Interfacial Fracture

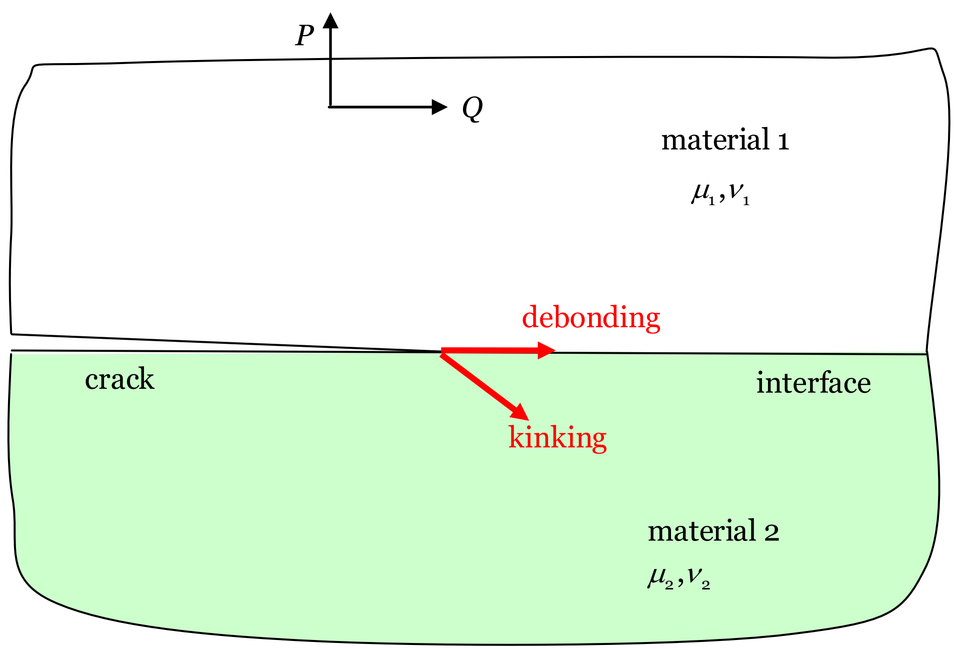
A body consists of two materials bonded at an interface. On the interface there is a crack. The body is subject to a load, causing the two faces of the crack to open and slide relative to each other. When the load reaches a critical level, the crack either extends along the interface, or kinks out of the interface.

A crack on the interface between two materials is similar to a mixed-mode crack in a homogeneous material. There is a significant difference, however. When a mixed-mode crack in a homogeneous material reaches a critical condition, the crack kinks out of its plane. By contrast, when a crack on an interface between two materials reaches a critical condition, the crack can extend along the interface, provided the interface is sufficiently weak compared to either material.

For the crack extending on the interface, the energy release rate is still defined as the reduction in the potential energy of the body associated with the crack advancing by unit area. The energy release rate characterizes the amplitude of the load. The critical condition for the extension of the crack also depends on the mode of the load. To characterize the mode of the load, we need the field around the tip of the crack.

Williams (1959) discovered that the singular field around the tip of a crack on an interface is not square-root singular, but takes a new form. The Williams field makes puzzling predictions: when the tip of the crack is approached, the stresses oscillate, and the faces of the crack interpenetrate. Interpenetration is clearly a wrong prediction. Is the Williams field useful?

In late 1980s, applications involving thin films and composites motivated Evans, Hutchinson, Rice and others to develop interfacial fracture mechanics. As pointed out by Rice (1988), so long as the Williams field is wrong only in a small zone around the tip of the crack, an annulus exits, within which the Williams field correctly predicts the field. The situation is analogous to using the square-root singular field under the small-scale yielding condition. Today the interfacial fracture mechanics on the basis of the Williams field is practiced routinely in industries.



An interfacial crack. A body consists of two dissimilar materials, labeled 1 and 2, bonded by a planar interface. Each material is homogeneous, isotropic, and linearly elastic. Let μ_1 be the shear modulus and ν_1 be Poisson's ratio of material 1. Let μ_2 be the shear modulus and ν_2 be Poisson's ratio for material 2. On the interface there exists a crack. Subject to a load, the body deforms under the plane-strain conditions. Denote the plane-strain moduli of the two materials by

$$\bar{E}_1 = \frac{E_1}{1-\nu_1^2} = \frac{2\mu_1}{1-\nu_1}$$

$$\bar{E}_2 = \frac{E_2}{1-\nu_2^2} = \frac{2\mu_2}{1-\nu_2}$$

When the load reaches to a critical condition, the crack either extends along the interface, or kinks out of the interface. The central object of the interfacial fracture mechanics is to formulate this critical condition.

Energy release rate. For the crack extending on the interface, the energy release rate G is still defined as the reduction in the potential energy associated with the crack advancing a unit area. All the familiar methods of determining the energy release rate still apply. For example, the energy release rate can be determined by measuring the load-displacement curves for bodes containing cracks of different sizes. As another example, the energy release rate can be calculated by using the J integral.

The energy release rate characterizes the amplitude of the load. It is evident that the critical condition for the extension of the crack also depends on the mode of the load. To characterize the mode of the load, we need to analyze the field near the tip of the interfacial crack.

The Williams (1959) field. By solving an eigenvalue problem, Williams (1959) discovered that the singular field around the tip of the interfacial crack is not square-root singular, but takes a new form. At a distance r ahead the tip of the crack, the stresses on the interface are given by

$$\sigma_{22} + i\sigma_{12} = \frac{Kr^{i\varepsilon}}{\sqrt{2\pi r}}.$$

At a distance r behind the tip of the crack, the two faces of the crack move relative to each other by the displacements

$$\delta_2 + i\delta_1 = \left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)\frac{Kr^{i\varepsilon}}{2(1+2i\varepsilon)\cosh(\pi\varepsilon)}\sqrt{\frac{2r}{\pi}}.$$

The notation of complex numbers is used. Recall $i = \sqrt{-1}$ and a mathematical identity:

$$r^{i\varepsilon} = \exp(i\varepsilon \log r) = \cos(\varepsilon \log r) + i\sin(\varepsilon \log r).$$

The constant ε is dimensionless and depends on the elastic constants of both materials:

$$\varepsilon = \frac{1}{2\pi}\log\left[\frac{(3-4\nu_1) + \dfrac{\mu_1}{\mu_2}}{(3-4\nu_2)\dfrac{\mu_1}{\mu_2} + 1}\right].$$

When the two materials have identical elastic constants, $\mu_1 = \mu_2$ and $\nu_1 = \nu_2$, the constant ε vanishes, and the singular field around the tip of the crack on the interface is similar to that around the tip of a crack in a homogeneous material.

The stress intensity factor K is complex-valued. An inspection of the above stress field shows that K is of dimension

$$K = [\text{stress}][\text{length}]^{\frac{1}{2}-i\varepsilon}.$$

One complex number corresponds to two real numbers. Thus, the crack on the interface is analogous to a crack in a homogeneous material under mixed-mode loading.

Stress intensity factor and energy release rate. The amplitude of the stress intensity factor is defined by

$$|K| = \sqrt{K\bar{K}}$$

This real-valued quantity has the familiar dimension:

$$|K| = [\text{stress}][\text{length}]^{1/2}.$$

Indeed, $|K|$ relates to the energy release rate G as (Malyshev and Salganik, 1965)

$$G = \frac{1}{2}\left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)\frac{|K|^2}{\cosh^2(\pi\varepsilon)}.$$

This relation can be proved by using the method due to Irwin (1957).

Critical conditions. Assume that an annulus exists, whose inside radius is large compared to small-scale events not modeled by the singular field, and whose outside radius is small compared to the length characteristic of the external boundary conditions. Inside the annulus, the Williams field prevails, characterized by the complex-valued stress intensity factor K. Consequently, K consists of the two messengers between the external boundary conditions and the crack-tip processes.

When the loads increase to some critical conditions, the crack pre-existing on the interface either extends along the interface, or kinks out of the interface. The critical conditions for debonding and kinking can be formulated in terms of the complex-valued stress intensity factor.

This idea was recognized almost immediately after Williams published his paper, but broad adoption of the idea in practice would take place after many years. The long delay was clearly not due to any lack of practical motivation: debonding of an interface and kinking out of an interface had been commonly observed. The delay was due to the peculiar form of the Williams singularity, in particular, due to this pesky constant ε.

The range of ε. The constant ε is monotonic in μ_1 / μ_2. When material 1 is much more compliant than material 1, in the limit $\mu_1 / \mu_2 \to 0$, we obtain that

$$\varepsilon \to \frac{1}{2\pi}\log(3 - 4\nu_1).$$

When material 1 is much stiffer than material 2, in the limit $\mu_1 / \mu_2 \to \infty$, we obtain that

$$\varepsilon \to -\frac{1}{2\pi}\log(3 - 4\nu_2).$$

If Poisson's ratios of the two materials are restricted in the interval $(0,1/2)$, the constant ε is bounded in the interval

$$|\varepsilon| < \frac{1}{2\pi}\log 3 \approx 0.175.$$

If Poisson's ratios of the two materials are restricted in the interval $(-1,1/2)$, the constant ε is bounded in the interval

$$|\varepsilon| < \frac{1}{2\pi}\log 7 \approx 0.310.$$

Phase angle of the stress intensity factor. Any complex number is associated with two real numbers. For example, we can write a complex number in terms of its amplitude and phase angle, namely,

$$K = |K|\exp(i\phi).$$

The stress-intensity factor K is complex-valued, but the amplitude $|K|$ and the phase angle ϕ are real-valued. As we have seen, the amplitude $|K|$ is related to the energy release rate G, and specifies the magnitude of the load. The phase angle ϕ specifies the mode of the load.

Oscillatory stresses. The Williams field predicts that the stresses a distance r ahead of the tip of the crack are given by

$$\sigma_{22} + i\sigma_{12} = \frac{Kr^{i\varepsilon}}{\sqrt{2\pi r}}.$$

Rewrite this equation by using $K = |K|\exp(i\phi)$, we obtain that

$$\sigma_{22} + i\sigma_{12} = \frac{|K|}{\sqrt{2\pi r}}\exp[i\phi + i\varepsilon\log r].$$

Separating the real and the imaginary parts, we obtain that

$$\sigma_{22} = \frac{|K|}{\sqrt{2\pi r}}\cos[\phi + \varepsilon\log r],$$

$$\sigma_{12} = \frac{|K|}{\sqrt{2\pi r}}\sin[\phi + \varepsilon\log r].$$

Thus, the Williams field predicts that the stresses are oscillatory as r approaches the tip of the crack.

The ratio of the shear stress to the tensile stress is

$$\frac{\sigma_{12}}{\sigma_{22}} = \tan[\phi + \varepsilon\log r].$$

When $\varepsilon = 0$, as for a crack in a homogenous material, the ratio σ_{12}/σ_{22} is independent of the distance r in the K-annulus, and the mode angle ϕ characterizes the relative proportion of shear to tension. When $\varepsilon \neq 0$, the ratio σ_{12}/σ_{22} varies with the distance r. In this case, what does ϕ signify?

Interpenetrating faces of a crack. The Williams field predicts that, at a distance r behind the tip of the crack, the two faces of the crack move relative to each other by the displacement

$$\delta_2 + i\delta_1 = \left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)\frac{Kr^{i\varepsilon}}{2(1+2i\varepsilon)\cosh(\pi\varepsilon)}\sqrt{\frac{2r}{\pi}} .$$

Rewrite this equation by using $K = |K|\exp(i\phi)$, we obtain that

$$\delta_2 + i\delta_1 = \left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)\frac{|K|}{2\sqrt{1+4\varepsilon^2}\cosh(\pi\varepsilon)}\sqrt{\frac{2r}{\pi}}\exp\left[i\phi + i\varepsilon\log r - i\tan^{-1}(2\varepsilon)\right].$$

The component of the displacement normal to the plane of the interface is

$$\delta_2 = \left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)\frac{|K|}{2\sqrt{1+4\varepsilon^2}\cosh(\pi\varepsilon)}\sqrt{\frac{2r}{\pi}}\cos\left[\phi + \varepsilon\log r - \tan^{-1}(2\varepsilon)\right].$$

When $\varepsilon = 0$, this expression is similar to that for a crack in a homogenous material. When $\varepsilon \neq 0$, this expression indicates that, for some values of *r*, the two faces of the crack interpenetrate.

Historically, this prediction disturbed many researchers, and perhaps was the single most significant objection against the Williams field. Recall that in posing the Williams problem, we assume that the two faces of the crack are traction-free. But the solution of this problem—the Williams field—tells us that the two faces of the crack interpenetrate.

To remove this contradiction, researchers allowed the faces of the crack to contact each other, and solved the resulting boundary-value problems.

On the virtues of taking $\varepsilon = 0$. All these troubles disappear when we set $\varepsilon = 0$. The stress field becomes square-root singular, namely,

$$\sigma_{22} + i\sigma_{12} = \frac{K}{\sqrt{2\pi r}} .$$

Separate the complex-valued *K* into the real and imaginary parts:

$$K = K_I + iK_{II} .$$

We obtain that

$$\sigma_{22} = \frac{K_I}{\sqrt{2\pi r}} ,$$

and

$$\sigma_{12} = \frac{K_{II}}{\sqrt{2\pi r}} .$$

The two parameters K_I and K_{II} measure the amplitudes of two fields. Consequently, we can treat a crack on an interface the same way as we treat a crack in a homogeneous material under mixed-mode loading.

Even when we set $\varepsilon = 0$, there is a significant difference between an crack on an interface and a crack in a homogeneous material. When a mixed-mode crack in a homogeneous material reaches a critical condition, the crack kinks out of its plane. By contrast, when a mixed-mode crack on an interface reaches a critical condition, the crack often extends along the interface, provided the interface is sufficiently weak compared to either material.

In reading the literature on interfacial fracture mechanics, you can simply set $\varepsilon = 0$. You will most likely miss nothing of substance. We will elaborate on what we mean by the phrase "most likely" shortly.

Hutchinson pointed out this simplification to me (Suo and Hutchinson, 1989), and I wrote up my PhD thesis without reading much of the literature on interfacial fracture mechanics. I met Fazil Erdogan, in January 1989, at a conference at UCSB. He said to me, "Young man, you should read old papers. Hutchinson may not need to read them, but you do."

Toward the end of the same conference, Erdogan gave a brief summary. He said, "All the old problems with interfacial fracture mechanics have recently disappeared, because ε is negligible by definition." In the audience were Ashby, Bassani, Evans, Hirth, Hutchinson, Lange, McMeeking, Rice, Ruhler…

Mode angle. We now return to the case $\varepsilon \neq 0$. The following few paragraphs draw upon Rice (1988). Recall the dimension of the complex-valued stress intensity factor,

$$K = [\text{stress}][\text{length}]^{\frac{1}{2} - i\varepsilon},$$

and the dimension of the magnitude of the stress intensity factor,

$$|K| = [\text{stress}][\text{length}]^{\frac{1}{2}}.$$

Write

$$K = |K| l^{-i\varepsilon} \exp(i\psi),$$

where *l* is an arbitrary length. We will call ψ the mode angle. Before we attach any intuitive significance to the length *l* and the phrase "mode angle", let us first take a look at what the definition does.

The choice of the length *l* is arbitrary. Let us examine the consequence of difference choices of the length, say l_A and l_B. The complex-valued *K* represents the external boundary conditions, the choice of an arbitrary length does not affect *K* and $|K|$. Consequently, we need to associate one mode angle ψ_A to one choice of the length l_A, and associate a different mode angle ψ_B with the other choice of the length l_B. To keep *K* and $|K|$ unchanged by the different choices of the length, we require that

$$l_A^{-i\varepsilon} \exp(i\psi_A) = l_B^{-i\varepsilon} \exp(i\psi_B).$$

Rearranging, we obtain that

$$\psi_B - \psi_A = \varepsilon \log\left(\frac{l_B}{l_A}\right).$$

This formula shows how the mode angle ψ changes with the length *l*. As a numerical example, let $\varepsilon = 0.1$, $l_A = 1\text{nm}$ and $l_A = 1\mu\text{m}$. This change in the length shift the mode angle by $\psi_B - \psi_A \approx 40^\circ$.

The phase angle ϕ was defined before by using the equation $K = |K|\exp(i\phi)$. The phase angle ϕ corresponds to the mode angle ψ associated with a special choice of the length $l = 1$. The unit of the length is left unclear. It is better to clearly specify a length *l*, and use ψ.

To see the significance of the mode angle, recall that the stresses a distance *r* ahead of the tip of the crack are given by

$$\sigma_{22} + i\sigma_{12} = \frac{Kr^{i\varepsilon}}{\sqrt{2\pi r}}.$$

Inserting $K = |K| l^{-i\varepsilon} \exp(i\psi)$, we obtain that

$$\sigma_{22} + i\sigma_{12} = \frac{|K|}{\sqrt{2\pi r}} \exp\left[i\psi + i\varepsilon \log\left(\frac{r}{l}\right)\right].$$

Separating the real and the imaginary parts, we obtain that

$$\sigma_{22} = \frac{|K|}{\sqrt{2\pi r}} \cos\left[\psi + \varepsilon \log\left(\frac{r}{l}\right)\right],$$

$$\sigma_{12} = \frac{|K|}{\sqrt{2\pi r}} \sin\left[\psi + \varepsilon \log\left(\frac{r}{l}\right)\right].$$

Thus, the Williams solution predicts that the stresses are oscillatory as r changes.

The ratio of the shear stress to the tensile stress is

$$\frac{\sigma_{12}}{\sigma_{22}} = \tan\left[\psi + \varepsilon \log\left(\frac{r}{l}\right)\right].$$

When $\varepsilon \neq 0$, the ratio σ_{12}/σ_{22} varies with the distance r. The variation is not rapid, because ε is small and because logarithm is a slowly varying function. Note that $\sigma_{12}/\sigma_{22} = \tan\psi$ when $r = l$. Thus, $\tan\psi$ approximates the ratio σ_{12}/σ_{22}, so long as r is not far from l.

For a brittle interface, for example, a natural choice is l = 1 nm, representative of the bond breaking zone size. With this choice, the mode angle ψ represents the relative proportion of shear to tension at the size scale of 1 nm.

Small-scale contact. As discussed before, the Williams field implies that for certain values of r the faces of the crack interpenetrate. This pathological behavior had been cited as a reason to oppose the use of the Williams field. Interpenetration clearly is a wrong prediction. But we have been so good at using solutions with wrong predictions. For example, linear elastic fracture mechanics has been based on the square-root singular field, which gives clearly wrong prediction for the magnitude of the stress at the tip of the crack. The trick has been to avoid using the solution where it is wrong. Indeed, the square-root singular field correctly predicts the stress in the *K*-annulus. To use the Williams field for an interfacial crack, all we really need to do is to ensure that the wrong part of the field occurs in a small zone around the tip of the crack, a zone excluded by the *K*-annulus .

In a homogeneous material, the faces of a crack come into contact when the remote load is compressive. By contrast, the faces of an interface crack may come into contact even the remote load has a tensile component. When the contact zone is large, one has to take into account of the forces on the faces of the crack in solving the boundary-value problem. In many situations, however, the contact zone is small compared to the overall dimension. Consequently, the K-annulus exists, with the inner radius enclosing the contact zone, as well as the bond-breaking process zone.

Rice (1988) has examined the condition for small-scale contact. The Williams field predicts that, at a distance r behind the tip of the crack, the two faces of the crack move relative to each other by the displacement

$$\delta_2 + i\delta_1 = \left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right) \frac{K r^{i\varepsilon}}{2(1+2i\varepsilon)\cosh(\pi\varepsilon)} \sqrt{\frac{2r}{\pi}}.$$

The component of the displacement normal to the interface is

$$\delta_2 = \left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)\frac{|K|}{2\sqrt{1+4\varepsilon^2}\cosh(\pi\varepsilon)}\sqrt{\frac{2r}{\pi}}\cos\left[\psi + \varepsilon\log\left(\frac{r}{l}\right) - \tan^{-1}(2\varepsilon)\right],$$

If the crack is required to be open within

$$l < r < 100l,$$

the mode angle must be restricted within

$$-\pi/2 + 2\varepsilon < \psi < \pi/2 + 2.6\varepsilon, \text{for} \varepsilon > 0$$

$$-\pi/2 - 2.6\varepsilon < \psi < \pi/2 + 2\varepsilon, \text{for} \varepsilon < 0$$

The number 100 is arbitrary, but the above conditions are insensitive to this number. When $\varepsilon = 0$, the above condition simply says that the contact does not occur when the crack is under tension, a known concluding for a crack in a homogeneous material.

Determine the complex-valued stress intensity factor. For a given configuration of an interfacial crack, the complex-valued stress intensity factor *K* is determined by solving the boundary-value problem within the theory of linear elasticity. Many other boundary-value problems have been solved; see a review by Hutchinson and Suo (1992). Finite element method and other numerical methods have been developed to determine the complex-valued stress intensity factor. Several examples are discussed below.

A small crack on an interface. A crack, length $2a$, pre-exists on the interface between two semi-infinite materials, under the plane-strain conditions. Remote from the crack, each material is in a state of homogeneous deformation. The remote tensile stress in both materials is σ, and the remote shear stress in both material is τ. The two homogeneous states must match transverse strains at the interface. Because the two materials have dissimilar elastic constants, we need to apply dissimilar remote transverse stresses to the two materials, $(\sigma_x)_1$ $(\sigma_z)_1$, $(\sigma_x)_2$, and $(\sigma_z)_2$. We regard σ as fixed. Under the plane strain condition, the out-of-plane stresses relate to the in-plane stresses as

$$(\sigma_z)_1 = \nu_1(\sigma + (\sigma_x)_1),$$

$$(\sigma_z)_2 = \nu_2(\sigma + (\sigma_x)_2).$$

To match ε_x in the two materials, we write

$$\frac{(\sigma_x)_1}{E_1} - \nu_1\frac{(\sigma_z)_1}{E_1} - \nu_1\frac{\sigma}{E_1} = \frac{(\sigma_x)_2}{E_2} - \nu_2\frac{(\sigma_z)_2}{E_2} - \nu_2\frac{\sigma}{E_2}.$$

The above three equations place constrains on the four transverse stresses $(\sigma_x)_1$ $(\sigma_z)_1$, $(\sigma_x)_2$, and $(\sigma_z)_2$. Consequently, when one of them is prescribed, the other three can be determined.

The transverse stresses, however, does not affect the stress intensity factor. Near the crack, both materials are in inhomogeneous state. This boundary-value problem has been solved, giving (Rice and Sih, 1965)

$$K = (1 + 2i\varepsilon)\sqrt{\pi a}(2a)^{-i\varepsilon}(\sigma + i\tau).$$

Write

$$\sigma + i\tau = Te^{i\omega},$$

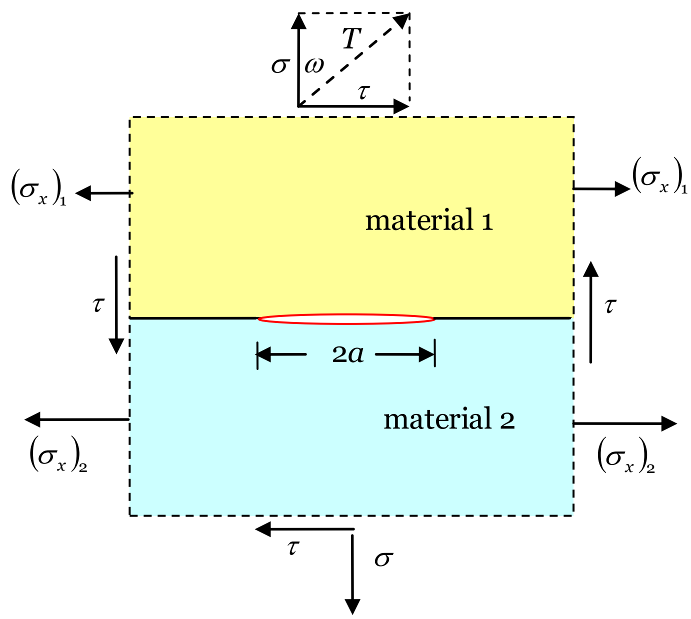
where *T* is the magnitude, and ω the direction, of the remote load. The stress intensity factor becomes

$$K = (1+2i\varepsilon)\sqrt{\pi a}(2a)^{-i\varepsilon} T e^{i\omega}.$$

Comparing this equation with $K = |K| l^{-i\varepsilon} \exp(i\psi)$, we obtain the mode angle:

$$\psi = \omega + \tan^{-1}(2\varepsilon) + \varepsilon \log\left(\frac{l}{2a}\right).$$

When $\varepsilon = 0$, the mode angle ψ equals the angle ω of the remote load. The two angles are different when $\varepsilon \neq 0$. Consider representative values, $\varepsilon = -0.05$, l = 1 nm, and 2a = 1 mm. Under remote tension, $\omega = 0$, one finds that $\psi = 34°$, indicating a significant shear component near the tip of the crack, at the size scale of l = 1 nm.



It may be tempting to choose the length of the crack, a, as the length l to define the mode angle. This choice goes against the spirit of fracture mechanics. We would like to use the mode angle to characterize the field near the tip of the crack. The same mode angle should mean the same local field. However, the length of the crack is a parameter that characterizes the geometry of the specimen.

It may also be tempting to choose the loading angle ω as the mode angle. This choice is bad for the same reason: ω is specific to the geometry of a particular specimen. The mode angle is supposed to characterize the local field. The above calculation shows that, even when the loading angle ω is fixed, a change in the length of crack a can cause the mode angle to change.

A thin film debonding from a substrate. A film, thickness h, is under a tensile in-plane stress σ, and is initially bonded to the substrate. The film may debond from the root of a channel crack, or from the edge of the film. When the debond length exceeds several times the film thickness, the debonding process attains a steady-state, in which the energy release rate is independent of the debond length. Under the plane strain conditions, an elementary consideration gives the energy release rate

$$G = \frac{\sigma^2 h}{2\bar{E}_f}.$$

The crack is under the mixed mode conditions. The mode angle must be determined by solving the boundary-value problem. When the film and the substrate have similar elastic constants, the stress intensity factors are given by the previous lecture, setting $P = \sigma h$ and $M = 0$. The mode angle is

$$\psi \approx 52^\circ.$$

Next examine the effect of the modulus mismatch on the mode angle. The complex stress intensity factor is given in the form

$$K = |K| h^{-i\varepsilon} e^{i\omega}.$$

The magnitude $|K|$ relates to the energy release rate by the Malyshev-Salganik relation. The angle ω depends on elastic mismatch between the film and the substrate, ranging between 40° to 60° if the mismatch is not excessive, as tabulated in Suo and Hutchinson (1990). The mode angle is

$$\psi = \omega + \varepsilon \log\left(\frac{l}{h}\right).$$

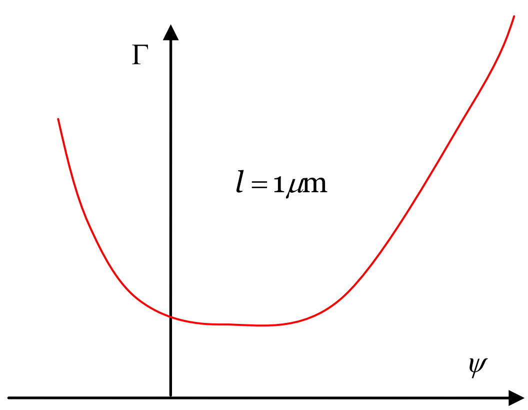
For representative values, $\varepsilon = -0.05$, l = 1 nm, and h = 1000 nm, the additional angle is $\varepsilon \log(l/h) = 20^\circ$.

Fracture energy is a function of the mode angle. When a crack extends on an interface between two materials, mixed-mode conditions prevail. The fracture energy is a function of the mode angle, $\Gamma(\psi)$. The crack extends when the energy release rate reaches the fracture energy:

$$G = \Gamma(\psi).$$

Due to elastic mismatch in the two materials, one should specify the length l in defining the mode angle ψ. The curve $\Gamma(\psi)$ has been measured for a few interfaces (e.g., Liechti and Chai, 1991).

The fracture energy tends to increase as the mode angle approaches $\pm\pi/2$. The energy cost for debond varies with the relative proportion of opening and shearing of the two faces of the crack. For example, a large amount of shearing may promote more inelastic deformation in the constituent materials, or promote near-tip sliding against roughness if the interface is not perfectly flat.



The curve $\Gamma(\psi)$ need not be symmetric with respect to $\psi = 0$, because the two materials on either side of the interface are dissimilar, breaking the symmetry between $\psi < 0$ and $\psi > 0$.

Sandwich specimens. A class of test methods has been particularly versatile for thin film structures. Suppose one needs to measure the fracture energy of an interface between a thin film of material 1 and a substrate of material 2. One can sandwich the thin film between two substrates of material 1. One may use a layer of adhesive, such as epoxy, to glue the bare substrate to the one covered with the film. The substrates are much thicker then the film and the adhesive, so that the whole specimen is easy to load. The interfacial fracture energy can be measured if the crack runs on the desired interface.

There are several advantages of methods of this kind. First, the methods are applicable for thin films of any thickness, and can even measure the interfacial energy between two thin films, when they are both sandwiched. Second, because the film is still bonded to one substrate in the crack wake, the residual stress in the film is unrelieved, and therefore does not contribute to the energy release rate. Third, the experimental data are relatively easy to interpret, as explained below.

Because the substrates are much thicker than the films, the stress field in the substrate, far away from the films, is unaffected by the presence of the films, and is the same as that around a crack in a homogeneous body. The energy release rate can be calculated from the homogeneous specimen by neglecting the thin films.

The mode angle, however, has to be determined by solving the boundary value problem that includes the thin films. The load on the substrate can be represented by the two stress intensity factors K_I and K_{II} for a crack in a homogenous body. Let ψ_∞ be the mode angle of the remote load, namely,

$$\tan \psi_\infty = K_{II} / K_I .$$

Near the crack tip, the stress field is that of an interfacial crack, characterized by the complex stress intensity factor *K*. The local mode angle ψ is defined using an arbitrary length *l*. The local mode angle relates to the remote mode angle as

$$\psi = \psi_\infty + \omega + \varepsilon \log\left(\frac{l}{h}\right),$$

where *h* is the thickness of one of the films, and ω is an angle that depends on elastic properties of the films and the substrates, as well as the ratios between various films.

Suo and Hutchinson (1989) solved the case of a single film between two substrates, and found that ω is less than 10° provided the elastic mismatch is not too large. It is rather cumbersome to keep track of the local mode angle, especially when several films are sandwiched, and when some of then deform plastically. A common practice has been to specify the mode angle for the sandwich specimens by the remote value ψ_∞.

Four-point bend. Consider a four-point bend specimen. The method was introduced by Charalambides et al. (1989) to determine interfacial fracture energy, and developed by Dauskardt et al. (1998) for thin film structures relevant for interconnects. The experimental procedure is as follows. Use two silicon wafers to sandwich films of interest. Notch the top wafer to within a few microns of the sandwiched films with a diamond wafering blade. Place the sample in a four-point bend fixture. Record load as a function of displacement, and observe the crack propagation in an optical microscope. At a certain load, a crack initiates from the notch root, approaches the interface, and then bifurcates into two cracks to propagate on the interface. When the interface cracks are long compared to the substrate thickness, the load-displacement curve exhibits a plateau—that is, the crack reaches a steady state, in which the energy release rate is independent of the crack length.

At the steady state, both the energy release rate and the mode angle are obtained analytically. The four-point bend configuration is the superposition of two other configurations, one being pure mode I, and the other being pure mode II. The energy release rates for the two configurations can be obtained from elementary considerations. Consequently, the energy release rate for the four-point bend specimen is

$$G = \frac{21}{4}\frac{M^2}{\bar{E}H^3},$$

where M is the moment per substrate unit width, H the thickness of each substrate, and $\bar{E}$ the plane strain modulus of the substrates. The mode angle for the four-point bend specimen is

$$\psi_\infty = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) \approx 41^\circ .$$

Once the plateau load is measured experimentally, one obtains the fracture energy Γ for the mode angle 41°.

A crack kinking out of an interface. A crack pre-exists on an interface between two materials. When the load increases to a critical level, the crack either extends and debonds the interface, or kinks out of the interface. This phenomenon has been analyzed by He and Hutchinson (1989).

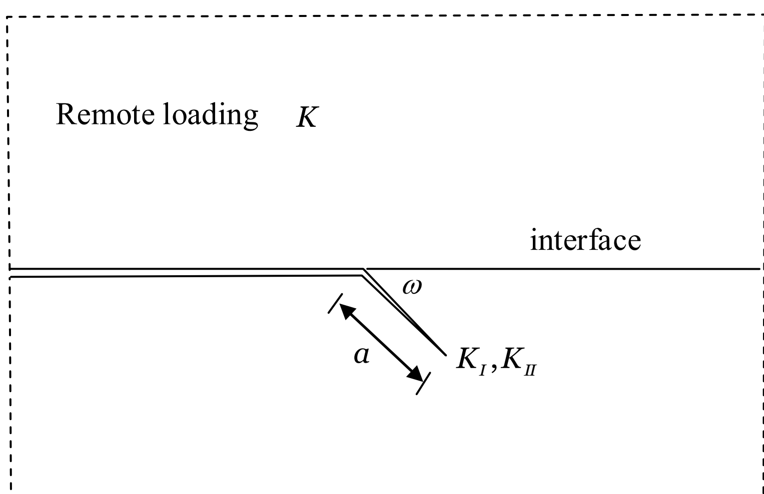
The condition for the crack to extend on the interface is that the energy release rate of the extension attains the fracture energy of the interface, namely,

$$G_{\text{debond}} = \Gamma(\psi).$$

The condition of the crack to kink into material 2 is that the energy release of the kink attains the fracture energy of material 2, namely,

$$G_{\text{kink}} = \Gamma_2 .$$

Debonding and Kinking are competing processes. The outcome depends on which of the above two critical conditions is first attained as the load ramps up.



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