Numerical analysis of golf club head and ball at various impact points

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Abstract

This paper investigates how the physical characteristics of a golf club head affect its performance, with the aim of developing a superior club head. The physical characteristics investigated were the magnitude of the moment of inertia and the location of the centre of gravity. The performances were classified as the release velocity, the spin rate of the ball and the size of the uniform restitution area. In the numerical analysis, several kinds of club head were modelled with different moments of inertia and different locations of the centre of gravity. For every club head model, the ball was hit at various impact points and the release velocity, spin rate and size of the uniform restitution area were calculated. The results are as follows. The bigger the moment of inertia I_{zz} about the vertical axis or the shallower the depth of the centre of gravity, the less side spin rate caused. I_{zz} has more influence on the side spin rate than the depth of the centre of gravity. The shallower the depth of the centre of gravity, the more back spin rate caused. The size of the uniform restitution area increases in proportion to the increase in I_{zz}, or in proportion to the decrease of the depth of the centre of gravity.

Keywords: golf club head, numerical analysis, restitution, spin rate, uniform restitution area

Introduction

The golf club is generally designed according to three criteria: the distance of flight, the direction of the flying ball and the 'feel' on impact. The following are studies on the design of golf clubs. Yamaguchi *et al*. (1984) and Iwatsubo *et al*. (1990) proposed the impedance matching condition. They showed that when the natural frequency of the ball matches that of the club head, the restitution coefficient at impact reaches its maximum. Grundy *et al*. (1990) investigated the movement of the club head during impact using a simple two-dimensional model. Friswell *et al*. (1998) improved a finite element model of a golf club by experimental modal analysis. Hashimoto & Nakasuga (1996) examined the restitution of the club head, considering the swing characteristics. Concerning the direction of the flying ball, the spin rate of the ball caused by the torsional deformation of the shaft was analysed by Iwatsubo *et al*. (1991); and concerning the 'feel' on impact, the torsional vibration of the shaft was analysed by Doi & Take (1991) and Jae-Eung (1991). In these studies, the sweet area is defined as the area over which the impact of the

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ball on the club head does not cause discomfort. Iwatsubo *et al.* (1998a) investigated the release velocity, spin rate and trajectory of the ball after impact, at various impact points, for a given club head. The release velocity, the spin rate and the size of the uniform restitution area of the ball are important characteristics in the design of a superior club head. A quantitative analysis of the effect of the characteristics of the club head, such as the moment of inertia and the location of the centre of gravity has, however, never been performed. Iwatsubo *et al.* (1998b) proposed a method of analysing the effect of the moment of inertia and location of the centre of gravity on the club head. The side spin rate after impact was calculated numerically with various impact points, for several kinds of club head, and some results were presented.

In this study, the effects of the characteristics of the club head on its performance were investigated numerically using impact analysis. Several kinds of club head were modelled with different moments of inertia and different locations of the centre of gravity. For every club head model, impact at various impact points was simulated and the release velocity and spin rate were calculated. In addition, the uniform restitution area was defined as the area where the release velocity of the ball was faster than the reference value. The size of the uniform restitution area was calculated for the various club heads. Finally, the correlation between the characteristics of the club head and its performance was investigated. Compared to the previous study (Iwatsubo *et al.* 1998b), more characteristics have been taken into account for a more precise design.

Analysis method

Assumptions

In this study, the analysis was carried out under the following assumptions, to investigate the effects of the characteristics of the club head on its performance.

1. The impact phenomenon can be described as the collision between the ball and the club head, because it is said that the shaft has negligible effect.
2. The effect of the dimples on the ball surface is ignored because the flying ball is not analysed.
3. The ball and the club head are made of linear elastic material, although the ball is actually made of viscoelastic material. This assumption is considered to satisfactorily model the characteristics of the club head.
4. The club head moves horizontally before impact because only the impact phenomenon is analysed.
5. Coulomb's friction law applies on the contact surface between the ball and the club head because the exact mechanism is not known.

Software used in the analysis

The software LUSAS ver.11 was used for the analysis of impact between the ball and the club head.

Coordinate system

Figure 1 shows the co-ordinate system used in the analysis. The origin *o* is the position of the centre of gravity of the ball before impact. The *y*-axis corresponds to the direction of club head motion before impact. The *x*-axis is perpendicular to the *y*-axis, while the *z*-axis completes a right-handed co-ordinate system.

Calculation of release velocity and spin rate

The flight distance and direction of the flying ball are the indices used to estimate the performance of

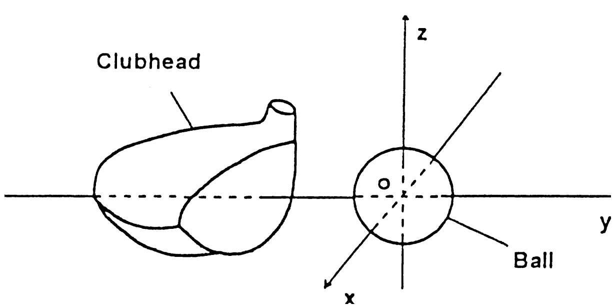

Figure 1 Coordinate system.

the club head. They are influenced by the release velocity and the spin rate of the ball after impact. Nodes in the ball model have different velocity vectors due to the spin after impact. The release velocity is defined as the average of the velocity vectors of all the nodes.

The spin rate is defined as follows: the ball is much deformed during impact. It almost returns, however, to its original spherical shape when it separates from the club head. When the ball is almost spherical in shape, the angular momenta of all elements are calculated with the centre of the ball as the origin. The spin velocities about the x, y, z axes are then calculated from the relationship between the sum of the angular momenta of all elements and the moment of inertia of the ball.

Definition of uniform restitution area

In this study, in order to define the sweet area, the uniform restitution area is introduced as that where the release velocity of the ball is faster than the reference value. The size of the uniform restitution area is calculated as follows:

Impact analysis between the ball and the club head is carried out at the various impact points on the surface where the velocity of the club head is v^h. The magnitude of the release velocity of the ball after impact, v^b, is calculated for every impact point and the ratio between v^h and v^b is calculated as

$$a_i = (v^b)_i / v^h \quad (i = 1, \ldots, n), \tag{1}$$

where n is the number of impact points and a_i is the meet ratio. The maximum value of the meet ratio is set at a_{max}. The area where the meet ratio decreases less than 1.0% compared to a_{max}, is defined as the uniform restitution area.

Results and discussion

Models for analysis

The ball and the club head are modelled using a finite element method.

The two-piece ball, with a diameter of 44 mm and weight of 43 g, shown in Fig. 2(a), is modelled

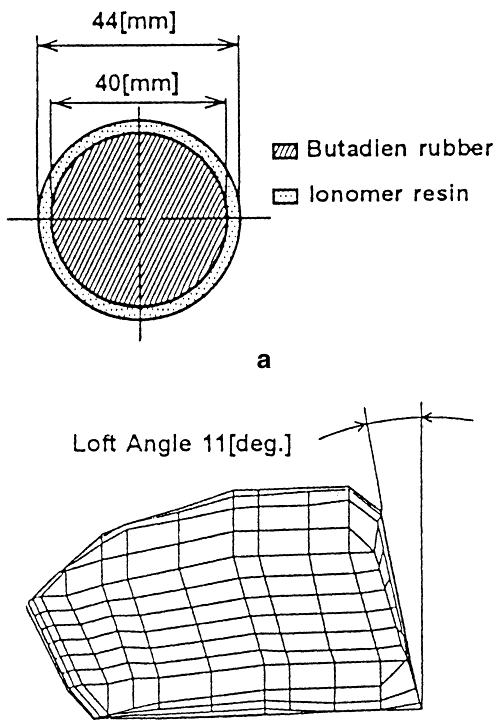

Figure 2 Analytical model. (a) Ball. (b) Club head.

using 840 tetrahedron elements with elastic property. The titanium driver head, whose loft angle is 11°, weight approximately 195 g and radius of curvature of the face shape approximately 370 mm, is modelled using 324 hexahedron elements with elastic property. Figure 2(b) shows a side view of the club head. The mesh pattern shown in Fig. 2(b) is designed to show the relatively accurate first vibration mode shape, as the first mode dominates the deformation. Table 1 shows the material data.

Seven kinds of club head, with different locations of the centre of gravity and different moments of inertia, were made by changing the distribution of the mass as follows: the head model was divided into 14 parts and the thickness of the

Table 1 Material data

Material	Young's Modulus E (N m^{-2})	Poisson's Ratio ν	Density ρ (kg m^{-3})
Butadien rubber	3.92×10^7	0.45	1150
Ionomer resin	2.94×10^8	0.40	950
Ti–6Al–4V	1.18×10^{11}	0.34	4507

Table 2 Specifications of club heads

	I_{zz} (g cm^{-2})	I_{xx} (g cm^{-2})	*Depth* (mm)
Head A	2069	1324	33.7
Head B	2107	1253	26.9
Head C	2025	1333	42.9
Head D	2255	1475	34.1
Head E	2219	1228	27.5
Head F	2170	1234	40.5
Head G	1888	1194	35.3

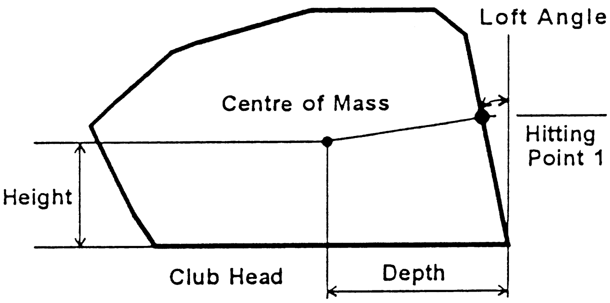

Figure 3 Characteristic terms of the club head.

parts adjusted to model seven kinds of club head with different characteristics. In all cases, the constraints were that the weight of the head model was 195 ± 1 g, and the eccentricity of the centre of gravity was less than 3 mm. Table 2 shows the characteristics of each club head. I_{xx} and I_{zz} are the moments of inertia about the x and z axes, respectively, and the 'Depth' is the depth of the centre of gravity shown in Fig. 3. Head model A is the standard design.

Analysis procedures

In this study, the factors related to the side spin, back spin and those related to the uniform restitution area were investigated.

To analyse these factors, the impact analysis was performed as follows. Firstly, the impact point 1 was set at the nearest node to the intersection point on the face with the perpendicular line from the centre of gravity shown in Fig. 3. Figure 4 shows 10 impact points. The release velocity, the spin rate and the meet ratio were calculated when the club head hit the ball at each impact point and the size of the uniform restitution area was calculated from

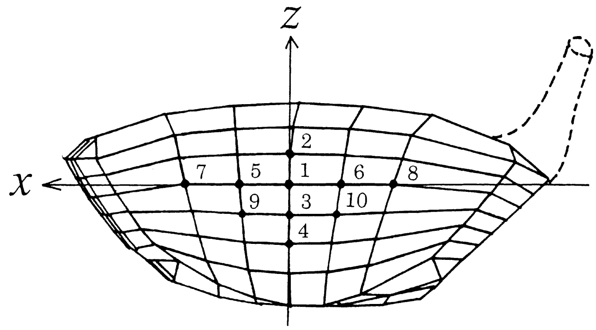

Figure 4 Impact points on the head face.

the meet ratio. The space between each impact point was approximately 7 mm in the vertical direction and approximately 11 mm in the horizontal direction. The velocity of the club head before impact, v^h, was set at 40 m s^{-1}, which is the average velocity of Japanese beginner and mid-level players. The coefficients of static and dynamic friction on the contact surface between the club head and the ball were set as 0.05, which is the estimated value between the ionomer resin body and smooth titanium plate. The time step of calculation is determined by the smallest mesh size and the velocity of wave propagation.

Deformations of head and ball during impact

Using the material data as shown in Table 1, the velocity of the club head at 40 m sec^{-1} and the coefficient of friction 0.05, a typical result is shown in Fig. 5, when the head model A hits the ball. The deformation of club head and ball during the impact can be seen. As shown in this figure, the ball is deformed much more than the head.

Factors related to side spin

The side spin rate of the ball after impact affects the direction of flight of the ball. The effect of I_{zz} and the depth of the centre of gravity on the side spin rate were investigated. To examine the side spin rate, the impact point was varied along the horizontal axis. The heights of the impact points 1, 5, 6, 7 and 8 were almost the same, in spite of the differences in horizontal eccentricities as shown in

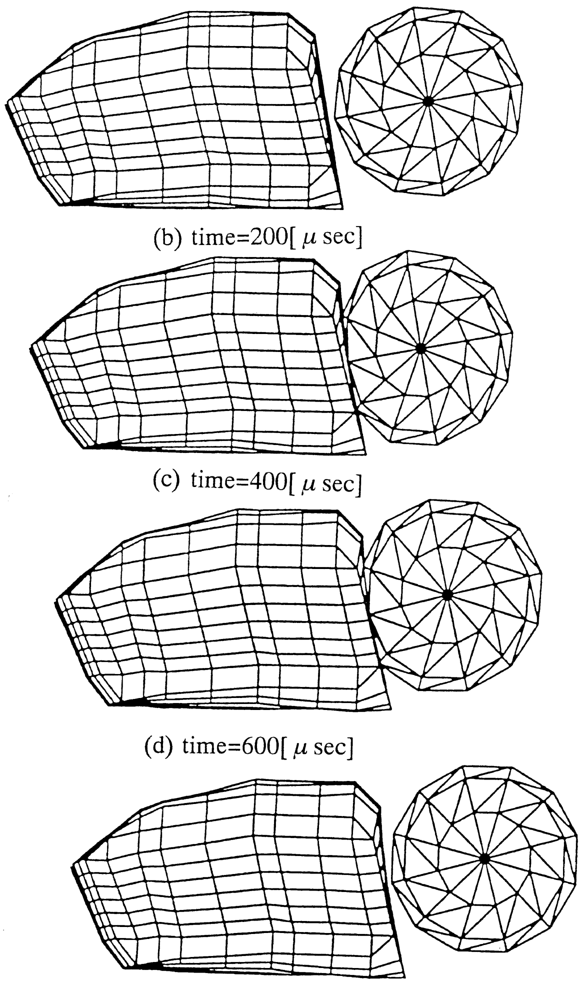

Figure 5 An example of impact analysis.

Fig. 4. Impact analysis was carried out at the impact points 1, 5, 6, 7 and 8.

Firstly, the effect of I_{zz} was investigated. In the case of head models A, D and G, the depths of the centre of gravity are almost the same, in spite of the difference in the moment of inertia. Figure 6(a) shows the side spin rate ω_z vs. the eccentricity of the impact point on the x-axis. In this figure, the impact points 7, 5, 1, 6 and 8 correspond to $x = -22, -12, 0, 12$ and 22 on the horizontal axis, i.e. the eccentricity in the horizontal direction. The direction of the x-axis is the same as that in Fig. 4. It is seen that the bigger I_{zz}, the smaller the magnitude of the side spin rate caused. It is also seen that the side spin rate increases in the case of small eccentricity from the heel side to toe side; it decreases, however, in the case of large eccentricity because of the round effect. Round effect is where the side spin is suppressed by the curved surface of the club head.

The effect of sensitivity on the club head performance is then examined. The sensitivity is an important factor in designing a superior club head. The sensitivity of I_{zz} to ω_z is examined as follows. The change ratio of I_{zz} is calculated as follows:

$$R_{zz} = \frac{(I_{zz})_{\mathrm{D}} - (I_{zz})_{\mathrm{G}}}{(I_{zz})_{\mathrm{A}}} = 0.177, \quad (2)$$

where the head model A is taken to be the reference. The change of ω_z for each head model is calculated as follows:

$$(R)_{\mathrm{i}} = \left\| \frac{(\omega_z)_{\max} - (\omega_z)_{\min}}{(\omega_z)_0} \right\|_{\mathrm{i}} \quad (i = A, D, G), \quad (3)$$

where $(\omega_z)_{\max}$ and $(\omega_z)_{\min}$ are the maximum and minimum ω_z among the results when the impact points are 1, 5, 6, 7 and 8. $(\omega_z)_0$ is the value when the impact point is 1, i.e. $x = 0$. This gives, $(R)_{\mathrm{A}} = 5.31$, $(R)_{\mathrm{D}} = 7.62$ and $(R)_{\mathrm{G}} = 4.04$. The change ratio of ω_z for the three head models is calculated as follows:

$$(R_{\omega z})_{zz} = \frac{(R)_{\mathrm{D}} - (R)_{\mathrm{G}}}{(R)_{\mathrm{A}}} = 0.674. \quad (4)$$

The sensitivity of I_{zz} to ω_z is therefore 3.81 (= 0.674/0.177).

The effect of the depth of the centre of gravity was then investigated. In the case of head models A, B and C, the moments of inertia were almost the same in spite of the differences in depth of the centre of gravity. Figure 6(b) shows the side spin rate ω_z vs. the eccentricity of the impact point on the x-axis. It is seen that the shallower the depth of the centre of gravity, the smaller the magnitude of the side spin rate caused and the smaller the change of side spin rate caused.

The sensitivity of the depth of the centre of gravity to ω_z was examined. Because the trend is

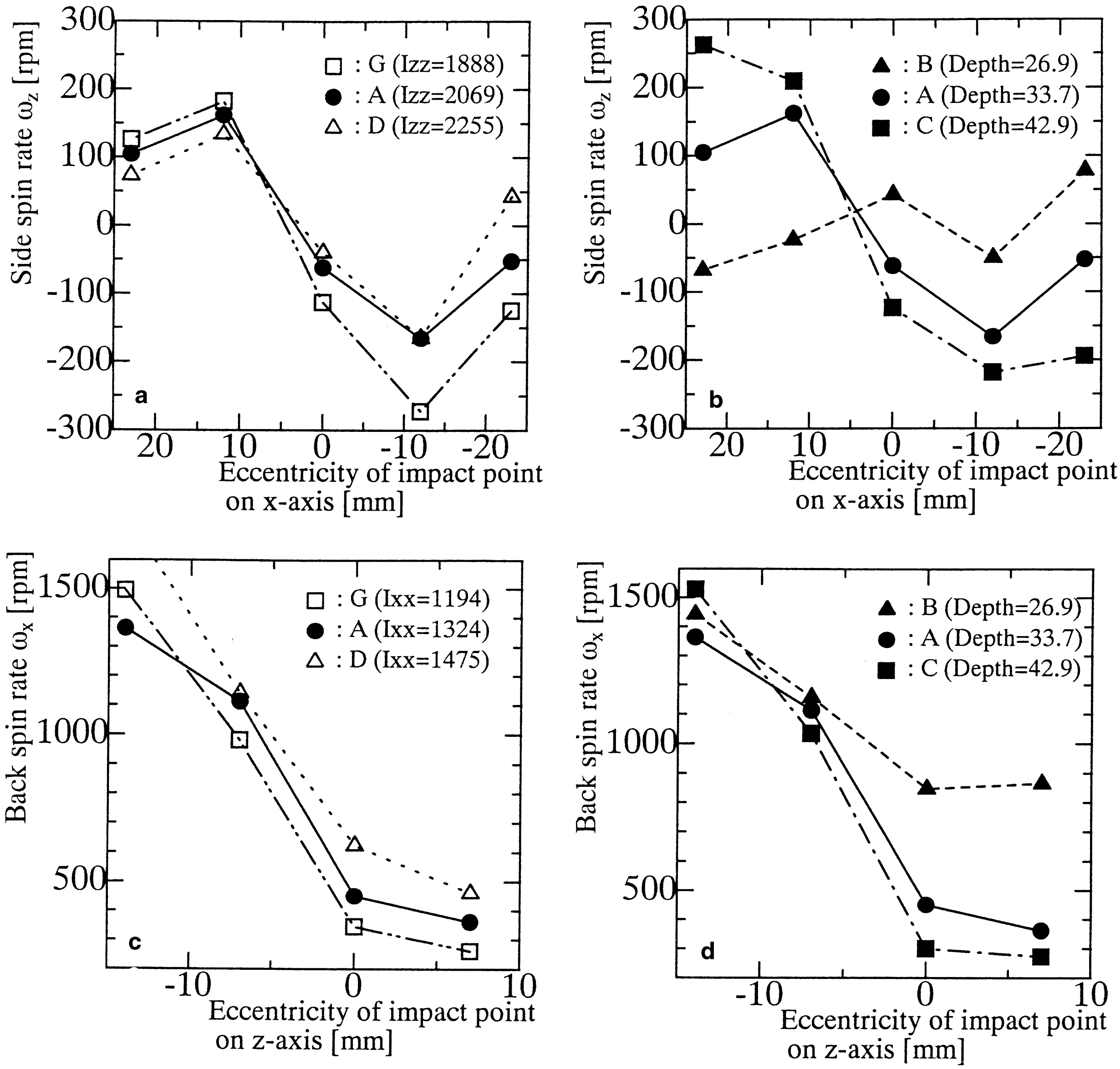

Figure 6 Spin rates for various restitution conditions. (a) Side spin rate for various *Depth*. (b) Side spin rate for various I_{zz}. (c) Back spin rate for various I_{xx}. (d) Back spin rate for various *Depth*.

different for head model B, the sensitivity was calculated using head models A and C. The change ratio of the depth of the centre of gravity was calculated as follows:

$$R_{\mathrm{Dep}} = \frac{(Depth)_{\mathrm{C}} - (Depth)_{\mathrm{A}}}{(Depth)_{\mathrm{A}}} = 0.273. \qquad (5)$$

The change of ω_z for each head model was calculated by using Eq. (3), i.e. $(R)_{\mathrm{A}} = 5.31$ and $(R)_{\mathrm{C}} = 3.90$. The change ratio of ω_z for the three head models is calculated as follows:

$$(R_{\omega z})_{\mathrm{Dep}} = \frac{(R)_{\mathrm{A}} - (R)_{\mathrm{C}}}{(R)_{\mathrm{A}}} = 0.266. \qquad (6)$$

The sensitivity of the depth of the centre of gravity to ω_z is therefore 0.97 (= 0.266/0.273).

It is recognized that the larger the moment of inertia or the shallower the depth of the centre of gravity, the lower the side spin rate caused. It is also recognized that I_{zz} has more influence on the side spin rate than the depth of the centre of gravity.

Factors related to back spin

The back spin rate of the ball after impact affects the distance of the flying ball. The effect of I_{xx} and the depth of the centre of gravity on the side spin rate were investigated. To examine the back spin rate, the impact point was varied along the vertical axis. The vertical eccentricities of the impact points 1, 2, 3 and 4 were different as shown in Fig. 4. Impact analysis is carried out at the impact points 1, 2, 3 and 4.

Firstly, the effect of I_{xx} is investigated. In the case of the head models A, D and G, the depths of the centre of gravity are almost the same in spite of the differences in the moment of inertia. Figure 6(c) shows the back spin rate ω_x vs. the eccentricity of the impact point on the z-axis. In this figure, the impact points 4, 3, 1 and 2 correspond to $z = -14$, -7, 0 and 7 in the vertical axis. It is seen that the larger the I_{xx}, the larger the magnitude of the back spin rate caused. This result is different from what would be expected, according to classical physics laws. A possible reason is thought to be that I_{zz} for the three head models are not the same. It is also seen that the back spin rate decreases according to the elevation of the impact point.

The sensitivity of I_{xx} to ω_x was examined. The change ratio of I_{xx} was calculated as follows:

$$R_{xx} = \frac{(I_{xx})_{\mathrm{D}} - (I_{xx})_{\mathrm{G}}}{(I_{xx})_{\mathrm{A}}} = 0.212. \qquad (7)$$

The change of ω_x for each head model was calculated by using Eq. (3), i.e. $(R)_{\mathrm{A}} = 2.22$, $(R)_{\mathrm{D}} = 2.07$ and $(R)_{\mathrm{G}} = 3.56$. The change ratio of ω_x for the three head models is calculated as follows:

$$(R_{\omega x})_{xx} = \frac{(R)_{\mathrm{G}} - (R)_{\mathrm{D}}}{(R)_{\mathrm{A}}} = 0.671. \qquad (8)$$

The sensitivity of I_{xx} to ω_x is therefore 3.16 (= 0.671/0.212).

Next, the effect of the depth of the centre of gravity was investigated. The head models A, B and C were used. Figure 6(d) shows the back spin rate ω_x vs. the eccentricity of the impact point on the z-axis. It is seen that the shallower the depth of the centre of gravity, the larger the magnitude of the back spin rate caused and the smaller the change in side spin rate caused. It is also seen that the back spin rate decreases according to the elevation of the impact point simulated to the case of I_{xx}.

To examine the sensitivity of the depth of the centre of gravity to ω_x, the change ratio of the depth of the centre of gravity was calculated as follows:

$$R_{\mathrm{Dep}} = \frac{(Depth)_{\mathrm{C}} - (Depth)_{\mathrm{B}}}{(Depth)_{\mathrm{A}}} = 0.475. \qquad (9)$$

The change of ω_x for each head model was calculated by using Eq. (3), i.e. $(R)_{\mathrm{A}} = 2.22$, $(R)_{\mathrm{B}} = 0.703$ and $(R)_{\mathrm{C}} = 4.18$. The change ratio of ω_x for the three head models was calculated as follows:

$$(R_{\omega z})_{\mathrm{Dep}} = \frac{(R)_{\mathrm{C}} - (R)_{\mathrm{B}}}{(R)_{\mathrm{A}}} = 1.566. \qquad (10)$$

The sensitivity of the depth of the centre of gravity to ω_x is therefore 3.29 (= 0.475/1.566).

It was found that the larger the moment of inertia or the shallower the depth of the centre of gravity, the higher the back spin rate caused. It is also found that the depth of the centre of gravity has a greater effect on the back spin rate than I_{xx}. It is noted that the results with respect to I_{xx} are different from physical knowledge.

Factors related to the uniform restitution area

Figure 7 shows the uniform restitution area of head model A, as an example when the meet ratio is set at $0.99 \times a_{\mathrm{max}}$. Table 3 shows the maximum release velocity and meet ratio.

Figure 8(a) and (b) show the width of the uniform restitution area in the x-direction and the size of the uniform restitution area vs. I_{zz}. As shown in the previous section, the larger I_{zz}, the smaller

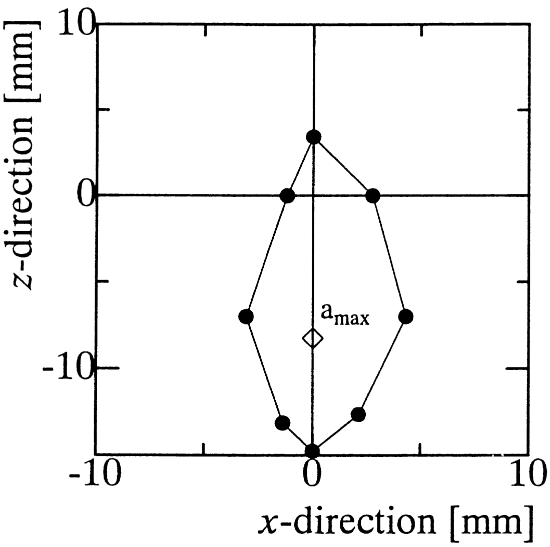

Figure 7 Sweet area of Head A.

Table 3 Maximum release velocity and meet ratio of club heads

	v_{max} (m sec^{-1})	a_{max}
Head A	54.28	1.36
Head B	55.84	1.40
Head C	53.93	1.35
Head D	54.90	1.37
Head E	55.13	1.38
Head F	53.92	1.35
Head G	54.01	1.35

the magnitude of side spin rate caused, so the release velocity increases in proportion to the increase in I_{zz}. The width of the uniform restitution area in the x-direction is therefore wide in proportion to the increase in I_{zz} as shown in Fig. 8(a). The area increases proportionately to the increase in I_{zz} as shown in Fig. 8(b); however, for some head models the area does not vary.

Figure 8(c) and (d) show the width of the uniform restitution area in the z-direction and the size of the uniform restitution area vs. I_{xx}. As shown in the previous section, the larger I_{xx}, the greater the magnitude of back spin rate caused, in spite of the fact that this is contrary to physical knowledge. The release velocity decreases in proportion to the increase of I_{xx}. The width of uniform restitution area in the z-direction therefore narrows in proportion to the increase in I_{xx} as shown in Fig. 8(c). The area also decreases in proportion to the increase in I_{xx} as shown in Fig. 8(d), although there is some scatter. It is noted that the results with respect to I_{xx} are different from physical knowledge.

Figure 8(e), (f) and (g) shows the width of the uniform restitution area in the x- and z-directions and the size of the uniform restitution area according to the depth of the centre of gravity. As shown in the previous section, the effect of the depth of the centre of gravity on the side spin is different from that on back spin. According to the numerical results of this study, the width of the uniform restitution area in the x- and z-directions decreases as the depth of the centre of gravity increases, as shown in Fig. 8(e) and (f). The size of the uniform restitution area also decreases in proportion to the increase in the depth of the centre of gravity as shown in Fig. 8(g).

Conclusions

In this study, the effects of the characteristics of the club head on its performance, such as the moment of inertia and the location of the centre of gravity, have been investigated numerically. Performance is indicated by the spin rate of the ball and the size of the uniform restitution area. The conclusions are as follows.

1. The larger the moment of inertia I_{zz} or the shallower the depth of the centre of gravity, the less side spin rate caused. I_{zz} has more influence on the side spin rate than the depth of the centre of gravity.
2. The larger the moment of inertia I_{xx} or the shallower the depth of the centre of gravity, the higher the back spin rate caused. The depth of the centre of gravity has more effect on the back spin rate than I_{xx}.
3. The size of the uniform restitution area is large in proportion to the increase in I_{zz}. On the other hand, it is small in proportion to the increase in I_{xx}. It also decreases in proportion to the increase in the depth of the centre of gravity.

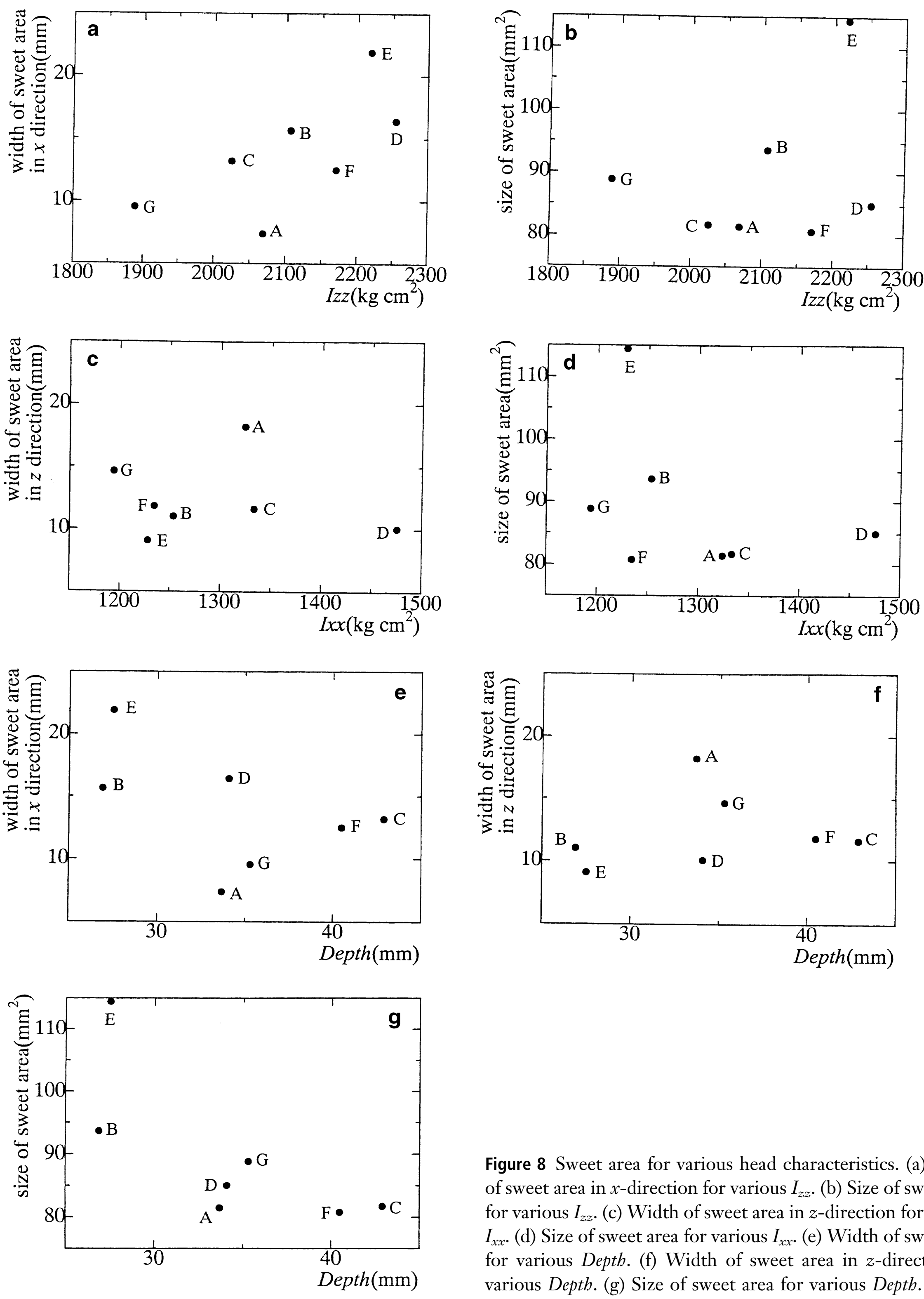

Figure 8 Sweet area for various head characteristics. (a) Width of sweet area in x-direction for various I_{zz}. (b) Size of sweet area for various I_{zz}. (c) Width of sweet area in z-direction for various I_{xx}. (d) Size of sweet area for various I_{xx}. (e) Width of sweet area for various *Depth*. (f) Width of sweet area in z-direction for various *Depth*. (g) Size of sweet area for various *Depth*.

4 The size of the uniform restitution area is large and the change of spin rate is small for the head model with a shallow depth of the centre of gravity.

5 It is noted that the results with respect to I_{xx} are different from physical knowledge.

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