**MACE 11010: Engineering Mechanics**

Lecture 2, Equilibrium of Many-Particle System: Force

Lecture 3, Equilibrium of Many-Particle System: Moment

5/10, Friday

**Some mathematics for N-particle system**

For a system of N particles at equilibrium, applying Newton's 1st law on the *i*-th particle

$$\boldsymbol{F}_i + \sum_{j \neq i} \boldsymbol{f}_{ij} = \boldsymbol{0}, \tag{1}$$

where $i = 1, 2, \cdots, N$, $\boldsymbol{F}_i$ is the external force acted on the *i*-th particle, $\boldsymbol{f}_{ij}$ is the internal force acted on particle *i* from particle *j*, $\sum_{j \neq i} \boldsymbol{f}_{ij}$ means the summation for all the particle *j* except when $j = i$.

Summation of the above equation over all particles, $i = 1, 2, \cdots, N$, gives

$$\sum_i \boldsymbol{F}_i + \sum_i \sum_{j \neq i} \boldsymbol{f}_{ij} = \boldsymbol{0}, \tag{2}$$

From Newton's $3^{rd}$ law, it can be proved (can you prove?) that

$$\sum_i \sum_{j \neq i} \boldsymbol{f}_{ij} = \boldsymbol{0}, \tag{3}$$

Therefore, from Eq. (2), one has

$$\sum_i \boldsymbol{F}_i = \boldsymbol{0}, \tag{4}$$

which means for a system of N particles at equilibrium, the resultant force $\sum_i \boldsymbol{F}_i$ (a vector) equals $\boldsymbol{0}$.

Further, choose a fixed point $\boldsymbol{O}$ in the space, the position of particle *i* to point $\boldsymbol{O}$ is denoted as $\boldsymbol{r}_i$. The cross product of $\boldsymbol{r}_i$ to equation (1) for each particle *i* gives

$$\boldsymbol{r}_i \times \boldsymbol{F}_i + \sum_{j \neq i} \boldsymbol{r}_i \times \boldsymbol{f}_{ij} = \boldsymbol{0} . \tag{5}$$

Summation of the above equation over all particles, $i = 1, 2, \cdots, N$, gives

$$\sum_i \boldsymbol{r}_i \times \boldsymbol{F}_i + \sum_i \sum_{j \neq i} \boldsymbol{r}_i \times \boldsymbol{f}_{ij} = \boldsymbol{0} . \tag{6}$$

From Newton's 3rd law, it can be proved (can you prove?) that

$$\sum_i \sum_{j \neq i} \boldsymbol{r}_i \times \boldsymbol{f}_{ij} = \boldsymbol{0} . \tag{7}$$

Therefore, from Eq. (6), one has

$$\sum_i \boldsymbol{r}_i \times \boldsymbol{F}_i = \boldsymbol{0} , \tag{8}$$

where $\boldsymbol{r}_i \times \boldsymbol{F}_i$, defined as $\boldsymbol{M}_i$, is the moment of the external force on particle *i* to the fixed point ***O***. For a system of N particles at equilibrium, the resultant moment $\sum_i \boldsymbol{M}_i$ (a vector) equals ***0***, i.e.,

$$\sum_i \boldsymbol{M}_i = \boldsymbol{0} , \tag{9}$$

To summary, for a system of N particles at equilibrium,

$$\sum_i \boldsymbol{F}_i = \boldsymbol{0} , \tag{10}$$

$$\sum_i \boldsymbol{r}_i \times \boldsymbol{F}_i = \boldsymbol{0} , \tag{11}$$