamond saw. The material is loaded with a combination of K_I and K_{II}, and the crack kinks—that is, the crack grows in a direction at an angle from the plane of the cut. Assume that the angle of the kink, θ_k, is determined by maximizing the hoop stress. Derive the relation between the initial kink angle θ_k and the mode angle, ψ, as defined by $\tan\psi = K_{II}/K_I$. Plot your result.

F. Erdogan and G.C. Sih, On the crack extension in plates under plane loading and transverse shear. Journal of Basic Engineering 85, 519-527 (1963).

44. Mode angle of an interface crack

An epoxy is bonded to a glass on a flat interface. Use representative elastic constants to calculate the parameter ε. A crack, length 2 mm, pre-exists on the interface. The size of the crack is small so you can regard the two materials as semi-infinite. A tensile stress σ is applied remotely from the crack. What is the mode angle at distance 1μm ahead of the crack tip?

The mode angle of an interfacial crack was carefully discussed in the following paper.

J.R. Rice. Elastic fracture mechanics concepts for interfacial cracks. *J. Appl. Mech.* 1988, **55**, 98-103. (http://esag.harvard.edu/rice/139_Ri_ElFracMechInterf_JAM88.pdf)