THE UNIVERSITY OF TEXAS AT AUSTIN
Department of Aerospace Engineering and Engineering Mechanics

EM 388F Fracture Mechanics: Term Paper

Fracture of Orthotropic Materials under Mixed Mode Loading

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Abstract: The objective of this paper is to analyze the fracture of orthotropic materials, with emphasis on wood. Wood is usually considered as a cylindrically orthotropic material, with the principal axes of orthotropy (R,T,L) given by the radial, tangential and longitudinal directions. There are large differences in stiffness and strength between these directions. Moreover, the fracture toughness is highly dependent on both the crack propagation direction and the crack plane orientation. Due to large variations in fracture toughness depending on the orientation, cracks usually propagate in the direction along the grain. Even when cross-grain notches are loaded in longitudinal tension, cracking occurs along the grain (perpendicularly to the notch). Cracks grow along the grain, irrespective of both the original orientation of the crack and the mode mixity. Therefore, mixed mode fracture criteria derived for homogeneous materials cannot be expected to be directly applied to wood. The following approaches for the fracture criteria within the framework of linear elastic fracture mechanics (LEFM) are presented: Critical Energy Release Rate, Critical Strain Energy Density, Critical In-Plane Maximum Principal Stress and Non-Local Stress Function. These criteria are compared to experimental results of mixed mode I/II fracture in different species of wood.

1. Introduction

Wood is usually considered as a cylindrically orthotropic material, with the principal axes of orthotropy (R,T,L) given by the radial, tangential and longitudinal directions (Figure 1):

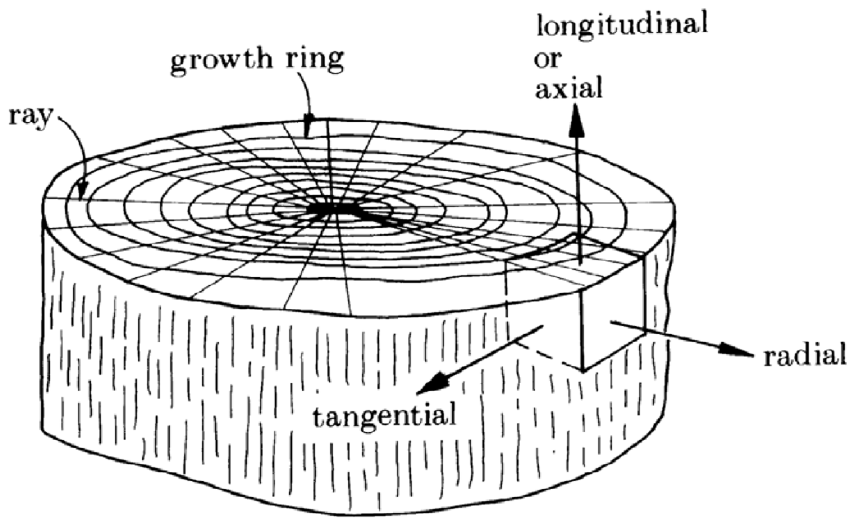

Figure 1: Principal axes of orthotropy [1]

There are large differences in stiffness between these directions. The moduli are typically ordered

$$E_L > E_R > G_{LR} \sim G_{LT} > E_T > G_{RT}$$

This anisotropy of wood can be largely understood in terms of the geometrical arrangement of an isotropic structural material. In this case, the tubular structure of wood cells [2].

The predominant type of cells, the tracheids, have a length to diameter ratio close to 100, and are closely aligned to the longitudinal direction of the tree trunk [3].

The tracheid walls can be viewed as a fiber-reinforced composite material, with strong fibrils wound in a helix along the cell.

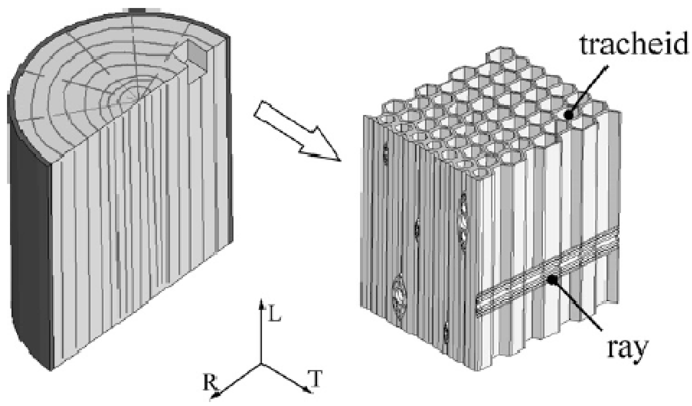

Figure 2: Macroscopic and microscopic structure of wood [4]

Table 1: Ratios of elastic moduli for clear wood in dry condition [5]

Species	Relative density *RD*	Moisture content *m* (%)	E_T/E_L	E_R/E_L	G_{LR}/E_L	G_{LT}/E_L	G_{RT}/E_L
Balsa	0.13	9	0.015	0.046	0.054	0.037	0.005
Spruce	0.37	12	0.041	0.074	0.050	0.061	0.002
Yellow-poplar	0.38	11	0.043	0.092	0.075	0.069	0.011
Douglas-fir	0.50	12	0.050	0.068	0.064	0.078	0.007
Mahogany	0.50	12	0.073	0.107	0.098	0.066	0.028
Sweetgum	0.53	11	0.050	0.115	0.089	0.061	0.021
Black Walnut	0.59	11	0.056	0.106	0.085	0.062	0.021
Alpine Maple	0.59	10	0.088	0.152	0.123	0.110	0.029
Yellow Birch	0.64	13	0.050	0.078	0.074	0.068	0.017

Table 2: Poisson's ratios for clear wood in dry condition [5]

Species	Relative density	Moisture content *m* (%)	ν_{LR}	ν_{LT}	ν_{RT}	ν_{TR}	ν_{RL}	ν_{TL}
Balsa	0.13	9	0.23	0.49	0.67	0.23	0.02	0.01
Spruce	0.37	12	0.44	0.56	0.57	0.29	0.03	0.01
Yellow-poplar	0.38	11	0.32	0.39	0.70	0.33	0.03	0.02
Douglas-fir	0.50	12	0.29	0.45	0.39	0.37	0.04	0.03
Mahogany	0.50	12	0.31	0.53	0.60	0.33	0.03	0.03
Sweetgum	0.53	11	0.32	0.40	0.68	0.31	0.04	0.02
Black Walnut	0.59	11	0.50	0.63	0.72	0.38	0.05	0.04
Alpine Maple	0.59	10	0.46	0.50	0.82	0.40	0.09	0.04
Yellow Birch	0.64	13	0.43	0.45	0.70	0.43	0.04	0.02

ν_{ij} is strain in the j direction due to unit strain in the i direction.

In a local Cartesian coordinate system whose axes coincide with the principal axes of orthotropy, the material constitutive relation can be written:

$$\varepsilon = C\,\sigma$$

or, in matrix form,

$$\begin{Bmatrix} \varepsilon_{LL} \\ \varepsilon_{RR} \\ \varepsilon_{TT} \\ \gamma_{LR} \\ \gamma_{LT} \\ \gamma_{RT} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_L} & \dfrac{-\nu_{RL}}{E_R} & \dfrac{-\nu_{TL}}{E_T} & 0 & 0 & 0 \\ \dfrac{-\nu_{LR}}{E_L} & \dfrac{1}{E_R} & \dfrac{-\nu_{TR}}{E_T} & 0 & 0 & 0 \\ \dfrac{-\nu_{LT}}{E_L} & \dfrac{-\nu_{RT}}{E_R} & \dfrac{1}{E_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{G_{LR}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{G_{LT}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{G_{RT}} \end{bmatrix} \begin{Bmatrix} \sigma_{LL} \\ \sigma_{RR} \\ \sigma_{TT} \\ \tau_{LR} \\ \tau_{LT} \\ \tau_{RT} \end{Bmatrix}$$

The compliance matrix is symmetric, i.e. $\nu_{kl}/E_l = \nu_{lk}/E_k$. For plane stress conditions, only the components C_{11}, C_{22}, C_{12}, C_{21}, and C_{66} are relevant. Moreover, for plane strain problems, the governing equations are the same as in plane stress, except that the in-plane compliances need to be replaced according to:

$$C'_{kl} = C_{kl} - C_{k3}\,C_{l3}/C_{33}\ \ (k = 1{,}2\ and\ l = 1{,}2)$$

The strength is markedly different depending on the stressing direction. In the radial and tangential directions the strength is 10-30% of that in longitudinal direction [1],[5]. There is also difference between tensile and compressive strengths.

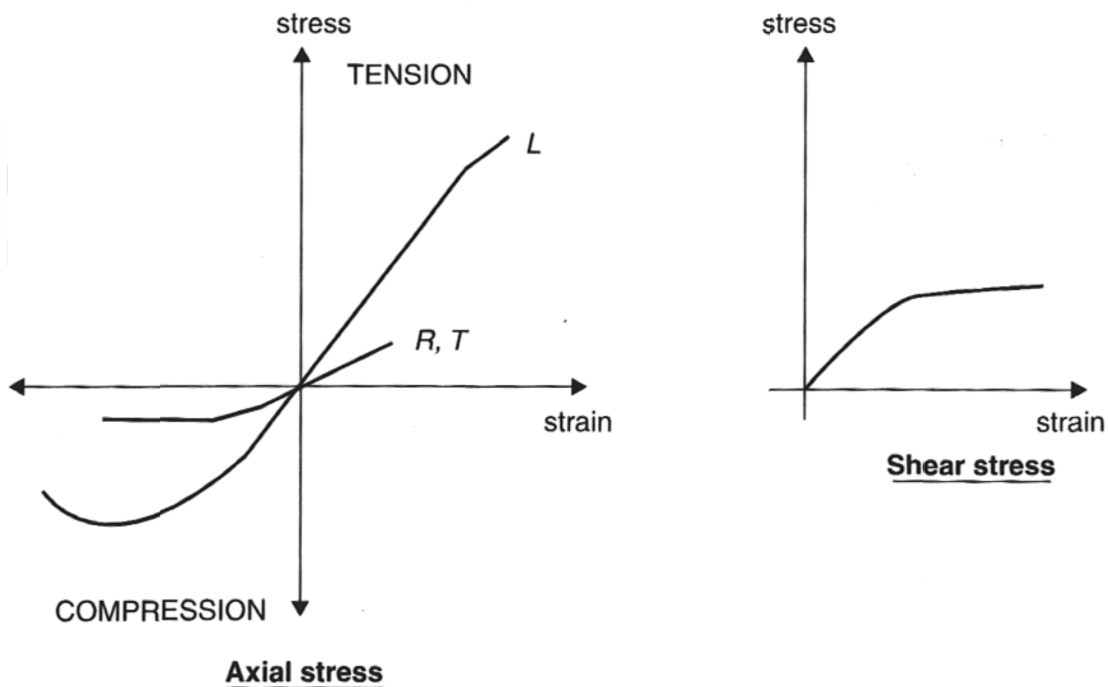

Figure 3: Stress-strain response of clear wood [5]

Moreover, the fracture toughness is highly dependent on both the crack propagation direction and the crack plane orientation.

Six principal systems of crack extension are usually discerned in wood. They are identified with a pair of letters, where the first letter specifies the direction perpendicular to the plane of the crack and the second letter specifies the direction of crack propagation.

Given that each of these six orientations can be subjected to three fracture modes (mode I, mode II and mode III), there is potentially a large number of fracture cases to consider for each species.

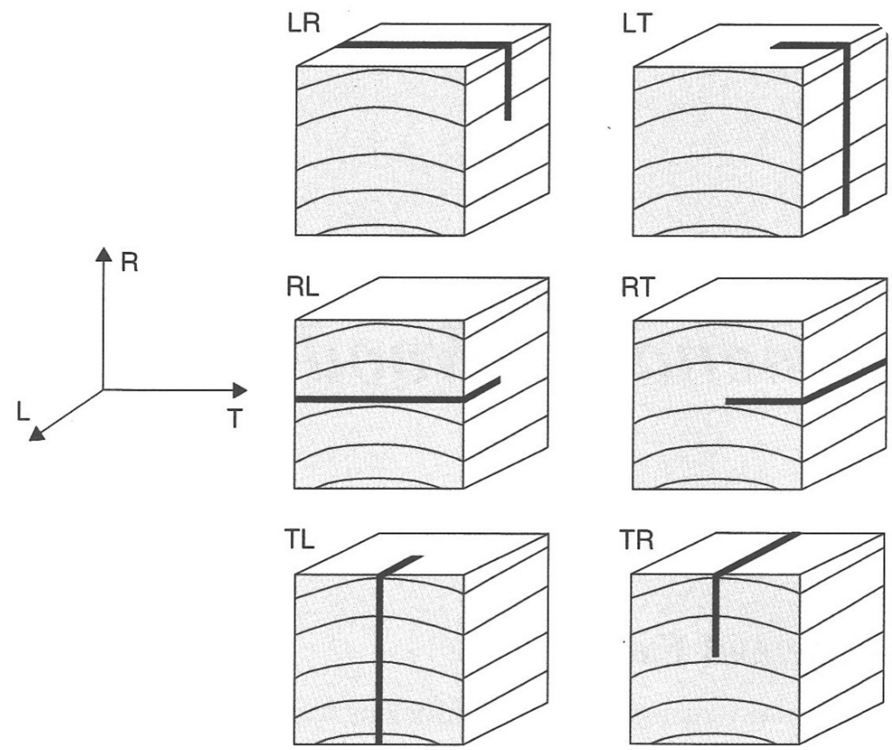

Figure 4: Fracture orientations relative to wood growth axis [5]

As cracks in wood generally grow along the grains irrespective of both the original orientation of the crack and the mode mixity, crack propagation along the grain (especially orientations RL and TL) is usually the primary focus.

Although fracture toughness has been measured for a variety of species under a range of fracture orientations, there is no standard method for determining fracture toughness in any mode. This makes comparison of data very difficult.

Most published fracture toughness data for wood is in terms of K_{IC} but there is also some information in terms of mode II, III and mixed mode. It is actually very difficult to produce pure fracture modes in wood and it should be emphasized that fracture in real structures is invariably a combinations of modes.

2. Mode I fracture: tension perpendicular to the grain

The direction of crack extension is governed not only by the direction that maximizes the energy release rate, but also by the planes of weakness in the material. Therefore, due to large variations in fracture toughness depending on the orientation, cracks usually propagate in the direction along the grain. This is the reason why mode I tension perpendicular to the grain has received the most attention in fracture mechanics applications to wood.

Table 3 presents a list of measured values of K_{IC} for TL and RL orientations.

Table 3: Sample of measured mode I fracture toughness values [5]

Species	K_{IC} ($kNm^{-3/2}$)		Specimen type
	TL	RL	
Douglas-fir	320	360	
	309^b	410^b	single edge notched beam
	260^c		double edge notched tension
	847^c		double cantilever beam
Western hemlock	375		
Western white pine	250	260	
Scots pine	440	500	
Southern pine	375		
Ponderosa pine	290		
Red spruce	420		
Northern red oak	410		
Sugar maple	480		
Yellow-poplar	517		

3. Mode I fracture: tension parallel to the grain

It is very difficult to produce a true mode I fracture condition with tension parallel to the grain (LR and LT orientations) without a mode II condition arising along the grain. As a result, published values of toughness are somewhat rare.

Some values published for Douglas fir (*Pseudotsuga menziesii*) give an idea of the order of magnitude of toughness in LR and LT orientations: K_{IC} is six to ten times greater than for tension perpendicular to the grain. [1],[5],[7]

Therefore, even when cross-grain notches are loaded in longitudinal tension, cracking usually occurs along the grain (perpendicularly to the notch).[1],[3],[6]

Figure 5 illustrates what happens with a notched beam. Splitting occurs along the grain, perpendicular to the grain.

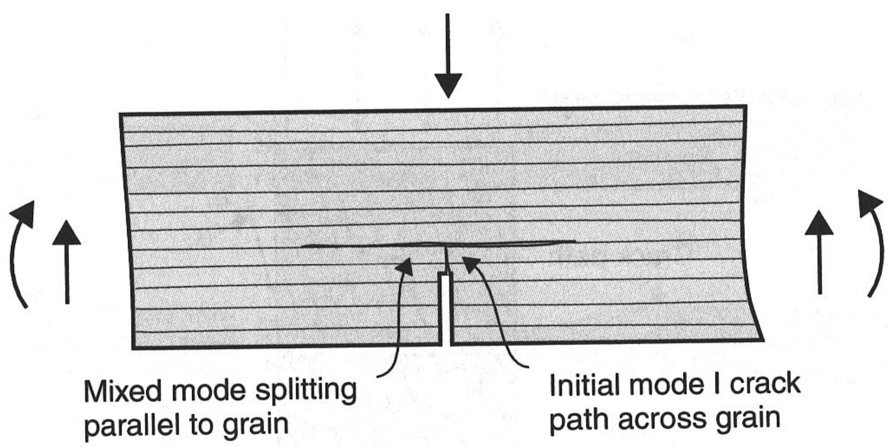

Figure 5: mixed mode fracture of wood loaded in flexure [5]

An example of a crack propagating at 90^{o} to the direction of the notch in Douglas-fir is presented below.

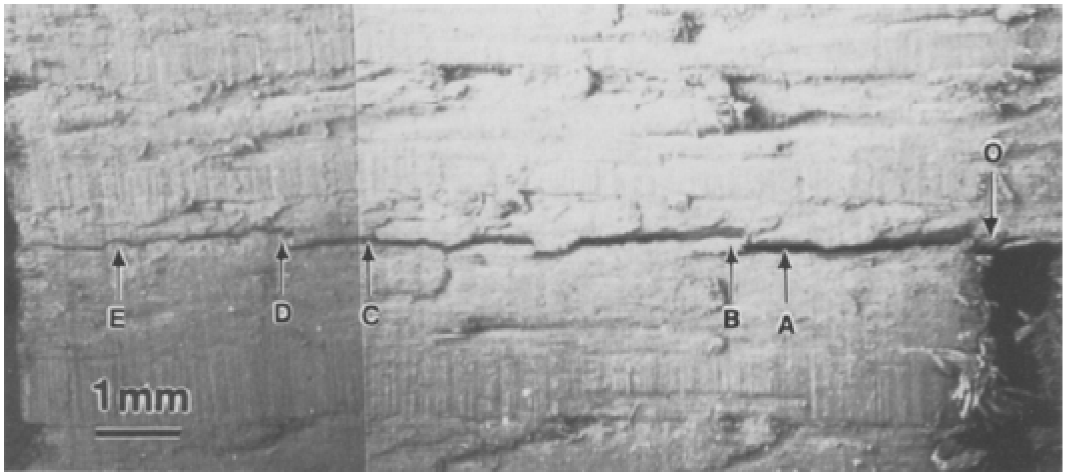

Figure 6: Crack induced in the first stage of loading in a specimen with the grain perpendicular to the notch [6]

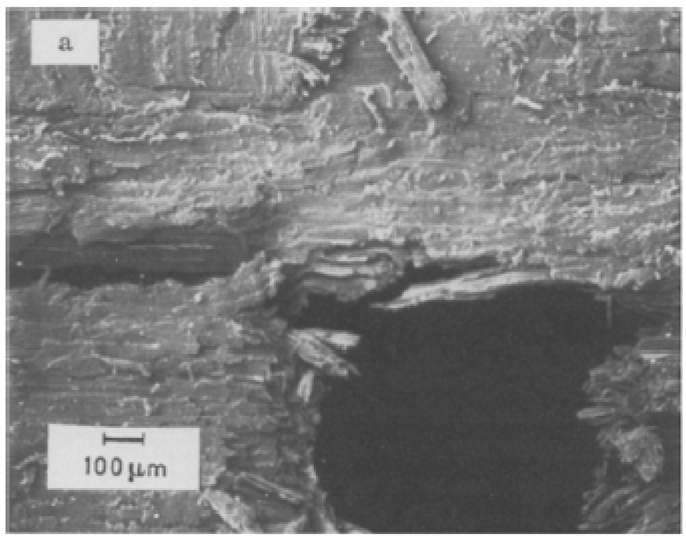

Figure 7: Higher magnification showing the origin of the crack [6]

Figure 8 shows the ratio of strain energy release rate for a cross-grain crack. It shows it is energetically more favorable for the crack to kink and propagate along the grain than to propagate across the grain.

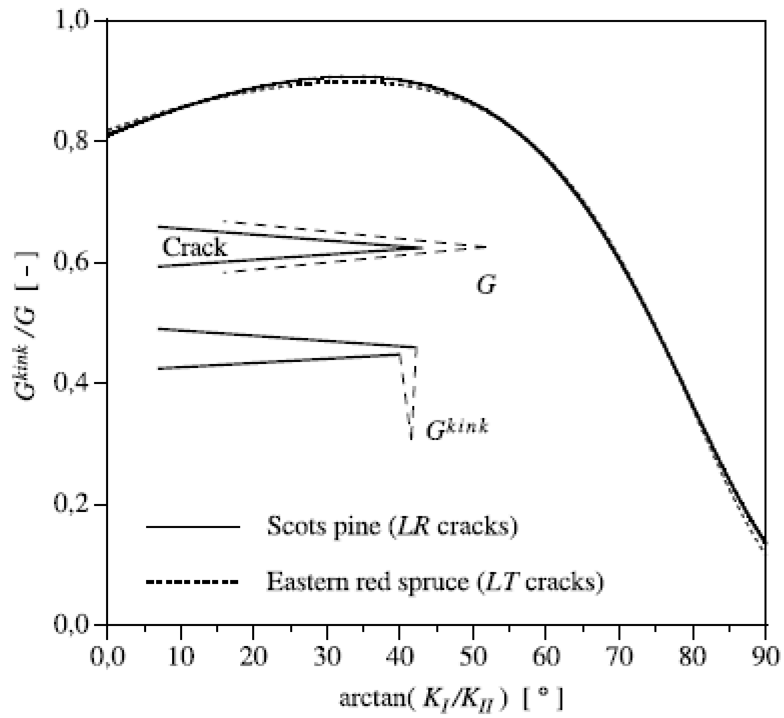

Figure 8: Ratio of strain energy release rates for different trajectories of a cross-grain crack [3]

The ratio of the energy release rates for competing trajectories can be shown to depend on the compliance matrix and on the relative proportion of K_{II} to K_I but not on their magnitudes [8]. This can be observed in the figure above.

In the case of the crack tip being subjected to a mode I loading, the condition for the crack to advance straight ahead is written as [8]:

$$\frac{G}{G^t} > \frac{\Gamma_0}{\Gamma_{90}}$$

Where Γ_0 is the toughness associated with straight ahead crack advance and Γ_{90} is that associated with crack advance by kinking.

The crack will kink if the inequality above is reversed.

4. Mode I fracture: tension at arbitrary angles relative to the grain

Because of the material anisotropy, it is essentially impossible to get a pure mode I. In such cases, a mixed mode condition arises.

Cracks will propagate along the weak axes of the material but frequently jump between grain lines when doing so maximizes the energy release [5].

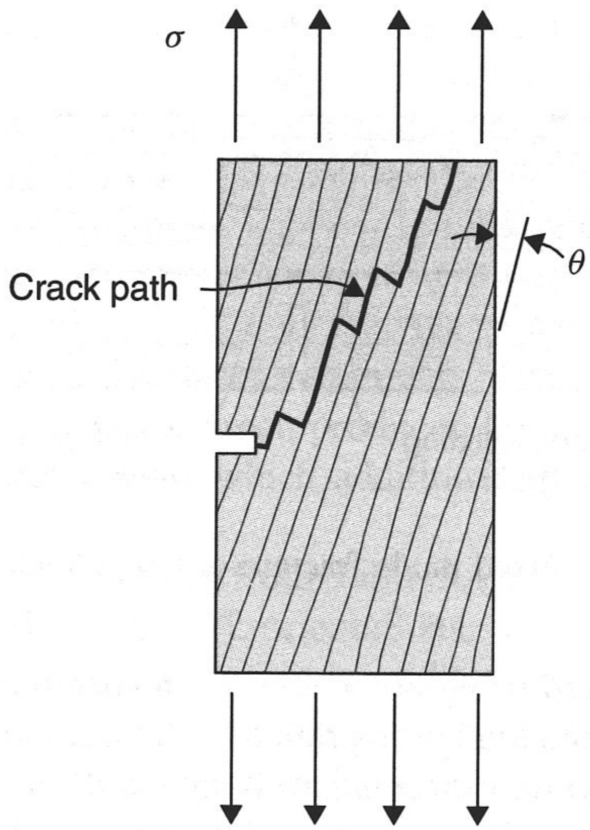

Figure 9: Jumping of a crack between growth layers (grain boundaries) [5]

5. Mode II fracture

It is particularly difficult to produce mode II stress at a tip of crack in an anisotropic material. Typically, stresses normal to the crack plane arise that add either a mode I opening component or a closing stress component that affect the results.

Mode II fracture is irrelevant in the LR and LT directions because there is no practical way to propagate a shear crack across the grain.

Some measured mode II fracture toughness values are presented below.

Table 4: A sample of measured mode II fracture toughness values [5]

Species	K_{IIC} (kNm$^{-3/2}$)		Specimen type
	TL	RL	
Douglas-fir		2230	
		1562/1746^{b}	center-slit beam
	1370^{d}		compact shear specimen
Western hemlock	2240		
	2420/2250^{c}		single end-notched beam
Western white pine			
Scots pine	2050		
Southern pine	2070		
	1930^{d}		compact shear specimen
Ponderosa pine			
Red spruce	2190	1665	
Poplar		2232^{e}	double edge notched tension

6. Mode III fracture

Mode III fracture has not traditionally been of great interest. Some investigations found the crack initiation energy in mode III to be over twice as high as mode I in both RL and TL directions.[5]

7. Mixed mode fracture

Mixed mode fracture conditions tend to be the dominant condition for real structures.

The general goal of the analysis is to apply the individually known mode I and mode II critical stress intensity factors K_{IC} and K_{IIC} to predict the fracture strength under mixed mode conditions.

A common way to produce mixed conditions is to put an inclined crack in a uniform tension field as shown in Figure 10. In isotropic materials, the crack will typically turn so that its plane is perpendicular to the load axis, becoming mode I. In case of wood, it will continue to propagate under mixed mode.

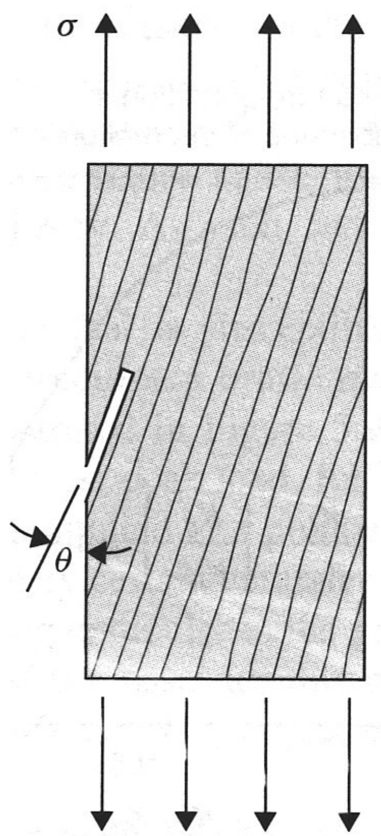

Figure 10: Single edge notched specimen for wood fracture testing [5]

The crack growth is dependent on not only the mode I and mode II fracture toughness values but also on the interaction between the two.

Several theories have been proposed for predicting mixed mode fracture in anisotropic homogeneous materials. They usually predict that a crack subjected to mixed mode loading will grow out of its original plane, with an inclination that depends on the material anisotropy and the degree of mixity[10]. They cannot be directly applied to wood, for which case the cracks generally grow along the grain, irrespective of the original orientation of the crack and the mode mixity.

A discussion of mixed mode fracture criteria follows.

8. Criteria for cracks oriented along the grain

Considering first a crack oriented along the grain, the singular stress at the crack tip is written as:

$$\sigma_{ij} = \frac{K_I\, f_{ij}(\theta)}{\sqrt{2\pi r}} + \frac{K_{II}\, g_{ij}(\theta)}{\sqrt{2\pi r}} \qquad (1)$$

Where f_{ij} and g_{ij} depend on the constitutive matrix [13].

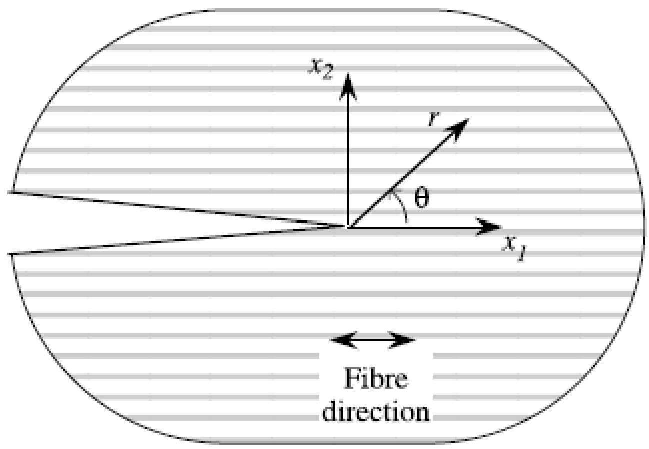

Figure 11: Crack oriented along the grain [3]

Cracks oriented in this manner propagate self-similarly, i.e. they do not leave their original plane.

Some criteria for cracks along the grain are compared and discussed below.

a. Empirical Criterion

The following empirical criterion for mixed crack growth has been proposed, based on experiments with balsa wood and fiber-glass-reinforced plastic plates (Scotchply) [9]:

$$\frac{K_I}{K_{IC}} + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1 \qquad (1)$$

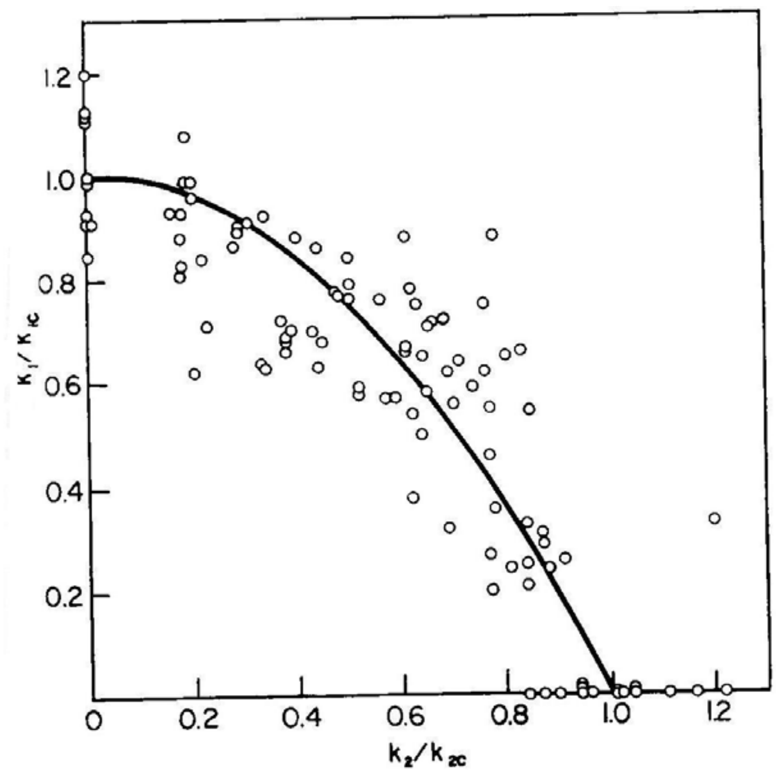

Figure 12: Interaction between stress-intensity factors K_{IC} and K_{IIC} for balsa wood [9]

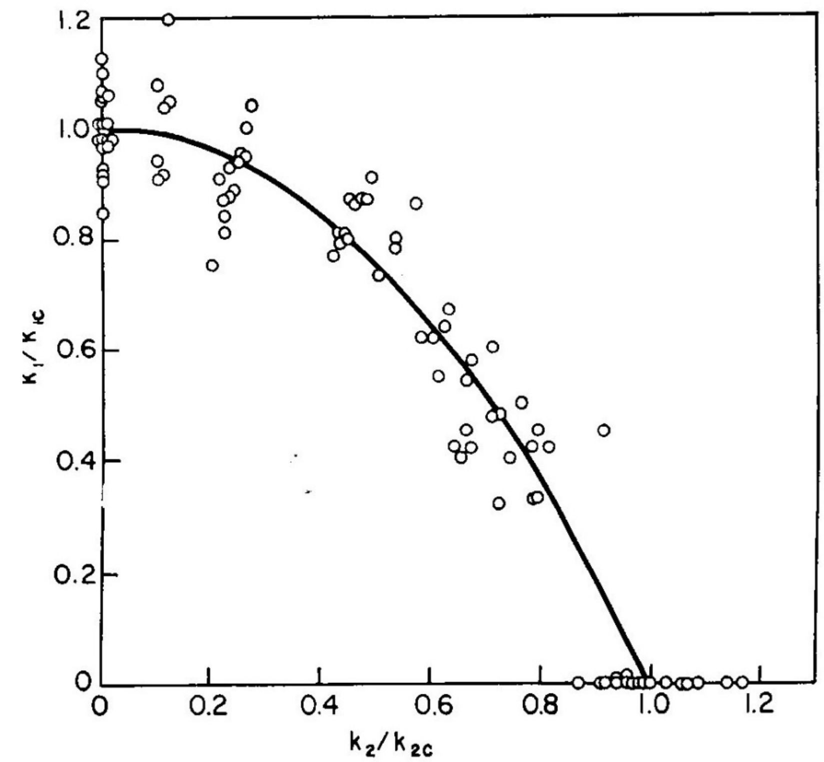

Figure 13: Interaction between stress-intensity factors K_{IC} and K_{IIC} for fiber-glass-reinforced plastic plates [9]

It has been proposed that mode II has no effect on fracture under mixed mode conditions. Different exponents have also been proposed as calibration constants, leading to the following alternative mixed mode criterion:

$$\left(\frac{K_I}{K_{IC}}\right)^a + \left(\frac{K_{II}}{K_{IIC}}\right)^b = 1$$

With a great number of experiments, it would be possible to fit the data, determining a, b, K_{IC} and K_{IIC}.

Some investigations using this empirical criterion [3] provide $a = 1$ and $2 < b < 3.4$.

b. Critical Energy Release Rate

This is the oldest and most widespread criterion, due to Griffith and Irwin. The assumption is that fracture takes place when the strain energy release rate during crack propagation equals the energy rate needed to tear the material apart. The mixed mode fracture criterion in terms of stress intensity factors can be written as [3]:

$$\left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1$$

$$\frac{K_{IIC}}{K_{IC}} = \left(\frac{C'_{22}}{C'_{11}}\right)^{1/4}$$

c. Critical Strain Energy Density

A fracture theory proposes [10] that crack propagation can be predicted based on the local strain energy density at the crack tip. Cracks subjected to mixed mode loading would propagate in a direction given by the local minimum of the strain energy density.

Crack growth would occur when the strain energy density at some distance from the crack tip in this direction reached a critical value.

Postulating that the crack should grow self-similarly i.e., in the direction of the grain, the criterion can, again, be written as:

$$\left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1$$

But the relationship between the toughness in mode I and mode II is now[3]:

$$\frac{K_{IIC}}{K_{IC}} = \left(\frac{C'_{11}\, f_{11}^2(0) + C'_{22}\, f_{22}^2(0) + 2C'_{12}\, f_{11}(0)\, f_{22}(0)}{C'_{66}\, g_{12}^2(0)}\right)^{1/2}$$

d. Critical In-Plane Maximum Principal Stress

In this case, the assumption is that fracture takes place as soon as the maximum principal stress at some distance in front of the crack tip reaches a critical value. The criterion becomes [3]:

$$\frac{f_{11}(0)+f_{22}(0)}{2f_{11}(0)}\frac{K_I}{K_{IC}}+\sqrt{\frac{2[f_{11}(0)-f_{22}(0)]^2}{[f_{11}(0)+f_{22}(0)]^2+2[f_{11}(0)-f_{22}(0)]^2}\left(\frac{K_I}{K_{IC}}\right)^2+\left(\frac{K_{II}}{K_{IIC}}\right)^2}=1$$

With

$$\frac{K_{IIC}}{K_{IC}}=f_{11}(0)$$

e. Non-Local Stress Fracture Criterion

A non-local stress fracture criterion based on the damage model of an elastic solid containing growing microcracks was recently proposed[4]. Crack initiation and propagation would occur when the mean value of the function $R(\sigma_n,\tau_n)$ of decohesive normal and shear stress over a segment *d*, the length of the damage zone, reaches its critical value:

$$\max_{\theta}\bar{R}(\sigma_n,\tau_n)=\max_{\theta}\left[\frac{1}{d}\int_0^d R(\sigma_n,\tau_n)\right]=1$$

Where $\bar{R}(\sigma_n,\tau_n)$ is called the non-local stress function. $R(\sigma_n,\tau_n)$ is the local stress function obtained using the microcrack damage model, which states that the propagation of microcracks takes place when the strain energy release rate equals the resistance to microcrack growth.

For cracks oriented along the grain, the criterion is written as:

$$K_I{}^2+\frac{c_{RL}}{c_R}K_{II}{}^2=K_{IC}{}^2$$

Where c_{RL} and c_R are the sliding and extensional compliances of the elastic solid weakened by microcracks oriented in the orthotropy plane of normal *R* (RL system).

Under the assumption of failure in pure mode I or pure mode II, the authors reduce the criterion to:

$$\left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1$$

This criterion used for cracks oriented along the grain. The full non-local stress fracture criterion for arbitrarily oriented cracks is discussed later in the present text.

f. Discussion

The criteria are compared to experimental results obtained for RL cracks in Scots Pine (*Pinus sylvestris*), TL cracks in Eastern Red Spruce (*Picea rubens*) and RL cracks in Norway Spruce (*Picea abies*).

Figure 14 presents a study of eastern red spruce [10]. The authors identify a definite interaction between K_{IC} and K_{IIC} and concluded the empirical criterion was the most adequate.

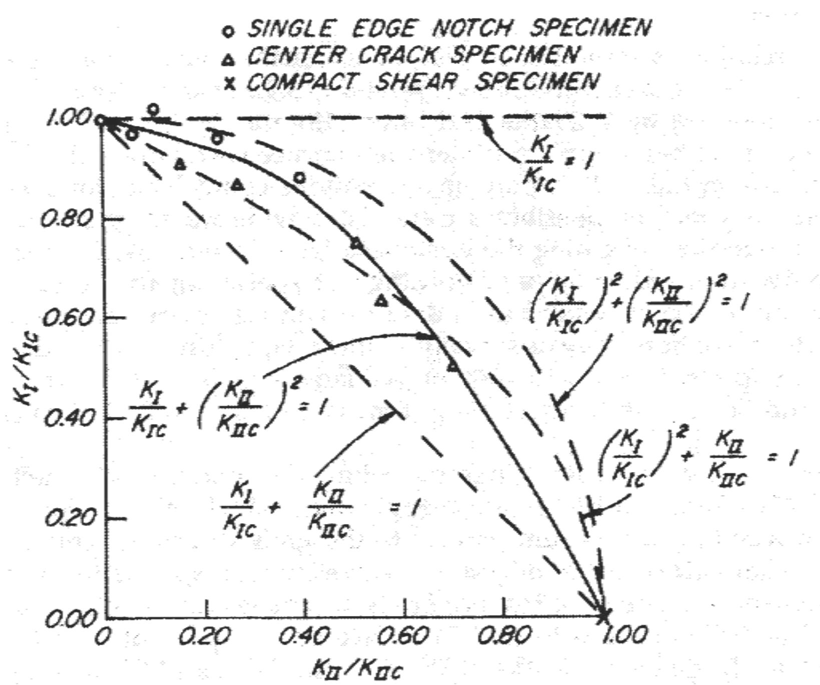

Figure 14: Mixed mode fracture criteria for cracks oriented along the grain (TL system) for Eastern Red Spruce [10]

Figure 15 and Figure 16 presents a comparison between three criteria: Critical Energy Release Rate, Critical Strain Energy Density and Critical In-Plane Maximum Principal Stress. The authors show that the criterion based on the maximum principal stress is better suited for predicting mixed mode fracture than the two energy criteria. They also present a conceptual model for mixed mode crack growth along the grain based on this conclusion.

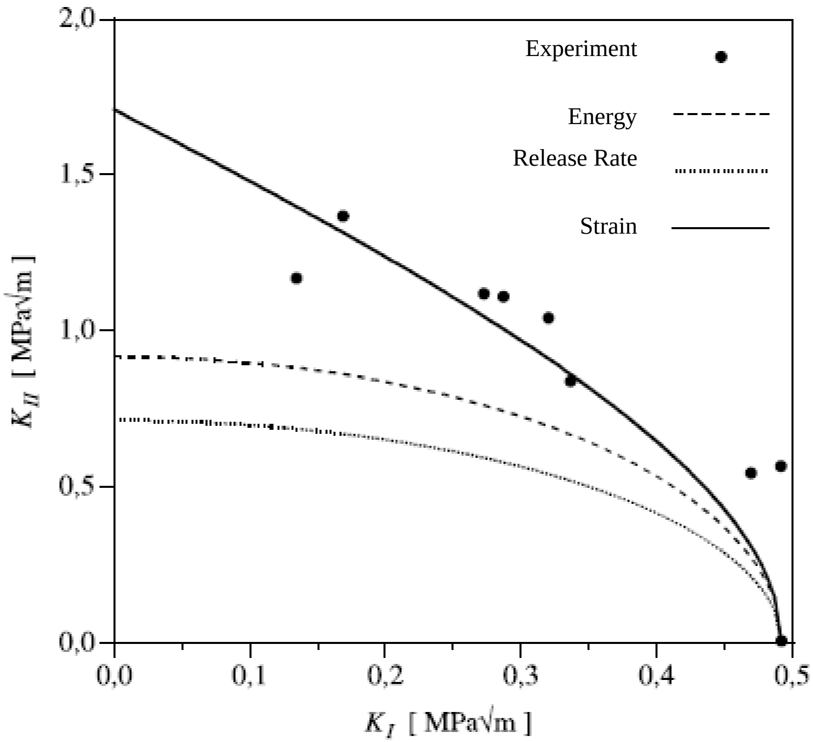

Figure 15: Mixed mode fracture criteria for cracks oriented along the grain (RL system) for Scots pine [3]

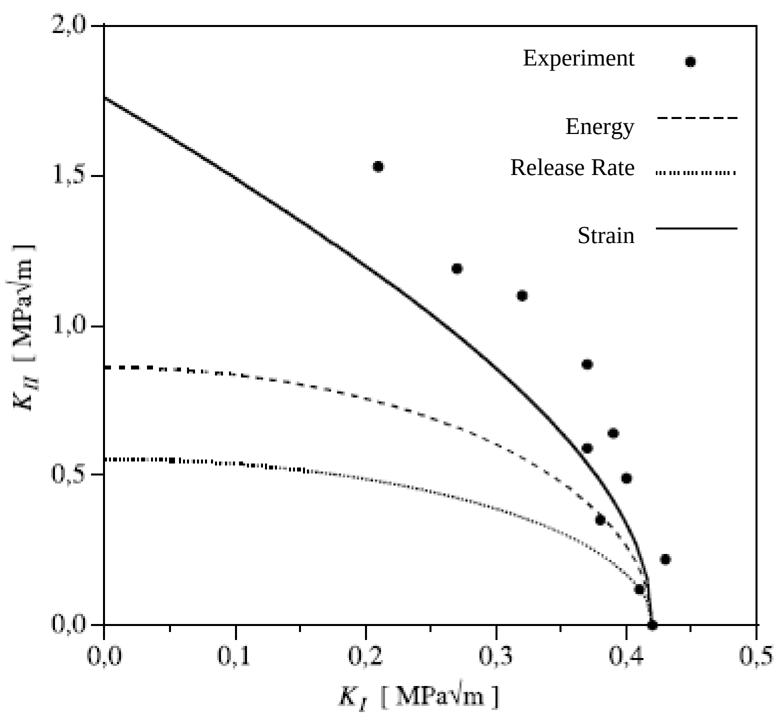

Figure 16: Mixed mode fracture criteria for cracks oriented along the grain (TL system) for Eastern Red Spruce [3]

Figure 17 compares the non-local stress fracture criterion with the energy based criteria and the critical in-plane maximum principal stress criterion. The authors conclude their proposed criterion predicts well the failure load for cracks oriented along the grain, but the real advantage of using this criterion is obtained for cracks arbitrarily oriented (discussed later in this text).

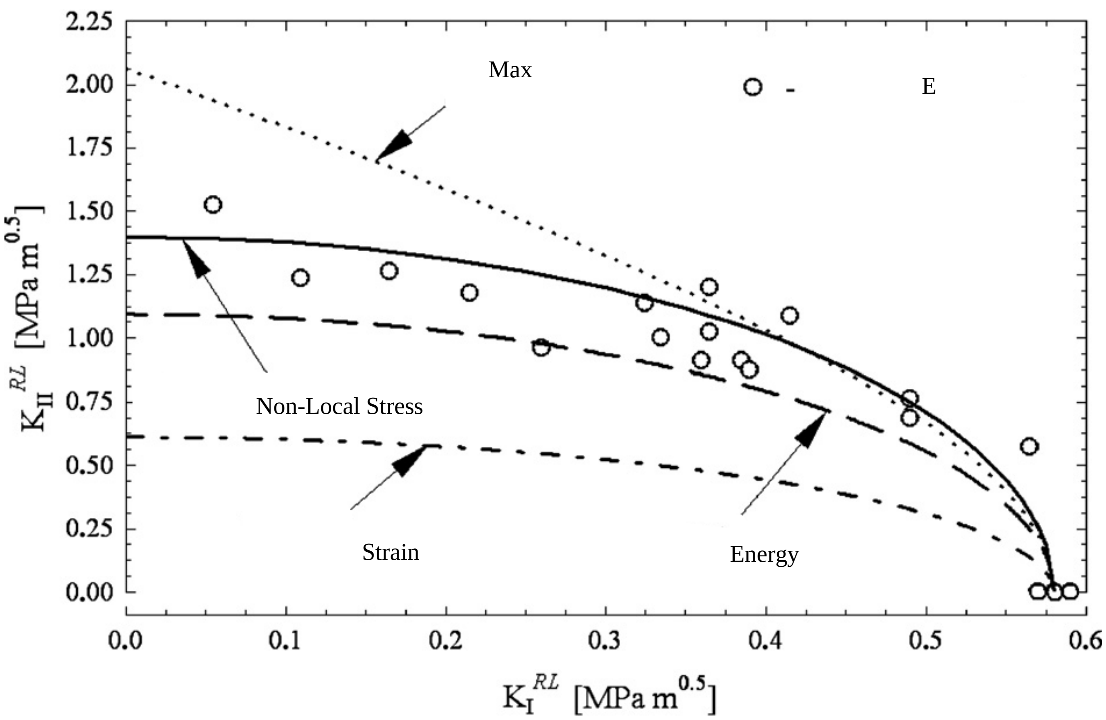

Figure 17: Mixed mode fracture criteria for cracks oriented along the grain (RL system) for Norway Spruce [4]

Remark: the energy release rate and the strain energy density curves shown in Figure 17 are defined differently: there is a term proportional to $K_I * K_{II}$ in the definition of strain energy release rate and strain energy density. However, this does not change the discussion and the conclusions presented here.

9. Criterion for cracks oriented across the grain

As mentioned before, crack growth at a cross-grain notch is known to take place along the grain, and the crack thus deviates perpendicularly from the original notch orientation. The assumption then is that the deviation takes place via a sharp kink, which grows along the grain when the toughness at the tip of the kink is exceeded.

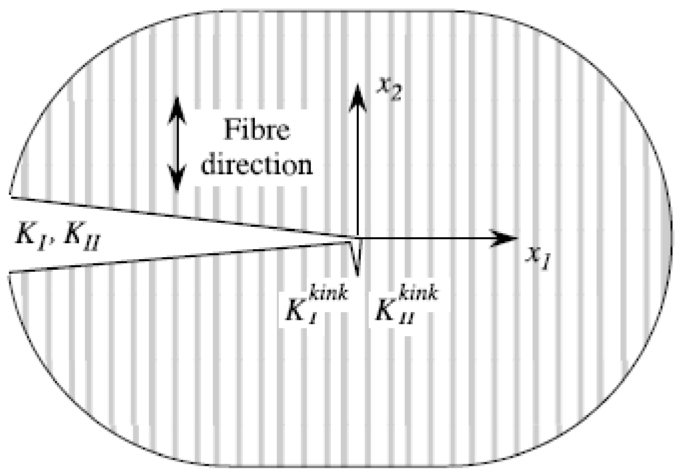

Figure 18: Crack oriented across the grain [3]

K_I^{kink} and K_{II}^{kink} are the local stress intensities at the tip of the kink. Since the kink is aligned with the grain, the same fracture criteria can be used, provided that K_I^{kink} and K_{II}^{kink} can be calculated. They can be expressed in terms of a linear combination of the stress intensity factors for the main crack:

$$K_I^{kink} = \alpha_{11} K_I + \alpha_{12} K_{II}$$

$$K_{II}^{kink} = \alpha_{21} K_I + \alpha_{22} K_{II}$$

The coefficients α_{ij} depend on the terms of the elastic compliance matrix [3].

Crack initiation is then predicted inserting K_I^{kink} and K_{II}^{kink} into the fracture criteria discussed for cracks oriented along the grain.

Note: The applicability of this criterion is restricted to configurations in which the T-stress, the non-singular stress, is very small.

The Critical In-Plane Maximum Principal Stress criterion was used for crack across the grain [12]. The authors state that the criterion is in fair agreement with the experimental data from the SENT and SENB test specimens. For the DCB tests, the divergence is explained by considering the influence of the crack tip T-stress.

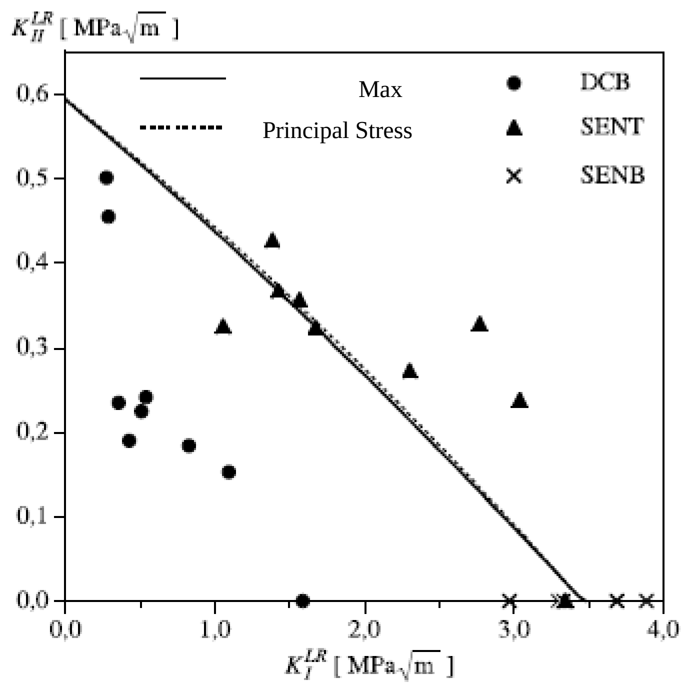

Figure 19: Mixed mode fracture in the LR system for Norway Spruce [11]

10. Criterion for cracks arbitrarily oriented

The Non-Local Stress Fracture Criterion for cracks arbitrarily oriented is written as:

$$\lambda_{11} {K_I}^2 + \lambda_{12} K_I K_{II} + \lambda_{22} {K_{II}}^2 = {K_{IC}}^2$$

Where the coefficients λ_{11}, λ_{12}, and λ_{22} are trigonometrical functions of the crack inclination angle [4].

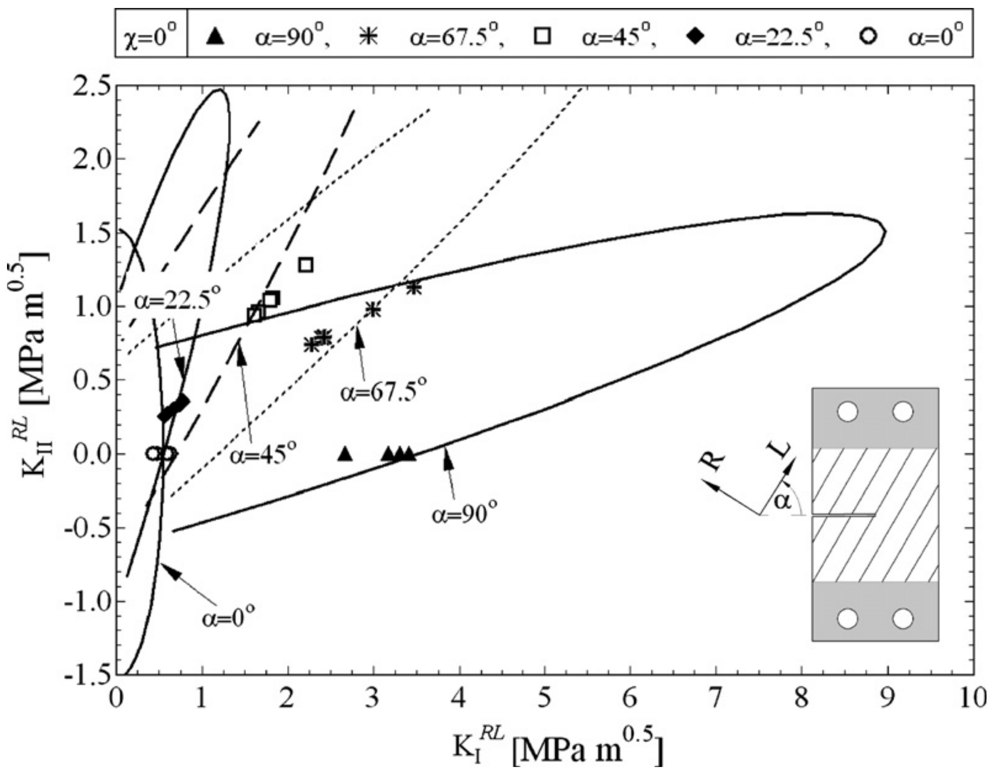

Figure 20: Mixed mode limiting fracture curves for cracks arbitrarily oriented propagating in the RL system [4]

Figure 20 shows the criterion compared to experimental data obtained with Pine wood (*Pinus silvestris*). The Non-Local Stress Fracture Criterion provides good results for all crack orientation angles.

11. Conclusions

Cracks oriented along the grain

Empirical criteria require a large number of material constants determined for each crack configuration. The energy based criteria don't predict well the mixed mode fracture in wood. The Critical In-Plane Maximum Principal Stress criterion [3] provides good estimates for low K_{II}/K_I rations. The Non-Local Stress Fracture Criterion [4] provides good predictions of the mixed mode fracture in wood.

Cracks oriented across the grain

Crack initiation is predicted inserting K_I^{kink} and K_{II}^{kink} into the fracture criteria discussed for cracks oriented along the grain. The Critical In-Plane Maximum Principal Stress criterion [3] provides good estimates as long as the T-stress is low.

Cracks arbitrarily oriented

It is important to note the need to include an additional term with the product K_I*K_{II}. The Non-Local Stress Fracture Criterion [4] provides the best prediction of the mixed mode fracture in wood for cracks arbitrarily oriented.

LEFM

Applications of LEFM theory to fracture of wood have been significant and developments have been useful but it should be noted that LEFM does not account for all physical phenomena associated with wood fracture. There are issues such as geometry and rate dependencies of the measured toughness. To address the drawbacks, researches started applying nonlinear fracture mechanics methods to fracture processes in wood in the late 1980s [5].

12. References

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