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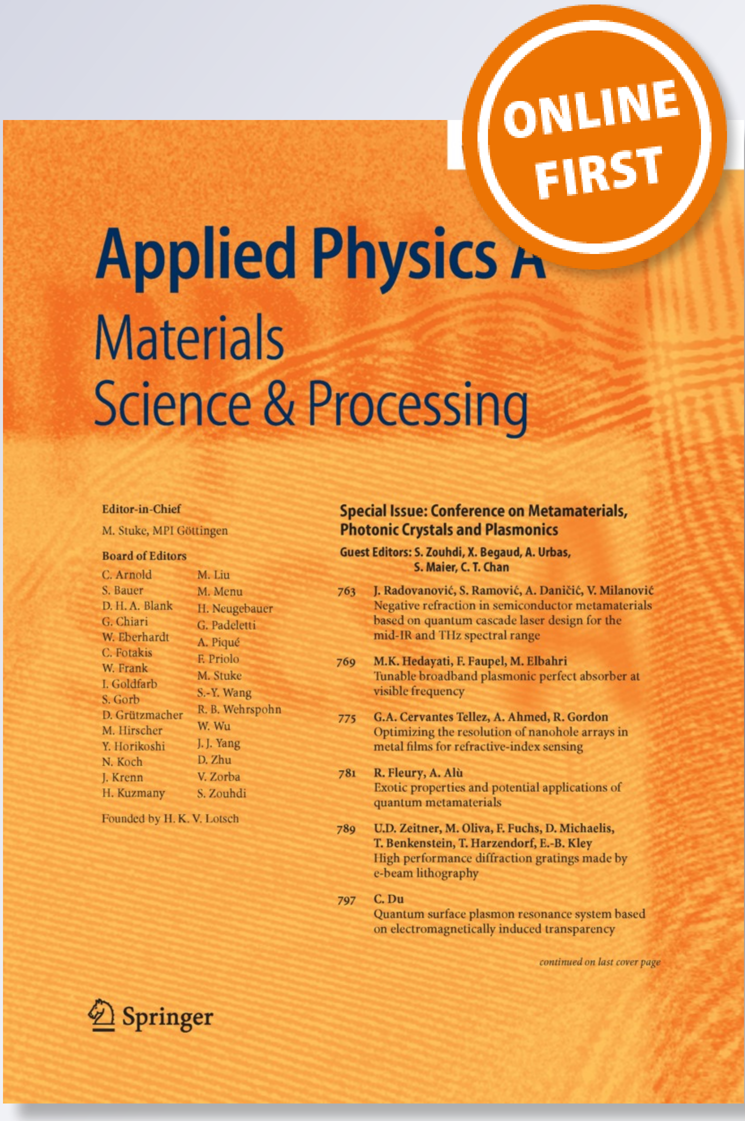

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RAPID COMMUNICATION

Modeling of the muscle-like actuation in soft dielectrics: deformation mode and electromechanical stability

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Abstract Soft dielectric elastomer is able to generate an electromechanical response in terms of reversible shape changing, which is a muscle-like behavior. The deformation and electromechanical stability of dielectric elastomers, classified by their deformation modes, uniaxial extension, equal biaxial expansion and pure shear, are investigated. Pull-in instability occurs in equal biaxial and uniaxial modes at a small stretch ratio, while the pure shear mode features wrinkling instability after a large stable deformation. The coupled stiffness of the voltage-activated material is established and studied for achieving the goal of high performance stretchable dielectric actuators.

Soft dielectric, or dielectric elastomer, is capable of large deformation over 100 % subject to an electric field, usually from a voltage-supply. Due to the fast, electrically activated strain, which is comparable to that of biological muscles, the dielectric elastomer-based actuators (DEAs) are considered to provide opportunities for biomimetic functions. This feature has been exploited in diverse devices, including soft robots, wearable human–machine interface, and artificial limbs [1–6].

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Among these applications, lifting weights is one of the tasks for the DEA to mimic the behavior of natural contractile muscles [6]. In the last decade, experimental investigations have been carried out to characterize the muscle-like deformation in DEAs of various shapes [7–10]. In theoretical studies, however, most efforts were devoted to modeling the equal biaxial deformation of DEAs in the configuration of either a planar circular plate, or a tube-shaped cylindrical shell [11, 12]. In these cases, the maximum deformation is limited by the pull-in instability [1, 3]. Pull-in instability can be understood as follows: When the applied voltage exceeds a critical level, the electrostatic stress due to the voltage prevails in the elastic stress due to the deformation of the elastomer. Therefore, the elastomer membrane becomes even thinner, leading to an increase in the electric field since the voltage is kept constant, which in turn further compresses the membrane. The significant growth of the electrostatic stress causes the film to expand drastically without the balance of elastic stress, until the electrical breakdown [12]. It is worth noting that for DEAs in applications, their deformation modes also include uniaxial extension and pure shear, according to the material property [13, 14]. Pull-in was frequently observed in the equal biaxial mode, but in other modes, the stability behavior is different. For example, in one experiment, a piece of dielectric sheet was lifted and subject to a weight (46 g) in tension. Under a voltage-produced electric field in the thickness direction, the DEA of uniaxial deformation is realized, withstanding voltages up to 10 000 V at strain <5 % [15], as Fig. 1(c) sketches. In another experiment, constrained by mechanical confinements at the top and bottom boundary, Fig. 1(d), the dielectric membrane deforms only in two directions, and this mode of deformation is defined as pure shear [14]. DEA in the pure shear-mode features a large strain >200 % without pull-in, as its unique advantage is that it offers a new

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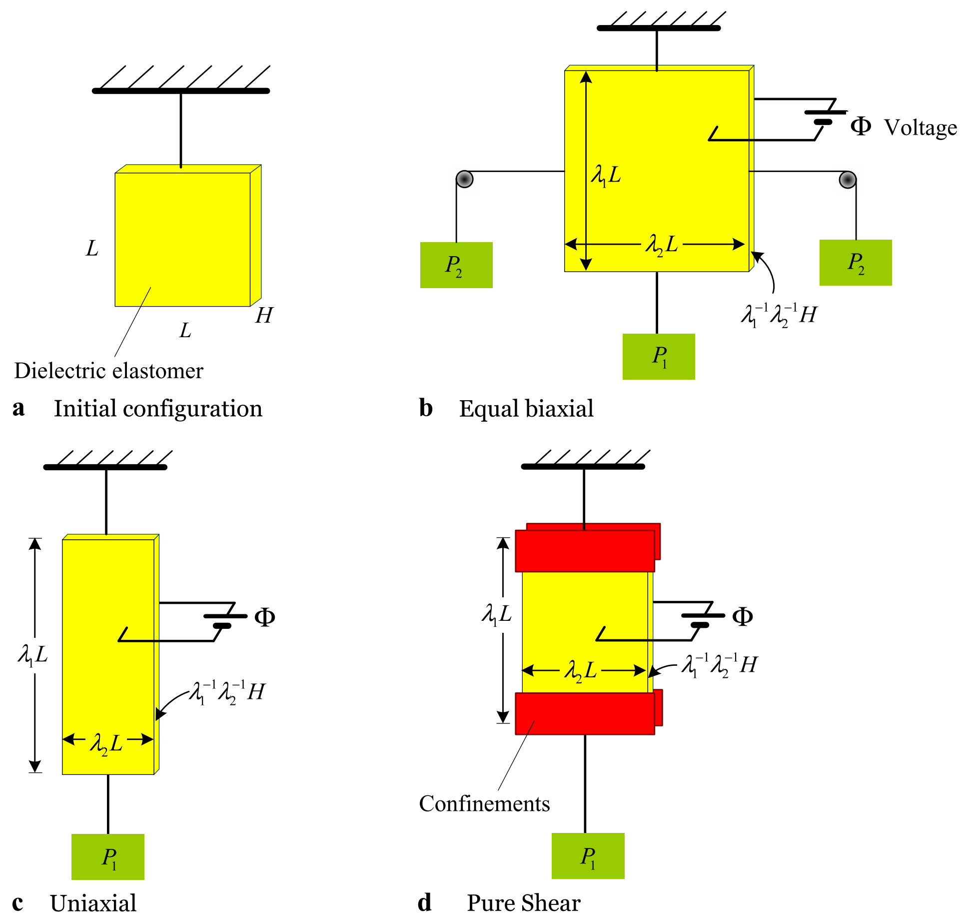

Fig. 1 Subject to mechanical weights and a voltage, a dielectric elastomer-based actuator deforms from (**a**) its original dimension $L \times L \times H$ to the current dimension $\lambda_1 L \times \lambda_2 L \times \lambda_1^{-1}\lambda_2^{-1} H$. According to the configurations of the actuator, the deformation modes are classified as: (**b**) equal biaxial extension, (**c**) uniaxial extension, and (**d**) pure shear

route for achieving giant deformation [16–19]. In different modes of deformation, DEAs exhibit a difference in stability behaviors. Nevertheless, in theoretical investigations, specific models and conclusions, concerning the conspicuous discrepancy, have not been established and fails to provide guidelines for DEAs of an upgraded high-performance. Therefore, in the current study, we probe the physical mechanisms in the coupled stability of DEA, under the modes of deformation. Using the thermodynamics-based model, we investigate the mechanism of stability for each mode of deformation.

As sketched in Fig. 1, different modes of deformation in DEAs can be achieved in the design of the actuators, all capable of elevating the weight under voltage Φ. The uniaxial elongation is realized in terms of a unidirectional extension by a single mechanical load P_1; while the biaxial expansion is attainable with an extra pair of load P_2. With confinements in the horizontal direction, DEA of pure shear deformation is obtained as well [16–19]. In order to study the nonlinear deformation in electromechanical coupling, we first establish the governing equations.

Without losing generality, we assume the dielectric deforms from a reference state of dimensions $L \times L \times H$ to the current state of dimensions $\lambda_1 L \times \lambda_2 L \times \lambda_1^{-1}\lambda_2^{-1} H$, under mechanical weights P_1 and P_2, as well as a voltage Φ. Here, λ_1 and λ_2 denote the stretch ratios of the two in-plane directions. The elastomer is taken to be incompressible, and consequently the stretch in the thickness is represented by $\lambda_1^{-1}\lambda_2^{-1}$.

Adopting the strain energy W_{stretch} function and the linear polarization form, we express the governing equations, according to the theory of dielectric elastomer [20]:

$$\sigma_1 = \lambda_1 \frac{\partial W_{\mathrm{stretch}}}{\partial \lambda_1} - \varepsilon E^2; \tag{1}$$

$$\sigma_2 = \lambda_2 \frac{\partial W_{\mathrm{stretch}}}{\partial \lambda_2} - \varepsilon E^2, \tag{2}$$

where ε is the permittivity, $\sigma_1 = \lambda_1 P_1/(LH)$ and $\sigma_2 = \lambda_2 P_2/(LH)$ are the true stresses, and $E = \Phi/(\lambda_1^{-1}\lambda_2^{-1}H)$ is the true electrical field at the deformed state. The prestretch by the mechanical weight would evidently modify the material property *via* strain-stiffening due to the finite contour length of polymer chains. A detailed investigation has been reported very recently using the Gent model to characterize the nonlinearity in elasticity [21]. In this study, we focus only on the fundamental mechanisms of electromechanical coupling in each deformation mode, ruling out the effect of prestretch. Thereby, we maintain the mechanical loads at a level that are sufficiently small to avoid

Table 1 The stretches and stresses in three modes of deformation

	Weight	Stretch
Equal biaxial	$P_1 = P_2 = P$	$\lambda_1 = \lambda_2 = \lambda_{EB}$
Uniaxial	$P_1 = P$, $P_2 = 0$	$\lambda_1 = \lambda_{UA}$, $\lambda_2 = \lambda_{UA2}$
Pure shear	$P_1 = P$	$\lambda_1 = \lambda_{PS}$, $\lambda_2 = 1$

stiffening. In this study, the Neo–Hookean model, $W_{\text{stretch}} = \frac{1}{2}\mu(\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2} - 3)$, is used to describe the hyperelasticity, with μ being the shear modulus.

For each deformation mode, Table 1 lists the geometrical relations of the stretches with a unified applied stress σ in vertical direction. Thus, we solve Eqs. (1) and (2), and then express the equations of state as:

$$\sigma_{EB} = \frac{P}{LH}\lambda_{EB} = \mu\left(\lambda_{EB}^2 - \lambda_{EB}^{-4}\right) - \varepsilon\left(\frac{\Phi_{EB}}{H}\right)^2 \lambda_{EB}^4, \tag{3}$$

$$\sigma_{UA} = \frac{P}{LH}\lambda_{UA} = \mu\left(\lambda_{UA}^2 - \lambda_{UA}^{-2}\lambda_{UA2}^{-2}\right) - \varepsilon\left(\frac{\Phi_{UA}}{H}\right)^2 \lambda_{UA}^2\lambda_{UA2}^2, \tag{4a}$$

$$0 = \mu\left(\lambda_{UA2}^2 - \lambda_{UA}^{-2}\lambda_{UA2}^{-2}\right) - \varepsilon\left(\frac{\Phi_{UA}}{H}\right)^2 \lambda_{UA}^2\lambda_{UA2}^2, \tag{4b}$$

$$\sigma_{PS} = \frac{P}{LH}\lambda_{PS} = \left(\lambda_{PS}^2 - \lambda_{PS}^{-2}\right) - \varepsilon\left(\frac{\Phi_{PS}}{H}\right)^2 \lambda_{PS}^2. \tag{5}$$

In Eqs. (3)–(5), the subscripts (EB, UA, and PS), represent equal biaxial expansion, uniaxial extension, and pure shear, respectively. Note here, (I) we only present the stretch λ_1 in vertical direction, to study the muscle-like effect of lifting weight; (II) in order to obtain λ_{UA}, one has to combine Eqs. (4a) and (4b) to solve λ_{UA2} as $\lambda_{UA2} = \sqrt{\lambda_{UA}^2 - \lambda_{UA}P/(\mu LH)}$.

We next employ the definition of coupled stiffness Y as [22]

$$Y = \frac{\partial\sigma}{\partial\lambda}, \tag{6}$$

to study the stability in the electromechanical coupling of DEA. Inspecting the governing equations, we apply Eqs. (6) to (3)–(5) by solving the voltage, and obtain:

$$Y_{EB} = \mu\left(-2\lambda_{EB} + 8\lambda_{EB}^{-5}\right) + 4\frac{P}{LH}, \tag{7}$$

$$\begin{aligned} Y_{UA} = \mu\Bigg(&\lambda_{UA} + \lambda_{UA2} - \lambda_{UA}^{-1}\lambda_{UA2}^2 - \lambda_{UA}^2\lambda_{UA2}^{-1} \\ &+ 4\lambda_{UA}^{-3}\lambda_{UA2}^{-2} + 2\lambda_{UA}^{-2}\lambda_{UA2}^{-3} \\ &\times\left(1 + \frac{2\lambda_{UA} - P/(\mu LH)}{2\sqrt{\lambda_{UA}^2 - \lambda_{UA}P/(\mu LH)}}\right)\Bigg) \\ &+ \frac{P}{LH}\left(1 + \lambda_{UA}\lambda_{UA2}^{-1}\right), \end{aligned} \tag{8}$$

$$Y_{PS} = \mu\left(4\lambda_{PS}^{-3}\right) - 2\frac{P}{LH}, \tag{9}$$

as the coupled stiffness of dielectric elastomer at the equilibrium state, with respect to each mode of deformation.

In the succeeding analyses, a small mechanical load ($P/(LH\mu) = 0.1$) is prescribed to maintain the DEA in tension to prevent buckling. Figure 2 plots the voltage *vs.* deformation curves using the normalized variables: $\Phi/(H\sqrt{\mu/\varepsilon})$ and λ, by solving Eqs. (3)–(5). The coupled stiffness is plotted correspondingly as well, when the material is electromechanically in equilibrium.

In the equal biaxial expansion, shown in Fig. 2(a), the pull-in instability occurs when the deformation curve reaches a local peak, denoted by $\Phi_{EB}/(H\sqrt{\mu/\varepsilon}) = 0.65$ as the critical voltage of instability. During the pull-in, the material thins down, which eventually causes an electrical breakdown failure due to the positive feedback in the coupled behavior [1]. Owing to the area expansion, the electrostatic stress grows proportionally with a quartic power of λ_{EB}, as the last item in the right-hand side of Eq. (3), $-\varepsilon(\Phi_{EB}/H)^2\lambda_{EB}^4$, indicates. Pull-in induces a remarkable compressive stress, denoted by the negative sign in Eq. (3), even at a small level of voltage, and leads to failure in consequence.

In Fig. 2(b), we illustrate the development of coupled stiffness with stretch. The line of $Y_{EB}/\mu = 0$ separates the diagram into two regions: unstable ($Y_{EB}/\mu < 0$) and stable ($Y_{EB}/\mu > 0$) regions. At the critical voltage of pull-in, the material expands with an entrance from stable to an unstable deformation state, when its coupled stiffness drops to zero. Subject to voltages exceeding the critical value, the dielectric elastomer fails to approach equilibrium, while under a voltage below the critical level, i.e., $\Phi_{EB}/(H\sqrt{\mu/\varepsilon}) = 0.5$ in Figs. 2(a) and 2(b), two equilibrium states are observed at stretch ratios: λ' and λ''. λ' corresponds to the stable deformation with a positive stiffness, while λ'' is unstable where the stiffness is negative. The negative coupled stiffness enables the elastomer great compliance, and results in the instability. Therefore, the stable deformation in this mode is attainable only if the coupled stiffness is positive.

Similar behaviors are displayed when the DEA undergoes uniaxial deformation, as can be observed in Figs. 2(c) and 2(d). Comparing Figs. 2(a) and 2(c), though pull-in is triggered in the uniaxial mode as well, the critical voltage is higher than that in the equal biaxial mode. In experiments of DEA in the uniaxial mode, the voltage was raised to 10 000 V to activate the deformation, which was only at a 5 % strain level. While in the equal biaxial mode, 3000 V of applied voltage could produce strain as large as 26 % [15, 16]. This difference is ascribed to the anisotropy of the elastomer in the uniaxial mode: elongation vertically and shrinking horizontally. This characteristic would be greatly amplified by the prestretch, as a conclusion in the reported discussion [21].

However, in the pure shear deformation, the dielectric elastomer exhibits a fluid-like deformation without pull-in.

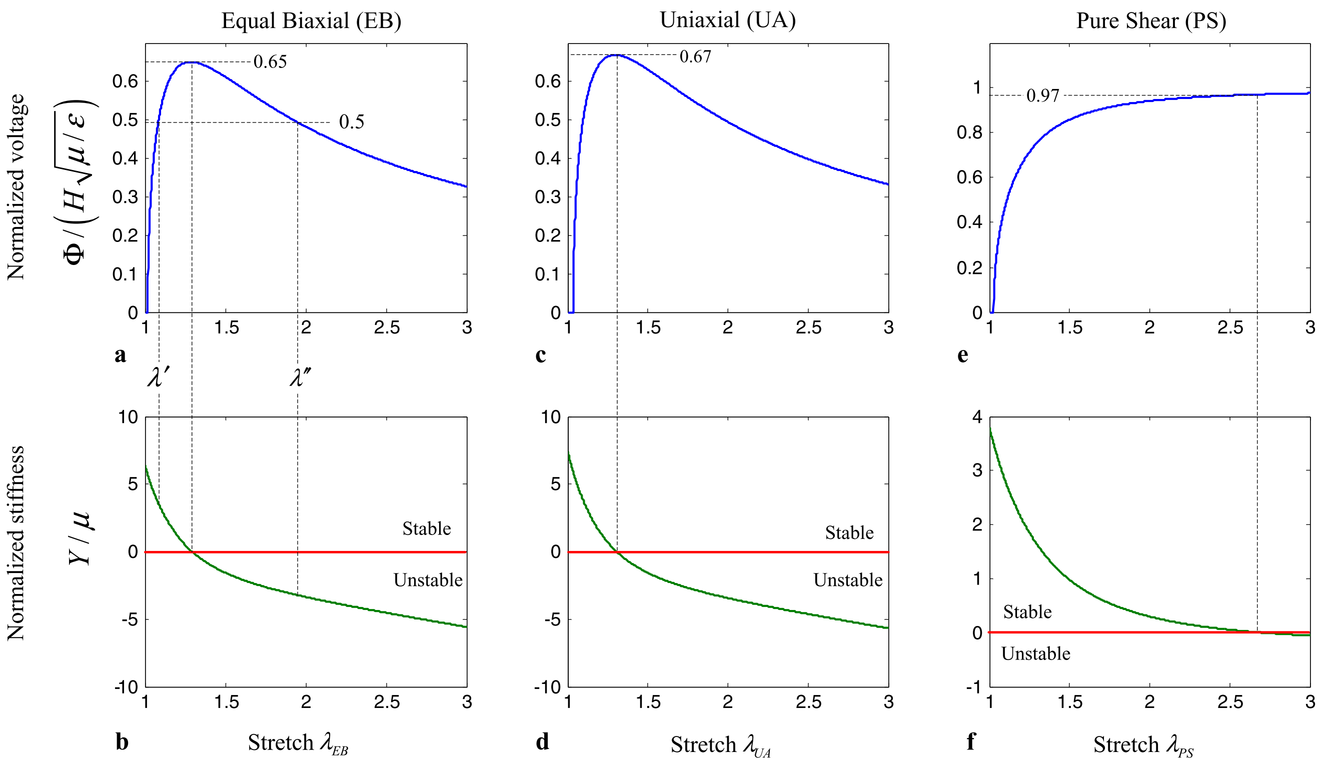

Fig. 2 Electromechanical actuation stretch and coupled stiffness of DEA, under different modes of deformation (column from left to right): equal biaxial, uniaxial, and pure shear

In Fig. 2(e), before the voltage-stretch curve approaching the plateau, denoted by $\Phi_{\mathrm{PS}}/(H\sqrt{\mu/\varepsilon}) = 0.97$, the material extends continuously without the request of an appreciable increase in voltage. This phenomenon resembles the property of dielectric liquid [23], and indicates the DEA as a giant deformation in a pure shear mode. In the experiment, a membrane of acrylic dielectric elastomer was fixed to rigid clamps and pulled vertically by a mechanical load. Subsequently, a voltage was applied to the dielectric elastomer [18]. A giant deformation was obtained in the vertical stretch over 3, which agrees with the prediction in the developed model: a large extension stretch of 2.6 before the plateau. The discrepancy between the theoretical and experimental results can be attributed to the different levels of mechanical load. Elastomer exhibits fluid-like deformation when its number of crosslink is low. The average length of polymer networks determines the maximum achievable stretch. Drastic thinning down in the pull-in instability hinders the stable extension of dielectric elastomer in the uniaxial and equal biaxial modes. In the pure shear mode, the confinements prevent the membrane from ceaseless thinning down, and consequently maintain the stable stretchability of the polymer.

In Fig. 2(f), at the voltage of $\Phi_{\mathrm{PS}}/(H\sqrt{\mu/\varepsilon}) = 0.97$, the coupled stiffness drops to $Y_{\mathrm{PS}}/\mu = 0$ at an associated stretch, which is located at the onset of the wrinkling instability. During the deformation in the pure shear mode, the dielectric extends in a large deformation state, where the confinement might fail to sustain the material in tension, generally the central part in the vertical direction. Due to the loss of tension, the material wrinkles as a result of mismatch in stresses. Wrinkling instability in soft membranes could potentially be used as a technique for pattern generation on polymer surfaces [24, 25]. Using the developed model, one can predict the appearance of wrinkle for a controllable surface pattern.

In summary, we studied the electromechanical stability of dielectric elastomer actuator at three modes of deformation: uniaxial extension, equal biaxial expansion, and pure shear. The pull-in instability occurs in equal biaxial and uniaxial modes at a small actuation stretch, though differs at each critical voltage, while the pure shear mode is able to produce a giant deformation before the wrinkling instability. The coupled electromechanical stiffness is defined and investigated. Stable deformation is attainable only if the material has a positive coupled stiffness. These conclusions provide the criteria and insights for soft dielectric based-actuators in biomimetic applications.

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