EM 388F – Fracture Mechanics
Term Paper

Fracture of Orthotropic Materials under Mixed Mode Loading

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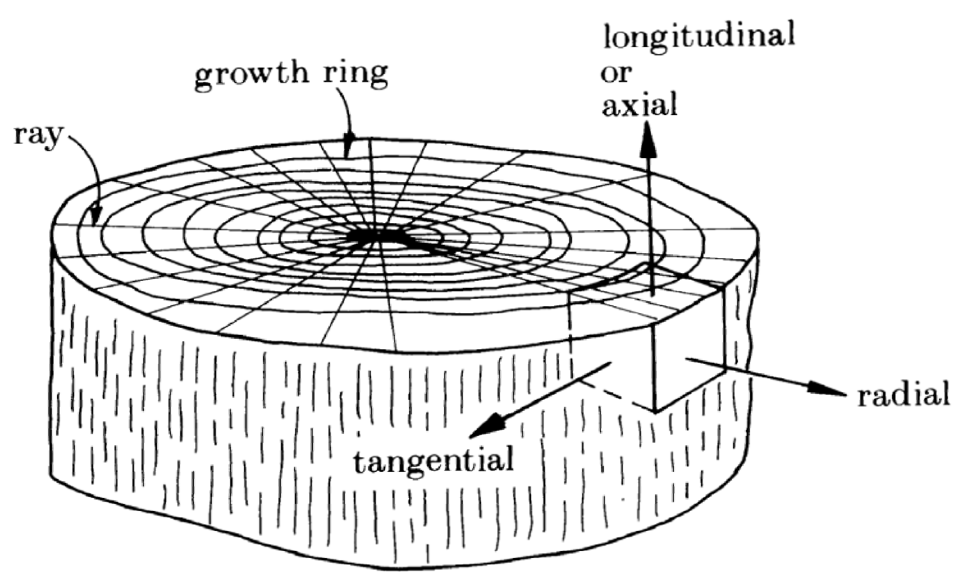
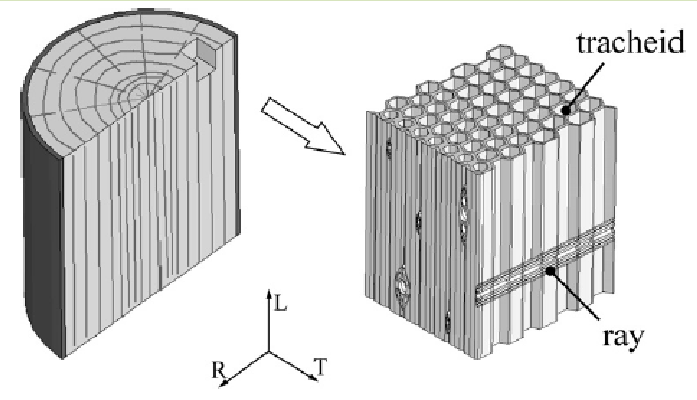
Outline

- Wood as orthotropic material
- Overview of fracture in wood
- Fracture toughness in mode I and mode II
- Mixed mode fracture criteria
- Comparison with experimental data
- Conclusions

Wood as Orthotropic Material

- Wood is generally considered a cylindrically orthotropic material, with the principal axes of orthotropy (R,T,L) given by the radial, tangential and longitudinal directions





Elastic Moduli

$$E_L > E_R > G_{LR} \sim G_{LT} > E_T > G_{RT}$$

Species	Relative density RD	Moisture content m (%)	E_T/E_L	E_R/E_L	G_{LR}/E_L	G_{LT}/E_L	G_{RT}/E_L
Balsa	0.13	9	0.015	0.046	0.054	0.037	0.005
Spruce	0.37	12	0.041	0.074	0.050	0.061	0.002
Yellow-poplar	0.38	11	0.043	0.092	0.075	0.069	0.011
Douglas-fir	0.50	12	0.050	0.068	0.064	0.078	0.007
Mahogany	0.50	12	0.073	0.107	0.098	0.066	0.028
Sweetgum	0.53	11	0.050	0.115	0.089	0.061	0.021
Black Walnut	0.59	11	0.056	0.106	0.085	0.062	0.021
Alpine Maple	0.59	10	0.088	0.152	0.123	0.110	0.029
Yellow Birch	0.64	13	0.050	0.078	0.074	0.068	0.017

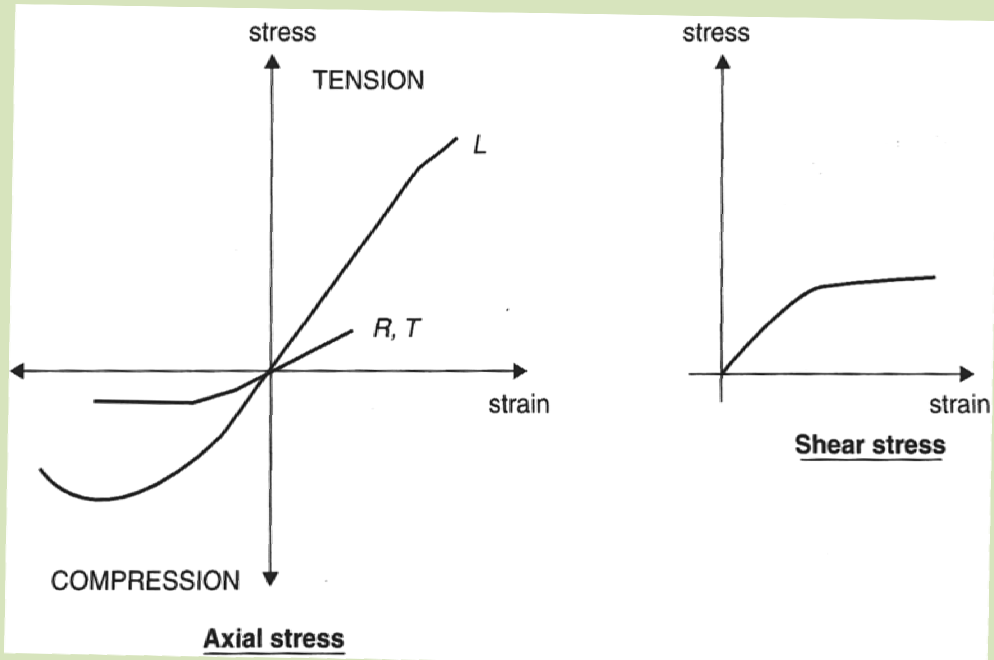
Constitutive Relation

$$\begin{Bmatrix} \varepsilon_{LL} \\ \varepsilon_{RR} \\ \varepsilon_{TT} \\ \gamma_{LR} \\ \gamma_{LT} \\ \gamma_{RT} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_L} & \frac{-\nu_{RL}}{E_R} & \frac{-\nu_{TL}}{E_T} & 0 & 0 & 0 \\ \frac{-\nu_{LR}}{E_L} & \frac{1}{E_R} & \frac{-\nu_{TR}}{E_T} & 0 & 0 & 0 \\ \frac{-\nu_{LT}}{E_L} & \frac{-\nu_{RT}}{E_R} & \frac{1}{E_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{LR}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{RT}} \end{bmatrix} \begin{Bmatrix} \sigma_{LL} \\ \sigma_{RR} \\ \sigma_{TT} \\ \tau_{LR} \\ \tau_{LT} \\ \tau_{RT} \end{Bmatrix}$$

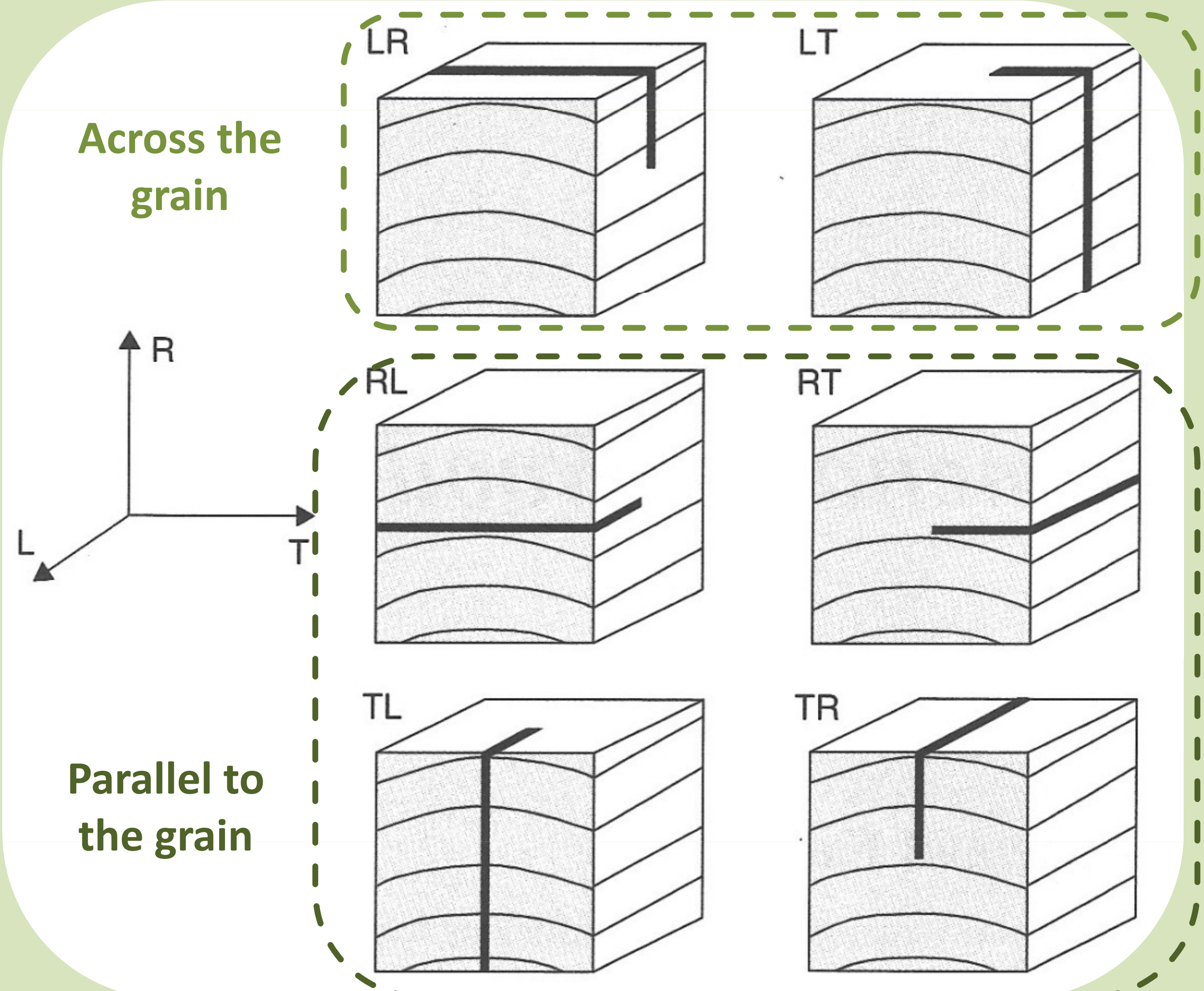
$$\nu_{kl}/E_l = \nu_{lk}/E_k$$

Strength

- In the radial and tangential directions the strength is **10-30%** of that in longitudinal direction
- There is also difference between tensile and compressive strengths.

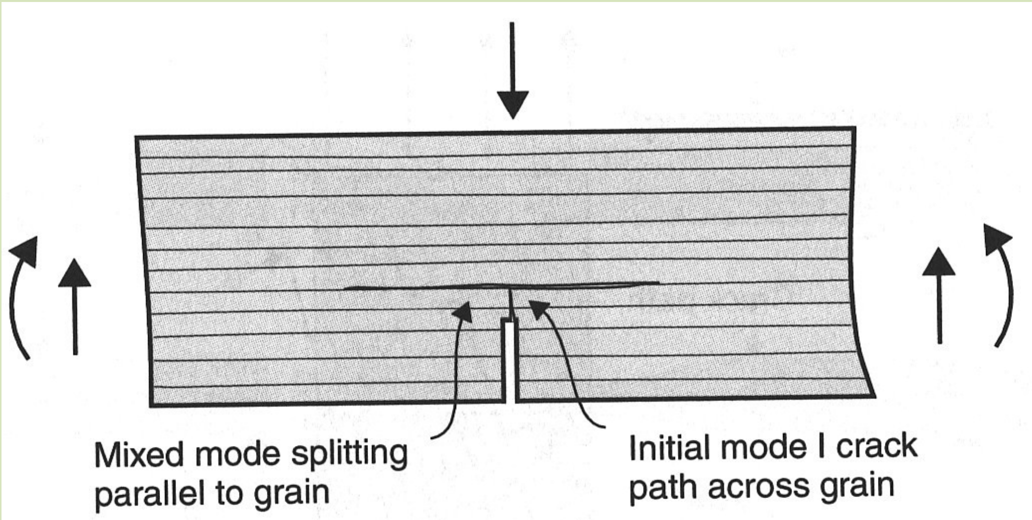


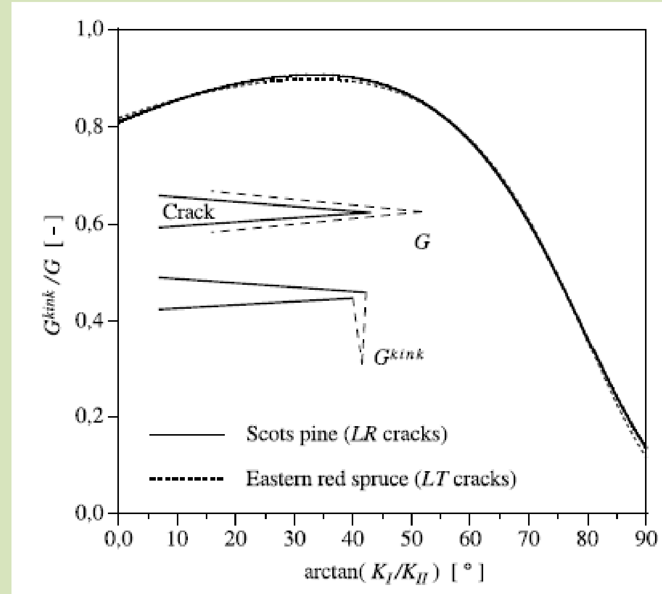
Fracture Orientations

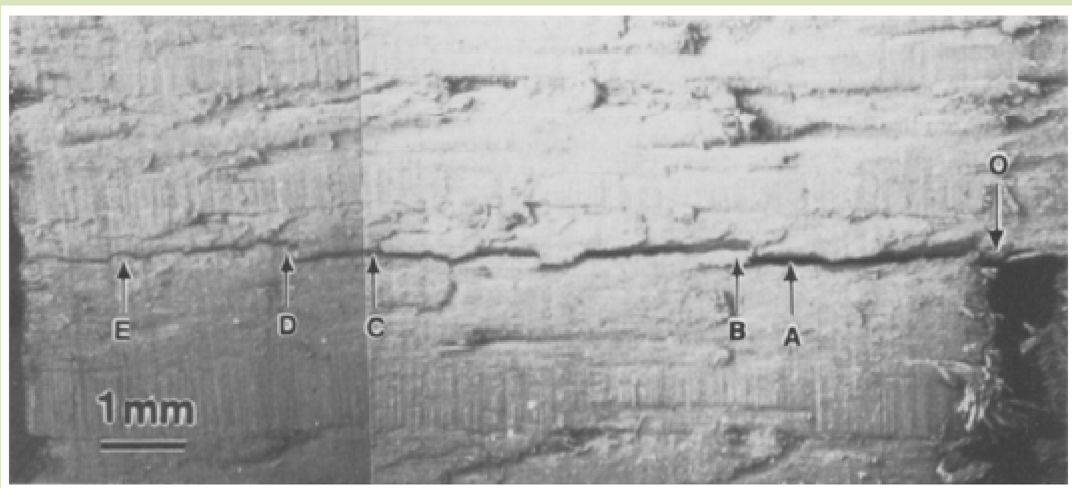


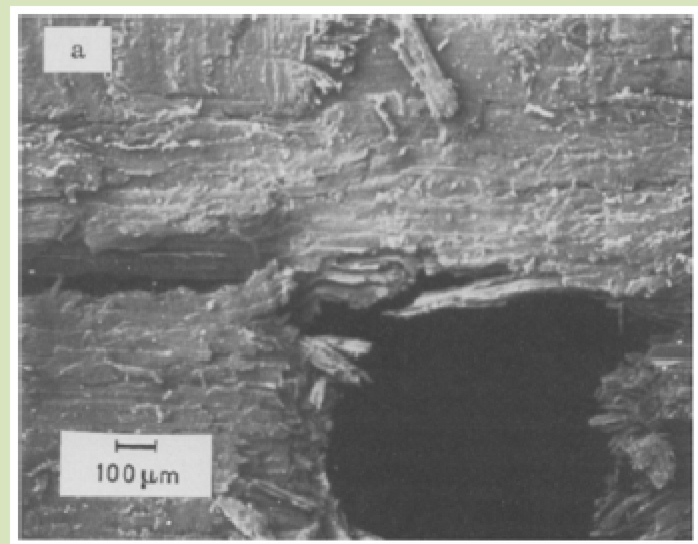
Fracture along the Grain

- Cracks in wood generally grow along the grains *irrespective of both the original orientation of the crack and the mode mixity*
- Even when cross-grain notches are loaded in longitudinal tension, cracking usually occurs along the grain (*perpendicularly to the notch*)
- Orientations RL and TL are usually the primary focus.


Mixed mode splitting parallel to grain
Initial mode I crack path across grain

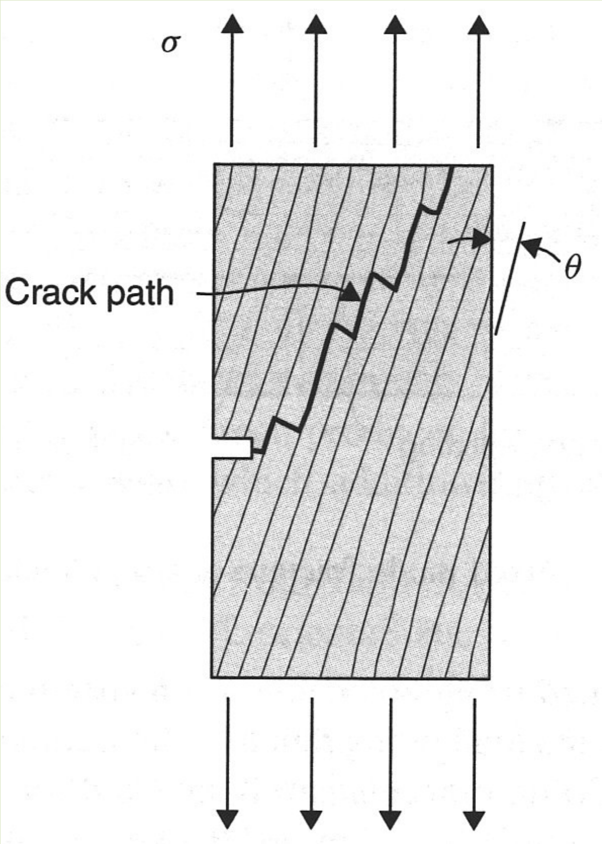

Crack
G
Gkink
Gkink /G [-]
arctan(KI/KII) [°]
Scots pine (LR cracks)
Eastern red spruce (LT cracks)


O
E
D
C
B
A
1 mm


a
100 µm

Tension at Arbitrary Angles Relative to the Grain

- Due to anisotropy, it is essentially impossible to get a pure mode I. *Mixed mode condition* arises!
- Cracks will propagate along the weak axes of the material but *frequently jump between grain lines* when doing so maximizes the energy release.



Fracture Toughness

Species	K_{IC} ($kNm^{-3/2}$)	
	TL	RL
Douglas-fir	320	360
	309[b]	410[b]
	260[c]	
	847[c]	
Western hemlock	375	
Western white pine	250	260
Scots pine	440	500
Southern pine	375	
Ponderosa pine	290	
Red spruce	420	
Northern red oak	410	
Sugar maple	480	
Yellow-poplar	517	

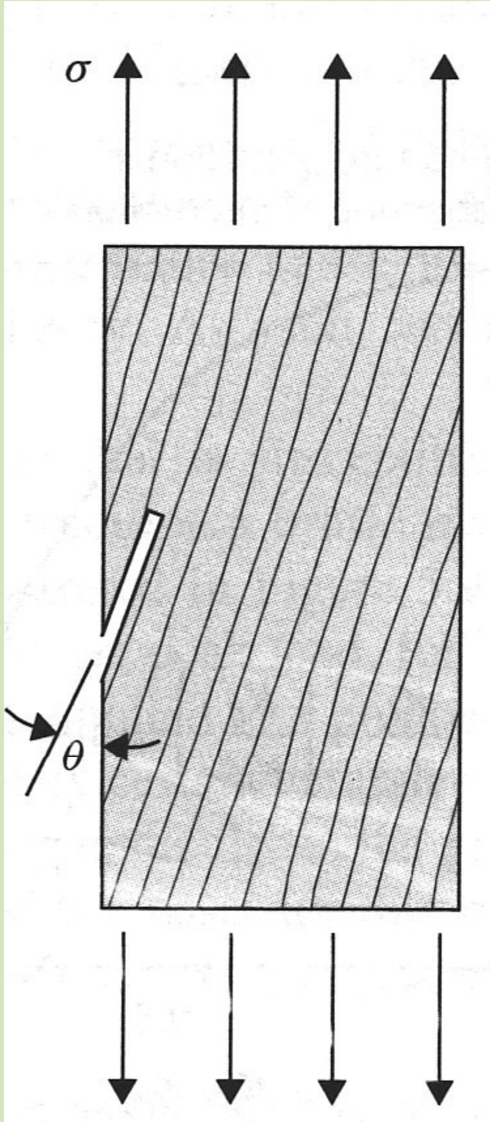
Species	K_{IIC} ($kNm^{-3/2}$)	
	TL	RL
Douglas-fir		2230
		1562/1746[b]
	1370[d]	
Western hemlock	2240	
	2420/2250[c]	
Western white pine		
Scots pine	2050	
Southern pine	2070	
	1930[d]	
Ponderosa pine		
Red spruce	2190	1665
Poplar		2232[e]

Mixed Mode Fracture

- Crack growth depends on not only K_{IC} and K_{IIC} but also on the interaction between the two
- Theories for predicting mixed mode fracture in anisotropic homogeneous materials usually predict that a crack subjected to mixed mode loading will grow out of its original plane. *They cannot be directly applied to wood*

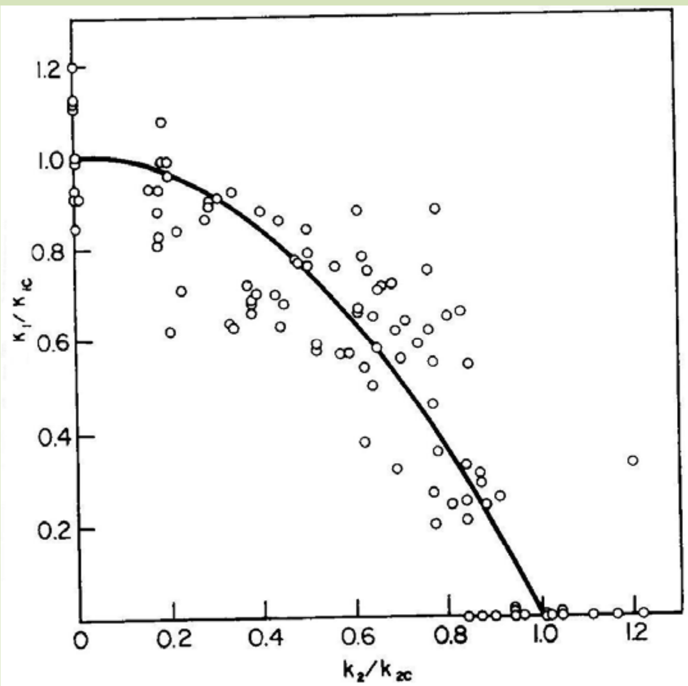
Mixed mode fracture

- In isotropic materials, the crack will typically turn so that its plane is perpendicular to the load axis, becoming mode I.
- In case of wood, it will continue to propagate under mixed mode.

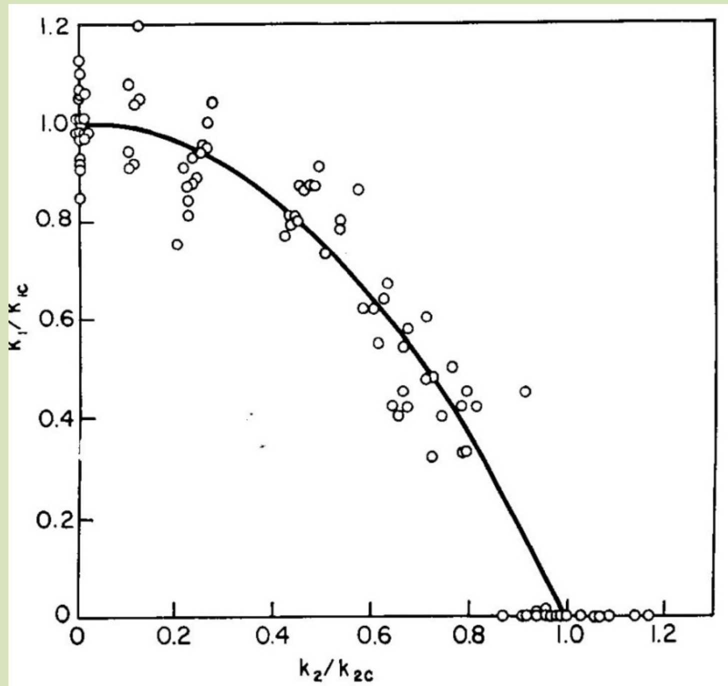


Mixed mode fracture criteria

- **Empirical criterion**
  - based on experiments with balsa wood and fiber-glass-reinforced plastic plates

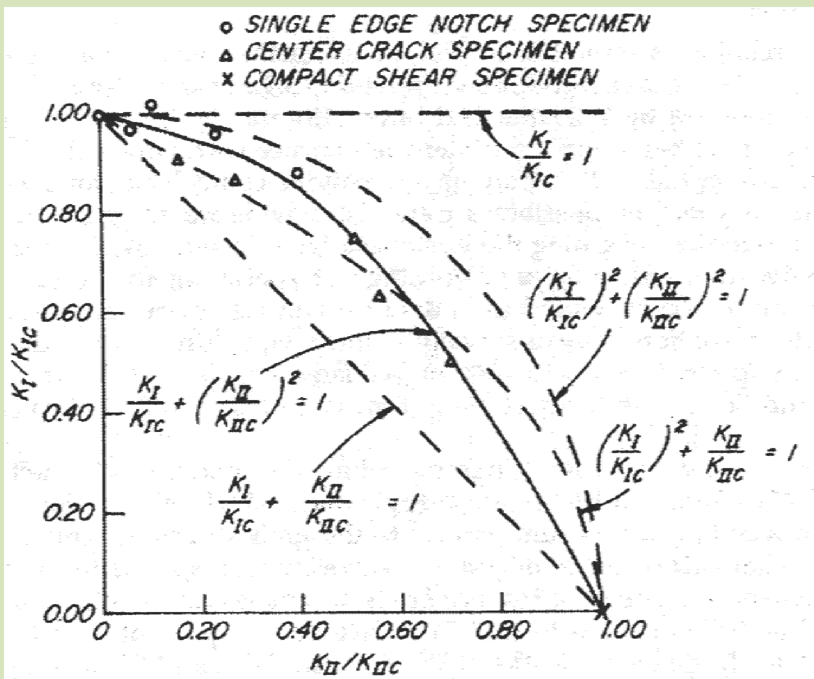


$$\frac{K_I}{K_{Ic}} + \left(\frac{K_{II}}{K_{IIc}}\right)^2 = 1$$



Mixed mode fracture criteria

- **Empirical criterion**
  – Eastern Red Spruce



$$\left(\frac{K_I}{K_{IC}}\right)^a + \left(\frac{K_{II}}{K_{IIC}}\right)^b = 1$$

2 < b < 3.5

Mixed mode fracture criteria

- **Critical Energy Release Rate**
  - fracture takes place when the strain energy release rate during crack propagation equals the energy rate needed to tear the material apart.

$$\left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1$$

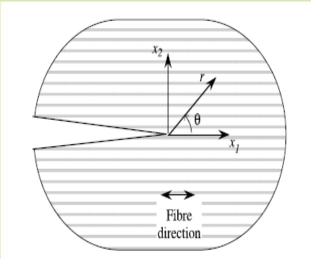
$$\frac{K_{IIC}}{K_{IC}} = \left(\frac{C'_{22}}{C'_{11}}\right)^{1/4}$$

Mixed Mode Fracture Criteria

- **Critical Strain Energy Density**
  - Crack growth occurs when the strain energy density at some distance from the crack tip reaches a critical value.

$$\left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1$$

$$\frac{K_{IIC}}{K_{IC}} = \left(\frac{C'_{11}\, f_{11}^2(0) + C'_{22}\, f_{22}^2(0) + 2C'_{12}\, f_{11}(0)\, f_{22}(0)}{C'_{66}\, g_{12}^2(0)}\right)^{1/2}$$



$$\sigma_{ij} = \frac{K_I\, f_{ij}(\theta)}{\sqrt{2\pi r}} + \frac{K_{II}\, g_{ij}(\theta)}{\sqrt{2\pi r}}$$

Mixed Mode Fracture Criteria

- **Critical In-Plane Maximum Principal Stress**
  - Fracture takes place when the maximum principal stress at some distance in front of the crack tip reaches a critical value.

$$\frac{f_{11}(0)+f_{22}(0)}{2f_{11}(0)}\frac{K_I}{K_{IC}}+\sqrt{\frac{2[f_{11}(0)-f_{22}(0)]^2}{[f_{11}(0)+f_{22}(0)]^2+2[f_{11}(0)-f_{22}(0)]^2}\left(\frac{K_I}{K_{IC}}\right)^2+\left(\frac{K_{II}}{K_{IIC}}\right)^2}=1$$

$$\frac{K_{IIC}}{K_{IC}}=f_{11}(0)$$

Mixed Mode Fracture Criteria

- **Non-local Stress Fracture Criterion (2008)**
  - Fracture takes place when the mean value of the function of decohesive normal and shear stress over a segment *d*, the length of the damage zone, reaches its critical value

$$\max_{\theta} \bar{R}(\sigma_n, \tau_n) = \max_{\theta} \left[\frac{1}{d} \int_0^d R(\sigma_n, \tau_n) \right] = 1$$

$$\left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1$$

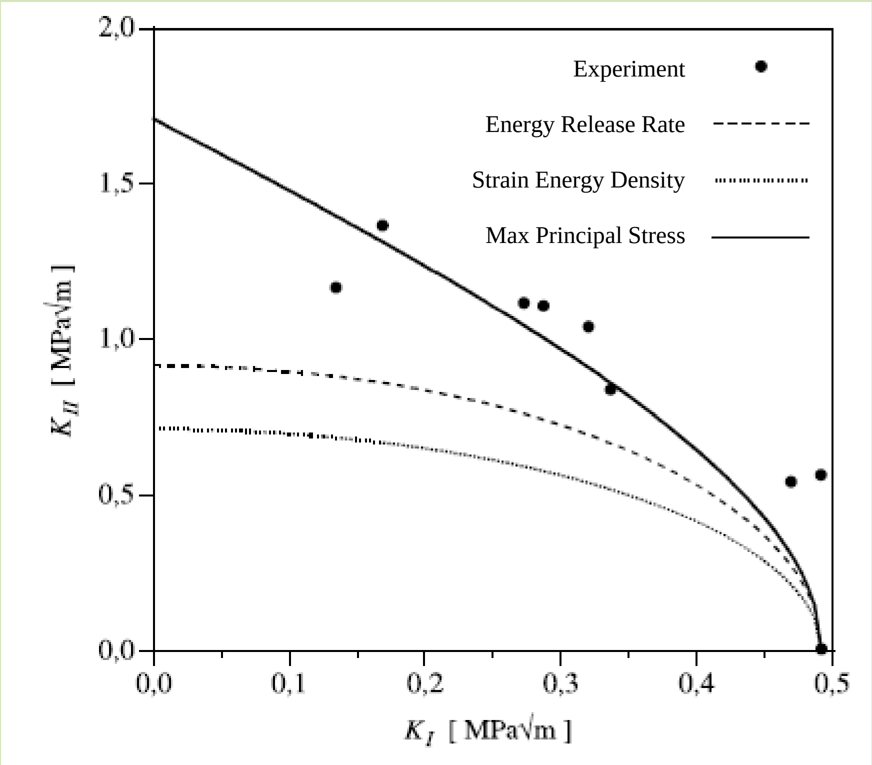
Crack parallel to the grain

$$\lambda_{11} K_I{}^2 + \lambda_{12} K_I K_{II} + \lambda_{22} K_{II}{}^2 = K_{IC}{}^2$$

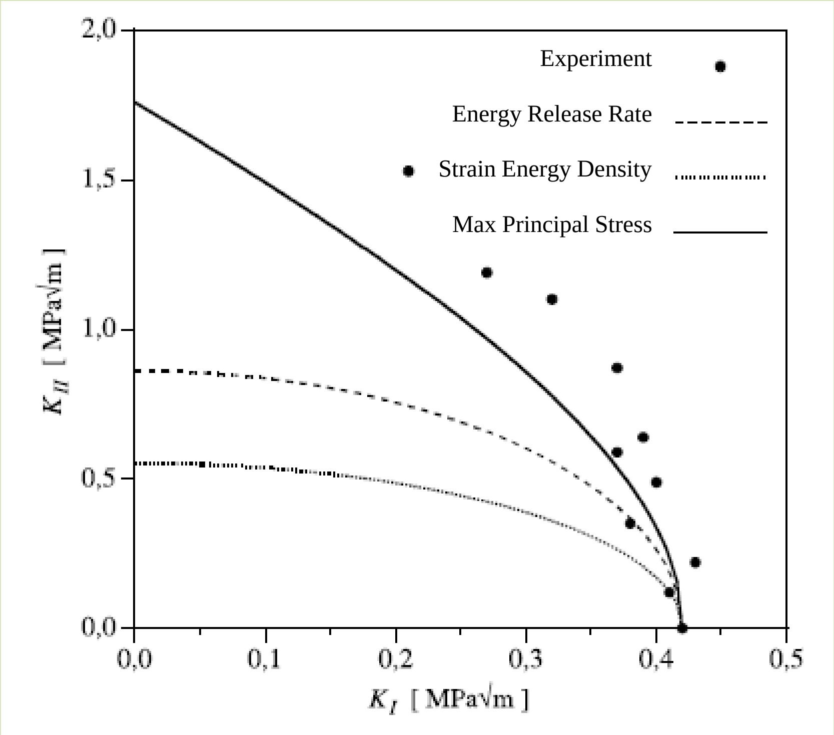
Crack arbitrarily oriented

Discussion

Scots Pine

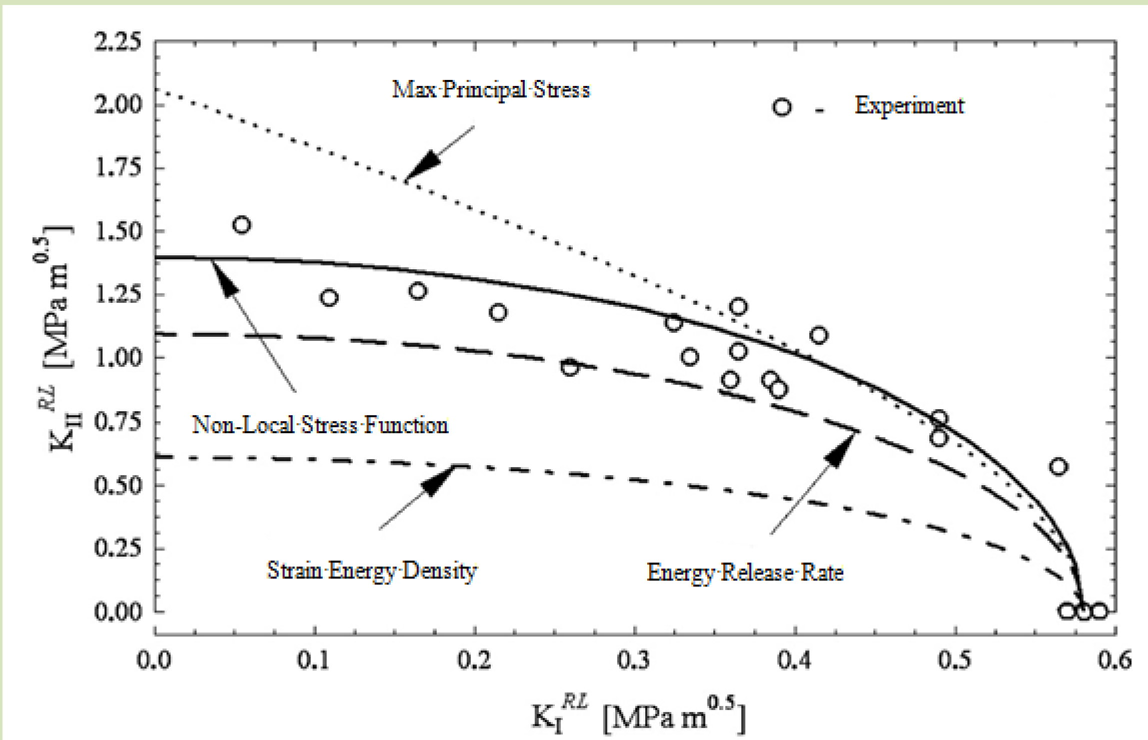


Eastern Red Spruce



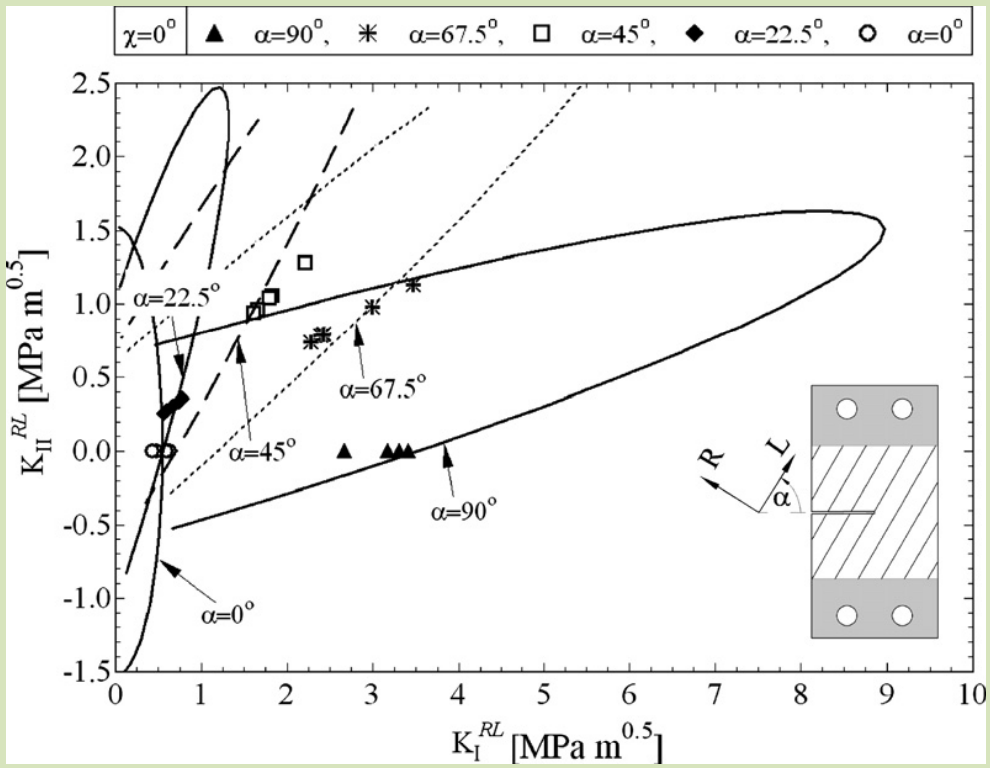
Discussion

Norway Spruce



Discussion

Pine Wood – crack arbitrarily oriented



Conclusions

- Empirical criteria require a large number of material constants determined for each crack configuration.
- The energy based criteria don't predict well the mixed mode fracture in wood.
- Cracks oriented along the grain
  - Critical In-Plane Maximum Principal Stress criterion
  - Non-Local Stress Fracture Criterion
- Cracks oriented arbitrarily
  - Non-Local Stress Fracture Criterion

References

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