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Vibration analysis of delaminated composite beams using analytical and FEM models

Hasan Çallioğlu & Gökmen Atlihan*
Department of Mechanical Engineering, Pamukkale University, 20020, Denizli, Turkey

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In this study, the effects of delamination length and orientation angle on the natural frequency of symmetric composite beams are investigated, analytically and numerically. The analytical method is developed using the Timoshenko beam theory. The transverse shear effect and the rotary inertia terms are taken into account in the governing equations of vibration. Two dimensional finite element models of the delaminated beams are established using contact element at the ANSYS. The values of the natural frequency in laminated beams are obtained using the normal penalty stiffness that is chosen as the elasticity modulus for the contact element of the delamination region. When the analytical results are compared with numerical results and the results in literature, it is seen that the results are very close to each other. It is shown that natural frequencies decrease when delamination length in the beam increases and the natural frequencies change with the change of orientation angle.

Keywords: Natural frequency, Vibration, FEM, Analytical model, Delamination

Composite materials have generally been used in structural designs of aircraft, ship and automobile because of high strength and stiffness to weight ratios. However, composites are very sensitive to the anomalies caused during their fabrication or service life. Delamination may arise either because of imperfect fabrication processes such as incomplete wetting and entrapped air pockets between layers, or because of certain in-service factors, such as low velocity impact by foreign objects. Although delamination damage is known to cause a degradation of overall stiffness and strength of the laminates, how delaminations cause change in vibration characteristics has already been investigated. The delaminated beam generally exhibits new vibration modes and frequencies that depend on the delamination length.

There are numerical studies using finite element method (FEM) in the literatures. Komur[1] presented buckling analysis of a woven-glass-polyester laminated composite plate with a circular/elliptical hole. However, dynamic behaviors of the laminated composite plates and beams without delamination[2-4] and with single and multiple delaminations can be found in the literatures. Topcu[5] studied the effects of stacking sequences on natural frequencies of laminated composite beams by using both experimental and theoretical approaches. Atlihan[6,7] investigated the effects of stacking sequences on natural frequencies of laminated composite beams via the differential quadrature method (DQM). They demonstrated that the effective stiffness of the laminated composite beam can be altered through a change in the stacking sequence.

Alnefaie[8] developed a three-dimensional (3D) finite element model of delaminated fiber-reinforced composite plates for dynamics analysis. Those models were used for detecting delaminations in composite plates. Ramtekkar[9] proposed two models, the unconstrained-interface model and the contact-interface model, for the computation of frequencies and the mode shapes of delaminated beams. Palacz[10] presented a spectral finite element model for analysis of flexural-shear coupled wave propagation in delaminated multilayered composite beams. Kumar and Shrivastava[11] developed a finite element formulation based on higher order shear deformation theory and Hamilton's principle for the free vibration response of thick square composite plates having a central rectangular cutout, with and without the presence of a delamination around the cutout. Della and Shu[12,13] developed analytical solutions for the free vibrations of multiple delaminated beams under axial compressive loadings via the Euler-Bernoulli beam

*Corresponding author (E-mail: gatlihan@pau.edu.tr)

theory. Lee[14] proposed an analytical formulation derived from the assumption of constant curvature at the multi-delamination for vibration analysis of multi-delaminated composite beams. Lee[15] presented free vibration analysis of a laminated beam with delamination using a layerwise theory. He derived equations of motion from the Hamilton's principle, and developed a finite element method to formulate the problem.

In this study, bending vibration behaviors of symmetric composite beams having various delaminations and orientation angles are investigated analytically and numerically. The analytical method for delaminated beams is developed using the Timoshenko beam theory, which takes into account shear effect and rotary inertia terms. The commercially available FEM program, ANSYS, is used for a numerical solution, and single-edge delaminated models with contact element for the laminated composite beams have been formed at ANSYS. As the analytic results are compared with the numerical results, it is found to be consistent with each other.

Material and Methods

Production of composite materials

In this study, glass fabric with 410 tows/m (Metyx® 600 tex) was supplied from Telateks. The E-glass/epoxy composite laminates were fabricated from unidirectional E-glass fabrics (270 g/m^2) by the hand lay-up method at Izoreel Firm, Izmir-Turkey. An epoxy resin matrix based on CY225 epoxy prepolymer and HY225 hardener supplied from Huntsman was used in the production of the composite laminates. The mixing ratio for resin-to-hardener in weight was 10:2 and fiber volume fraction in all laminates was about 60%. The composite laminates were cured in a lamination press for 2 h, at 120°C, at a constant pressure of 0.3 MPa. The composite laminates were then cooled down to the room temperature maintaining the pressure. Composite specimens used in order to obtain the mechanical properties were cut from the fabricated composite plates.

Table 1—Material properties and dimensions of the laminated composite beam

Properties	Symbol	Present	Ref.[19-21]
Longitudinal elasticity modulus	E_1 (MPa)	44150	144800
Transverse elasticity modulus	E_2 (MPa)	12300	9650
Shear modulus	G_{12} (MPa)	4096	4140
Poisson's ratio	υ_{12}	0.2	0.3
Density	ρ(kg/m^3)	2026	1389.23
Length	L (mm)	400	381
Height	h (mm)	3.3	25.4
Width	b (mm)	20.5	12.7

An Instron® 8801 dynamic fatigue testing machine with Instron® 2663-821 advanced video extensometer was used for tensile test to determine mechanical properties of the composites such as elasticity modulus, rigidity modulus and Poisson's ratio. The mechanical properties obtained of the composites were given in Table 1.

Analytical method

Free-body diagram and geometry for a beam element are shown in Fig. 1 for the Timoshenko beam theory, which accounts for both the rotary inertia and the shear deformation of the beam. When the symmetric beam is vibrating transversely, the relation between the moment equilibrium condition and the inertial moment of the beam element yields by using the dynamic equilibrium condition for shearing forces in the w-direction[16]

$$-Vdx+\frac{\partial M}{\partial x}dx-\rho_m I_{yy}\frac{\partial^3 w}{\partial x\partial t^2}dx=0 \qquad \dots (1)$$

where V is shearing force, M is internal moment, ρ_m is density and I_{yy} is inertia moment.

From elementary flexural theory, the relationship between the bending moment and the curvature can be written as

$$M=\frac{E_{ef}I_{yy}}{\rho}=E_{ef}I_{yy}\frac{d^2 w}{dx^2} \qquad \dots (2)$$

where the effective elasticity modulus E_{ef} is[4]

$$E_{ef}=\frac{8}{h^3}\sum_{j=1}^{\frac{m}{2}}(E_x)_j(z_j^3-z_{j-1}^3) \qquad \dots (3)$$

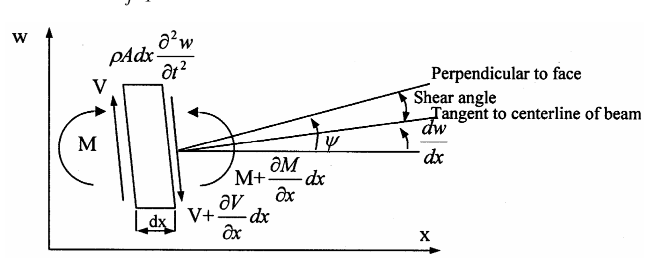

Fig. 1–Free body diagram (FBD) of a deformed beam element

where E_x is elasticity modulus of j^{th} layer, m is the number of layer of the beam, z_j is distance between the outer face of j^{th} layer and the neutral plane, h is high of the beam, as seen in Fig. 2.

The differential equation for the transverse vibration of the prismatic beams can be written as

$$E_{ef} I_{yy} \frac{\partial^4 w(x,t)}{\partial x^4} = \rho_m I_{yy} \frac{\partial^4 w}{\partial x^2 \partial t^2} - \frac{\rho_m A \partial^2 w(x,t)}{\partial t^2} \quad \ldots (4)$$

where A is cross-sectional area of the beam, the first term on the right side represents the effect of rotary inertia.

The slope of the deflection curve depends not only on the rotation of cross-section of the beam but also on the shearing deformations. If ψ denote the slope of the deflection curve when the shearing force is neglected and β the angle of shear at the neutral axis in the same cross-section, the total slope can be found as

$$\frac{dw}{dx} = \psi + \beta \quad \ldots (5)$$

The expressions for the bending moment and the shearing force are

$$M = E_{ef} I_{yy} \frac{d\psi}{dx} \quad \ldots (6)$$

$$V = -k^{'} \beta A G = -k^{'} \left(\frac{dw}{dx} - \psi \right) A G \quad \ldots (7)$$

in which $k^{'}$ is the form factor of the cross-section, $k^{'}$ is 6/5 for rectangular cross-sectional beam, G is the rigidity modulus.

Eliminating ψ, the differential equation for rotation and the translatory motion of an element can be given by

$$E_{ef} I_{yy} \frac{\partial^4 w}{\partial x^4} + \rho_m A \frac{\partial^2 w}{\partial t^2} - \rho_m I_{yy} \left(1 + \frac{E_{ef}}{k^{'} G} \right) \frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\rho_m^2 I_{yy}}{k^{'} G} \frac{\partial^4 w}{\partial t^4} = 0 \quad \ldots (8)$$

When it is considering a simply-supported beam, this equation and the boundary conditions are satisfied by taking

$$w_n = \left(\sin \frac{n \pi x}{L} \right) \left(A_n \cos p_n t + B_n \sin p_n t \right) \quad \ldots (9)$$

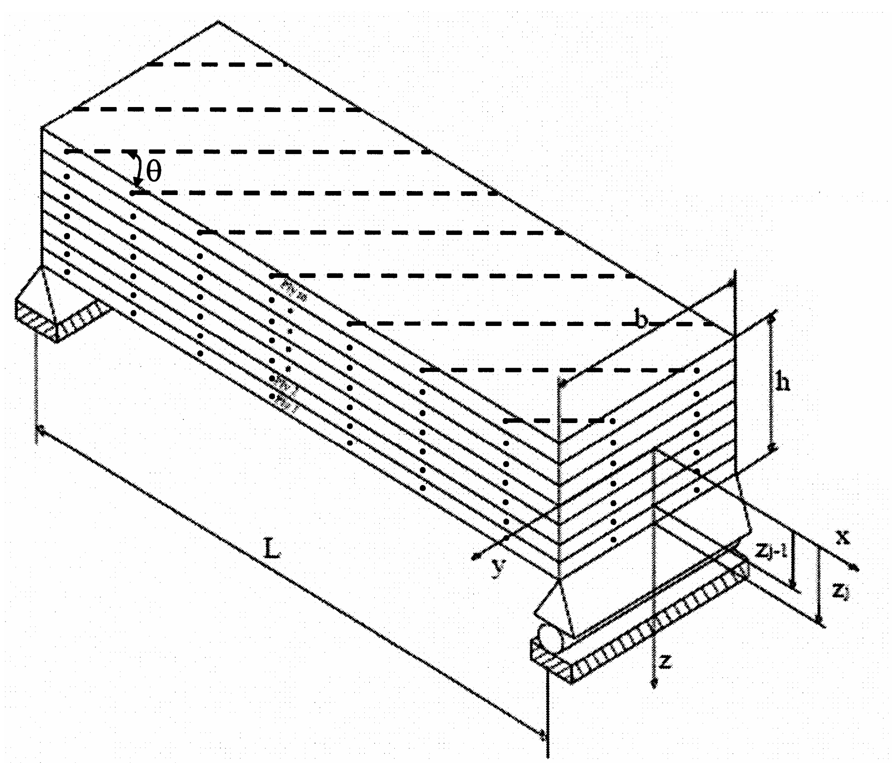

Fig. 2–Simply-supported laminated composite beam

where n is number of mode. A_n and B_n are the constants.

$$A_n = \frac{2}{l}\int_0^l f_1(x)\sin\frac{n\pi x}{l}dx$$
$$B_n = \frac{2}{l\,p_n}\int_0^l f_2(x)\sin\frac{n\pi x}{l}dx \quad \ldots (10)$$

where $y_0 = f_1(x)$ and $\dot{y}_0 = f_2(x)$ represent the initial transverse displacement of any point on the beam (at time $t=0$) and initial velocity, respectively. Substituting Eq. (9) into Eq. (8), the following equation for calculating the frequencies can be obtained

$$a^2\frac{n^4\pi^4}{L^4} - p_n^2 - p_n^2\frac{n^2\pi^2 r_g^2}{L^2} - p_n^2\frac{n^2\pi^2 r_g^2}{L^2}\frac{E_{ef}}{k^{'}G} + \frac{r_g^2\rho_m}{k^{'}G}p_n^4 = 0 \quad \ldots (11)$$

where $r_g = \sqrt{\frac{I_{yy}}{A}}$, $a = \sqrt{\frac{E_{ef}\,I_{yy}}{\rho_m A}}$

When the last term, which is a smaller quantity from the other terms that are the second-order, is neglected in order to simplify the solution and, to obtain the effects of rotary inertia and shearing deformations, the angular frequencies can be written as

$$p_n = \frac{a\pi^2}{\lambda_n^2}\left[1 - \frac{1}{2}\frac{\pi^2 r_g^2}{\lambda_n^2}\left(1 + \frac{E_{ef}}{k^{'}G}\right)\right] \quad \ldots (12)$$

Where $\lambda_n = \frac{L}{n}$, L is the length of the beam.

The natural frequencies are

$$\omega_n = \frac{p_n}{2\pi} \quad \ldots (13)$$

If there is a delamination in the laminated composite beam, the following equations can be used[4]:

$$E_{zd} = \frac{\sum_{j=1}^{s} E_{ef}\, z_j}{z} \quad \ldots (14)$$

$$E_d = \left(E_{zd} - E_{ef}\right)\frac{A_d}{A_t} + E_{ef} \quad \ldots (15)$$

where E_{zd} is longitudinal Young's modulus of a laminate totally delaminated along one or more interfaces (imperfect effective elasticity modulus), s number of sublaminates formed by the delamination, z_j thickness of the j^{th} sublaminate, E_d longitudinal Young's modulus of a laminate partially delaminated along one or more interfaces (imperfect effective elasticity modulus), A_d delaminated area and A_t total interfacial areas.

When the symmetric composite beams are totally or partially delaminated, instead of the effective elasticity modulus E_{ef} in equations, E_{zd} in Eq. (14) or E_d in Eq. (15) is used.

Finite element method (FEM)

Finite element method (FEM) known as a powerful tool for many engineering problems has been used to compute such as elastic-plastic, residual and thermal stresses, buckling and vibration analysis. Because of this, Ansys software that is a commercial FEM program was preferred for the vibration analysis of the laminated composite beams.

The Shell 99 element type was selected for 3-D modeling of solid structures in Ansys 10.0[17]. Initially, the beams are modeled in order to get a initial estimation of the undamped natural frequencies ω_n and mode shape n. Element type of Shell 99 may be used for layered applications of a structural shell model. The element has six degrees of freedom at each node; translations in the nodal x, y and z directions and rotations about the nodal x, y and z axes. This element is constituted by layers that are designated by numbers (LN–layer number), increasing from the bottom to the top of the laminate; the last number quantifies the existent total number of layers in the laminate (NL–total number of layers). The geometry, node locations and the coordinate system for this element are shown in Fig. 3.

The boundary conditions have been applied on the nodes, that is to say the dimensions in the x, y and z coordinates of these nodes are, respectively, 400 mm, zero and 20.5 mm for 2D, and the displacements and rotations of all nodes about the y-z plane are also taken as zero. The model of the laminated composite beam with sixteen layers is generated. This beam has delaminated ratios (*a*/*L*) varying from 0.1 to 0.9, where *a* is delamination length and *L* length of the beam. It can be seen from Fig. 4 that the laminated composite beams having edge or middle delamination were consisted of three or four areas.

It has firstly been glued area A1 with both areas A2 and A3, but interface of A2 and A3 is not glued in the beams with edge delamination[18]. In the same way, for the beams having the middle delamination it has been glued areas A1 and A4 with areas A2 and A3. Therefore, delamination has been formed between areas A2 and A3, as seen in Fig. 4. The double areas occur at the same coordinates of the interfacial areas while areas are been meshing. The last, the composite beams are formed a contact element between areas A2 and A3 so that the areas with the delamination can be moved together. The contact element is selected as Contact 174 element type, as seen in Fig. 5. Thus, the laminated composite beams have been had both edge and middle delamination. In the same manner, the composite beams with both edge and middle multiple delamination can also be consisted.

After the mesh generation process, a delaminated composite beam with delamination ratio *a*/*L*=0.2 has 1000 elements and 2976 nodes. By increasing delamination ratios, numbers of elements and nodes of the beams increase. Normal penalty stiffnesses of the contact element are chosen between 10^4 and 10^9.

Results and Discussion

In this study, vibration behaviors of laminated composite beams having simply supported, single-edge delamination and various stacking sequences are investigated analytically and numerically. In analytical solution, Timoshenko beam theory is valid and in the calculation of the natural frequency, the effective elasticity modulus E_{ef} for a non-delaminated beam or the imperfect effective elasticity modulus E_d or E_{zd} for a partially or totally delaminated beam is used instead of elasticity modulus E in a beam manufacturing isotropic material. The laminated composite beams are modeled as single edge delamination by using Ansys that is a commercial software program used in the numerical solution (FEM). Material properties and dimensions of the laminated composite beam are given in Table 1. The orientation angles of the beams, as can be seen in Fig. 2, are chosen as $[\theta°]_{16}$ where θ varies from 0° to 90°.

Figure 6 shows the effect of the contact element on the natural frequency. It can be seen from Figs 6 (a)

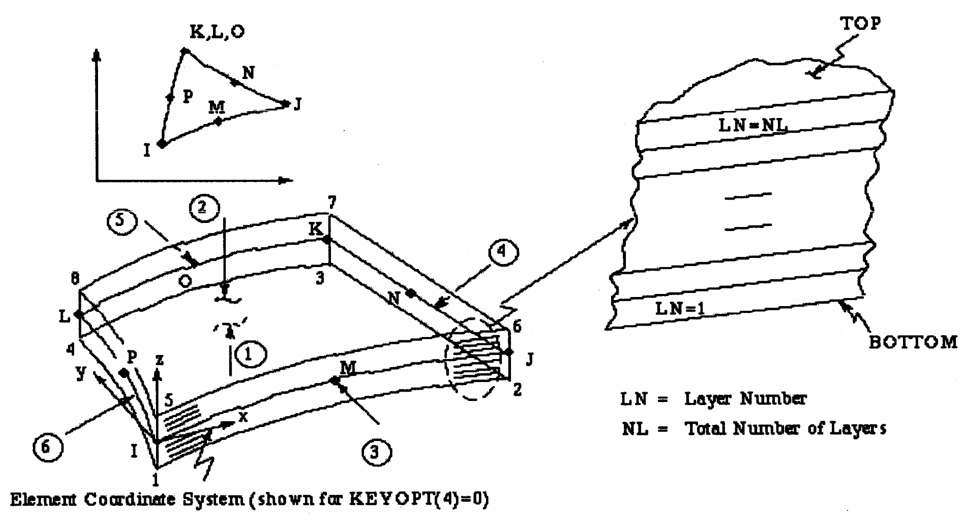

Fig. 3–Element coordinate system

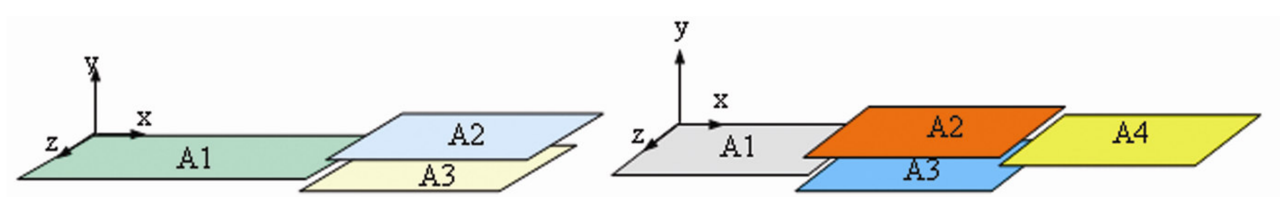

Fig. 4–Laminated composite beams with (a) edge and (b) middle delaminations

and (b) the variation of the natural frequencies for the composite delaminated beams without and with contact elements. As seen from the figures, while the delaminated laminates of the beams without contact element separate from one another, the delaminated laminates of the beams with contact element move with each other. For example, the frequency values are obtained as 28.202 Hz and 48.148 Hz in the delaminated beams with θ=45°, a=320 mm and n=2 for non-contact and contact element, respectively. When the frequency values are compared with the ones obtained from analytical solution, the value for the beam with contact element is very close to the ones with analytical solution, 47.758 Hz. Thus, as the delaminated composite beams are modeled, it must surely be utilized contact elements. It is also very important choice of the normal penalty stiffness when the contact element is used.

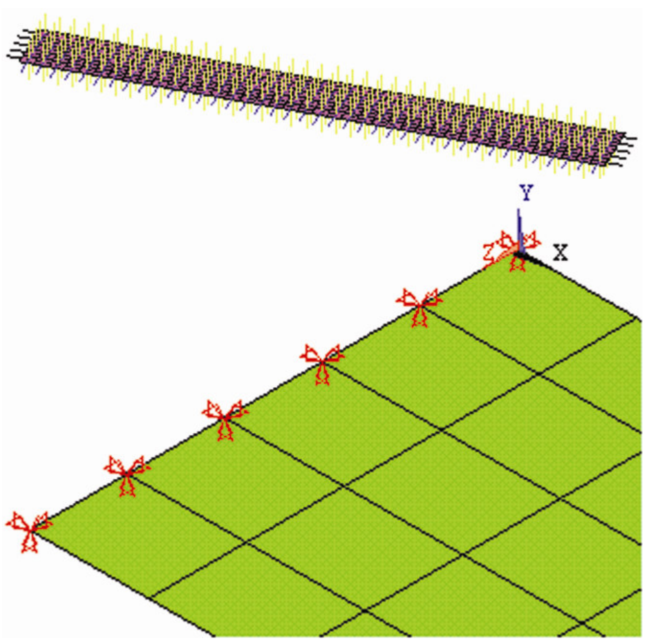

Fig. 5–View of a laminated composite beam with (a) contact element and (b) boundary conditions and mesh shape

The effects of the normal penalty stiffnesses on the natural frequency of a laminated composite for number of mode n=1 are given in Table 2. The beam is of orientation angle 45°, delamination length a=40 mm. Normal penalty stiffnesses of the contact element are chosen between 10^4 and 10^9. As seen in Table 2, the most compatible normal penalty stiffness value can approximately be chosen as effective elasticity modulus of the beam in order to find the more exact natural frequency and reduce the solution time. When the stiffnesses are chosen as lower or larger values than the effective elasticity modulus, the natural frequencies are obtained as wrong values for large values of n (as n= 2, 3), and also if the stiffness values are chosen a larger value the solution time increases.

Table 3 gives comparison of the natural frequency values obtained from present analytical method and in Ref.[19-21] for a $[0°]_4$ simply supported composite beam without delamination. The frequency values are quite close to each other.

Variations of the natural frequency values versus changing orientation angles in the laminated composite beams with various delamination lengths are shown in Fig. 7 for number of modes n= 1-3. It can be seen from the figure that the analytical and

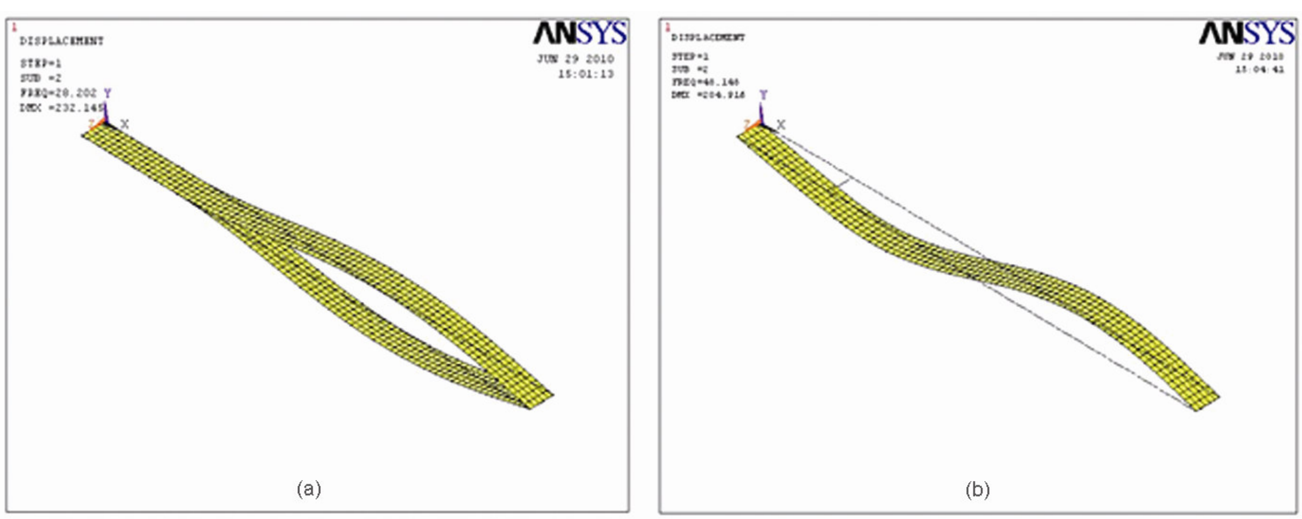

Fig. 6–Mode shape of a laminated composite beam with (a) noncontact element and (b) contact element for the mode number n=2

Table 2—Natural frequency values for different normal penalty stiffness for θ=45°, a=40 mm and n=1

Normal penalty stiffness (FKN)	Natural frequency (Hz)
10^4	23.036
10^5	23.036
10^6	23.037
10^7	23.047
10^8	23.104
10^9	24.140

Table 3—Comparison of natural frequencies of a $[0°]_4$ composite beam without delamination

Mode (n)	ω (kHz)			
	Ref.[19]	Ref.[20]	Ref.[21]	Present
1	0.753	0.755	0.756	0.756

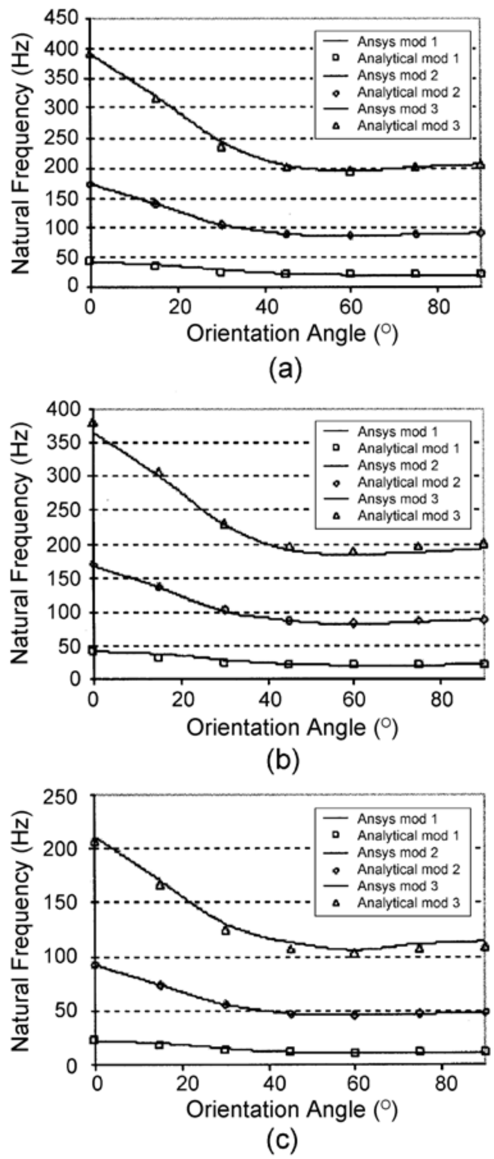

Fig. 7–Variations of the natural frequencies with orientation angle for beams with (a) a=0 mm, (b) a=40 mm and (c) a=320 mm

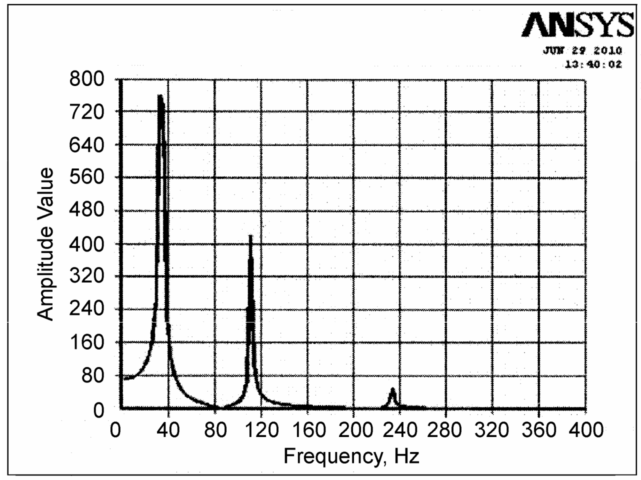

Fig. 8–The natural frequency values in the delaminated beam having θ=0° and a=320 mm for n=1-3

numerical frequency values are very close to each other. The values in non-delaminated and delaminated beams decrease more and more by increasing orientation angle from 0° until about 52° whereas after θ=52° the values increase very slowly, as seen in Fig. 7. As expected, the natural frequencies decrease with increase in delamination lengths. The largest frequency value is obtained for both non-delaminated and delaminated beams with orientation angle 0°. For example, the frequency ω=43.225 Hz. can be seen from Fig. 7b for θ=0°, n=1 and a=40 mm.

Figure 8 shows the natural frequencies obtained from the numerical analysis (ANSYS) for the beam with orientation angle θ=0°, delamination length a=320 mm and number of modes n=1-3. It is seen from this figures that when number of modes increase, amplitudes decrease whereas natural frequencies increase. It is obtained that the natural frequency values for n=1-3 are, respectively, 23.239 Hz., 109.625 Hz. and 232.873 Hz.

Conclusions

In this study, the natural frequencies of the simply supported composite beams with single-edge delamination are investigated analytically and numerically. In the analytical solution, Timoshenko beam theory is used. In numerical solution, ANSYS finite element code is used and when the beams are modeled, the contact elements and the values of the normal penalty stiffness are defined. The following conclusion can be drawn:

(i) It is obtained that the natural frequencies decrease when length of delaminations on the beam increase.

(ii) It is also obtained that the natural frequencies change with the change of orientation angle.

(iii) In the finite element solution, the contact elements must surely be used in delaminated region in order to obtain natural frequency in the delaminated beams.

(iv) The normal penalty stiffness of the contact element can be chosen as the effective elasticity modulus of the laminated composite beams.

(v) It is found that the frequency values obtained from the numerical result with contact element and normal penalty stiffness, which is equal to the effective elasticity modulus, are approximately the same with the ones in the analytical solution.

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References

1 Komur M A, Sen F, Ataş A & Arslan N, *Adv Eng Software*, 41 (2009) 161-164.
2 Jones RM, *Mechanics of composite materials*, (Philadelphia, USA), 1999, 187-275.
3 Vinson J R & Sierakowski R L, *The Behavior of structures composed of composite materials*, (New Jersey, USA), 2002, 155-200.
4 Gibson RF, *Principles of composite material mechanics*, (Philadelphia,USA) 1994, 190-267.
5 Topcu M, Atlihan G, Callioglu H & Conkur E S, *Adv Compos Lett,* 17 (2008) 7-13.
6 Atlihan G, Callioglu H, Conkur E S, Topcu M & Ugur Y, *J Reinf Plast Compos*, 28 (2009) 881-892.
7 Atlihan, G, Demir E, Girgin Z & Çallıoğlu H, *Adv Compos Lett,* 18 (2009) 37-44.
8 Alnefaie K, *Compos Struct,* 90 (2009) 21-27.
9 Ramtekkar G S, *J Sound Vib*, 328 (2009) 428-440.
10 Palacz M, Krawczuk M & Ostachowicz W, *Compos Struct,* 68 (2005) 45-51.
11 Kumar A & Shrivastava RP, *Compos Struct,* 70 (2005) 317-333.
12 Della C N & Shu D, *J Reinf Plast Compos,* 28 (2009) 1365-1380.
13 Della C N & Shu D, *Compos Pt B:Eng,* 38 (2007) 109-118.
14 Lee S, Park T & Voyiadjis G Z, *Compos Pt B:Eng*, 34 (2003) 647-659.
15 Lee J, *Compos Struct,* 74 (2000) 121-129.
16 Timoshenko S, *Vibration problems in engineering*, (New York, USA), 1990 432-435.
17 *Ansys 10.0 User's Manual*, (2005).
18 Atlihan G, *Vibration analysis of the delaminated composite beams,* Ph. D. Thesis, Pamukkale University, Denizli, Turkey, 2010, 50-95.
19 Yıldırım V & Kıral E, *Compos Struct*, 49 (2000) 313-320.
20 Eisenberger M, Abramovich H & Shulepov O, *Compos Struct*, 31 (1995) 265-271.
21 Chandrashekhara K & Bangera K M, *Comput Struct*, 43 (1992) 719-727.