

MODELLING OF THE FREE BENDING BEHAVIOR OF A MULTILAYER CABLE TAKING INTO ACCOUNT THE TANGENTIAL COMPLIANCE OF CONTACT INTERFACES

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Introduction

The bending stiffness of helical strands is an important parameter in the study of their dynamic behavior, such as wind-induced motion of overhead line conductors. A common approach is to model the conductor (or simple straight strand) using a constant stiffness value chosen between two limiting states known as EI_{min} (independent wire behavior) and EI_{max} (wires acting together as a solid) [1]. Because of the presence of friction between the cable's wires, its flexural bending stiffness varies during the bending process according to the cable's deformation, leading to a non-linear behavior [2].

In this paper, the model presented in [2] is extended in order to take into account the shearing, micro-slipping and ultimately the gross slipping at the interlayer wire contact interfaces, as opposed to only considering a stick-slip system. Other cable models considering the tangential compliance of contact interfaces can be found in the literature, most notably Hardy and Leblond [3], for a cable having only the outer layer wires behaving independently, and more recently Rawlins [4], for a cable vibrating in sine-shaped loops. The model presented in this paper is formulated to be implemented into a finite-element program to evaluate the effect of the tangential compliance of contact interfaces on the free bending behavior of multilayer conductors.

DESCRIPTION OF THE MODEL

An in-depth review of the various approaches proposed for the bending stiffness evaluation of taut simple straight strands is presented in [5]. As reported by Cardou, one of the best model available for the prediction of the free bending stiffness of multilayer simple straight strands is the one developed and implemented into a simple finite-element program by Papailiou [2], [6]. While Papailiou relied on a dedicated finite-element program to validate his theory through a comparison with quasi-static test results, using the cable's secant bending stiffness, Dastous [7] made the demonstration that Papailiou's analytical model can effectively be integrated into a generic finite-element program to model dynamic problems, using the cable's tangent bending stiffness.

This section describes the main elements underlying the analytical model used for the computation of the bending stiffness of a simple straight strand under tension. Many of these elements are directly taken from [2], and also from Hong *et al.* [8], who reconsidered some simplifications made by Papailiou regarding the pressure transmission from outer layer to inner ones.

Cable geometry

As for the all of the publications referenced so far in this paper, this new model, described in depth in [9], is aimed at analyzing the bending behavior of simple straight strands, that is, cables made of a central core wire surrounded by N concentric layers of helically wounded round section wires. The

model can be applied to both heterogeneous (e.g. ASCR conductor) and homogeneous constructions (e.g. AAC, AAAC conductors; conventional earth wires), as long as the tension applied to the cable is distributed among all the constituent wires (i.e. self-supported conductors, like ACSS, are not covered by the model). In all cases, the cables considered in this study have a reversed lay construction.

Figure 1 shows the geometry of a wire in a bent cable element and the cross-section of a $N = 4$ layers conductor. The wire j of layer i is wrapped around layer $i-1$ at a lay angle β_i with respect to the conductor central axis. Layer i comprises n_i wires of diameter d_i and their central axis are positioned on a cylinder of radius R_i . In a given cross-section of the cable, the angular position φ_{ij} of wire j of layer i is measured with respect to the bending axis of the cable. The wires of layer i are characterized by a cross-section area A_i and a Young modulus E_i .

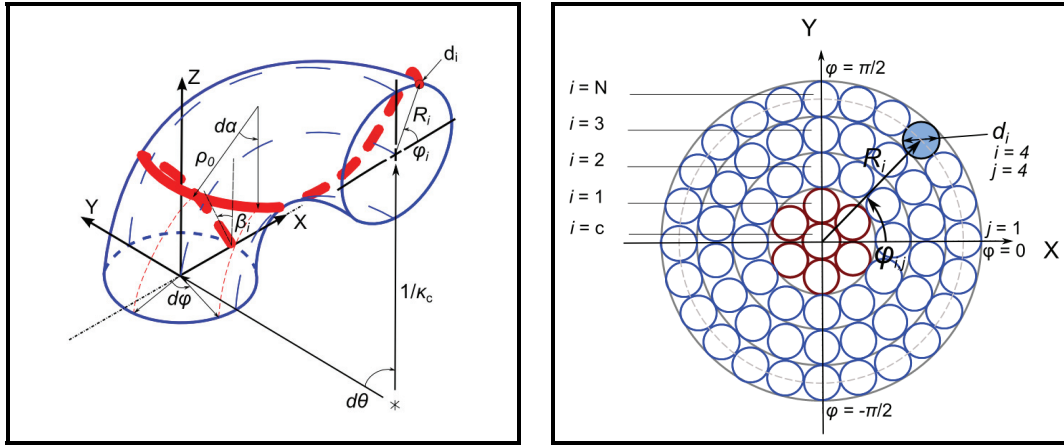


FIGURE 1 – Description of a cable's geometry

Cable under axial loading

Let's first consider a cable element of unit length. When a tensile load H is applied to this element, in the absence of any transversal load (cable curvature $\kappa_c = 0$), the tensile load in the wires of layer i is distributed uniformly and is given by [2]

$$T_{0i} = \frac{E_i A_i \cos^2(\beta_i)}{\sum_{\text{all wires}} E_k A_k \cos^3(\beta_k)} H \quad (1)$$

The tension T_{0N} in the outer layer wires gives rise to a radial force dP_{0N} between the outer layer N and layer $N-1$ [2]

$$dP_{0N} = T_{0N} \sin(\beta_N) d\varphi_N \quad (2)$$

As we move towards the inner layers of the cable element, the radial force dP_{0i} between layer i and layer $i-1$ arises from tension T_{0i} but also from tensions $T_{0i+1..0N}$ of layers $i+1$ to N . Hong *et al.* [8] brought slight modifications to the model presented in [2], that are integrated in the present work for the calculation of radial forces between layer i and $i-1$

$$dP_{0i} = \left(T_{0i} \sin(\beta_i) + \sum_{k=i+1}^N 2T_{0k} \frac{R_i n_k \tan \beta_k}{R_k n_i \tan \beta_i} \right) d\varphi_i \quad (3)$$

While [2] and [8] consider a line contact between a wire and the layer underneath, the present model requires the computation of the normal force applied on elliptical contact patches between layers in order to compute their geometry (except for the six wires layer wrapped around the central core wire). The number of contact points per lay length of a wire of layer i with layer $i-1$ is given by [6]

$$npc_{i,i-1} = n_{i-1} \left(\frac{R_i \tan \beta_{i-1}}{R_{i-1} \tan \beta_i} + 1 \right) \quad (4)$$

Using the geometric relation $\Delta\varphi_i = 2\pi/npc_{i,i-1}$, the normal force at contact areas is computed by letting $d\varphi_i \rightarrow \Delta\varphi_i$ in (2) and (3)

$$P_{0N} = T_{0N} \sin(\beta_N) \Delta\varphi_N \quad (5)$$

$$P_{0i} = \left(T_{0i} \sin(\beta_i) + \sum_{k=i+1}^N 2T_{0k} \frac{R_i}{R_k} \frac{n_k}{n_i} \frac{\tan \beta_k}{\tan \beta_i} \right) \Delta\varphi_i \quad (6)$$

Geometry of contact ellipses

The dimensions of the contact ellipses (Fig. 2) are computed assuming Hertzian contact [10], [11].

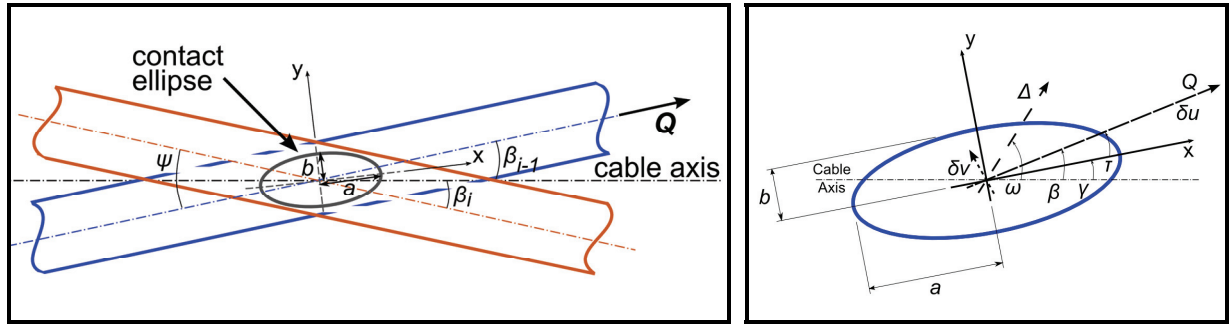


FIGURE 2 – Contact ellipse geometry

Major semi-axis a is given by [12]

$$a_i = \left[\left(3E(e) P_{0i} \frac{1}{A_{cp} + B_{cp}} \right) / \left(2\pi E^* (1-e^2) \right) \right]^{1/3} \quad (7)$$

With $E(e)$ the elliptical integral of the second kind of argument $e = 1 - (b/a)^2$ and $1/E^* = (1-\nu_i^2)/E_i + (1-\nu_{i-1}^2)/E_{i-1}$, where ν corresponds to the Poisson ratio of materials in the two layers in contact. The ellipticity b/a is taken from [13]

$$\frac{b}{a} \cong \left(A_{cp} / B_{cp} \right)^{2/3} \quad (8)$$

Curvature parameters A_{cp} and B_{cp} are related to the curvature of the two surfaces in contact and are given by (9)-(10), assuming $B_{cp} > A_{cp}$ and with R' and R'' representing respectively the maximum radius of curvature of the surface (curvature due to helicity of the wire, taken as $\rho_{0i} = R_i / \sin^2(\beta_i)$, negative value for concave surface) and the minimum radius of curvature of the surface ($r_i = d_i/2$). The angle ψ represents the angle between the two cylinders (Fig. 2) [11]:

$$A_{cp} = \frac{1}{2}(U - V) ; B_{cp} = \frac{1}{2}(U + V) \quad (9)$$

$$U = \frac{1}{2} \left(\frac{1}{R_i'} + \frac{1}{R_i''} + \frac{1}{R_{i-1}'} + \frac{1}{R_{i-1}''} \right) ; V = \frac{1}{2} \left[\left(\frac{1}{R_i'} - \frac{1}{R_i''} \right)^2 + \left(\frac{1}{R_{i-1}'} - \frac{1}{R_{i-1}''} \right)^2 + 2 \left(\frac{1}{R_i'} - \frac{1}{R_i''} \right) \left(\frac{1}{R_{i-1}'} - \frac{1}{R_{i-1}''} \right) \cos(2\psi) \right]^{1/2} \quad (10)$$

The above calculations are based on Hertz elastic contact theory. Following the example of Rawlins [4], the present model considers the yielding of the contact ellipses once the average bearing stress $\sigma_c = P_{0i}/\pi ab$ at the contact interface exceeds the material bearing yield strength, taken as $R_{cY} = 240$ MPa. In all cases, the ellipticity is assumed to remain the same as with Hertzian contact (eq. 8), while the contact area is computed using $\pi ab = P_{0i}/\sigma_c$.

Cable bending

The basic theory used in the current model to predict the helical strand bending behaviour has already been explained in detail in [2], [6]-[8]. Because of space constraints, the following description of the model is restricted to the elements essential to the implementation of the extended model. The complete analysis is presented in [9].

For a given value of curvature κ_c , the tangent stiffness of a cable is computed using [7]

$$EI = \sum_{i,j=stick} EI_{i,j}^{stick} + \sum_{all\ wires} EI_{i,j}^{wire} \quad (11)$$

While in [2], [6]-[8], EI^{stick} is computed using $EI_{i,j}^{stick} = E_i A_i R_i^2 \sin^2(\varphi_{i,j}) \cos^3(\beta_i)$, the present model takes into account the effect of the tangential compliance by letting $EI^{stick} \rightarrow EI^{elast}$. Before computing the effective bending stiffness of the cable, the status of each wire has to be established. To this end, the criteria for gross slipping, taken from [8], is verified for each wire of the cross section of the unit length cable element

$$\begin{aligned} \text{Gross slipping} \quad \Leftrightarrow \quad dT_{i,j} \geq \mu dP_{i,j} \\ E_i A_i R_i \cos^2(\beta_i) \cos(\varphi_{i,j}) \kappa_c \geq \mu \left(T^{stick}(\varphi_{i,j}) \sin(\beta_i) + 2 \sum_{k=i+1}^N T_k^{slip}(\varphi_{i,j}) \sin(\beta_k) \frac{R_i}{R_k} \frac{n_k}{n_i} \frac{\tan \beta_k}{\tan \beta_i} \right) \end{aligned} \quad (12)$$

Where $T^{stick}(\varphi_{i,j})$ is given by

$$T_{i,j}^{stick} = T_{0i} + E_i A_i R_i \sin(\varphi_{i,j}) \cos^2(\beta_i) \kappa_c \quad (13)$$

And $T^{slip}(\varphi_{i,j})$ is taken as the last value of $T^{stick}(\varphi_{i,j})$ computed before the onset of gross slip. Referring to Fig. 2, the tangential traction force across a contact interface is evaluated for wires which haven't reached the gross slip state using

$$Q_{i,j} = E_i A_i \cos^2(\beta_i) R_i \cos(\varphi_{i,j}) \kappa_c \Delta \varphi_i \quad (14)$$

The normal force on the contact interface P_{ij} it then computed using (6) and (13). The angle $\tau_i = \beta_i - \gamma_i$ between $Q_{i,j}$ and the contact ellipse's principal axis is calculated with $\gamma_i \cong \beta_i - \psi_i/2$. The wire's displacement along its axis δu is then computed using the relation derived in [14]

$$\delta u_{i,j} = \frac{3\mu P_{i,j}(2-\nu_i)}{16G_i a} \left[1 - \left(1 - \frac{Q_{i,j}}{\mu P_{i,j}} \right)^{2/3} \right] \eta_i \quad (15)$$

With $G_i = E_i/2(1+\nu_i)$ and η_i defined in [14]. The displacement δu , induced by the shearing and the micro-slipping of the contact interface, is then used to establish the contribution of each wire to the overall bending stiffness of the cable. In a perfect *stick* state, the additional wire elongation due to the cable bending is given by [2]

$$\varepsilon_{i,j}^{stick} = R_i \sin(\varphi_{i,j}) \cos^2(\beta_i) \kappa_c \quad (16)$$

Across a contact patch, the *virtual* elongation associated with displacement δu may be evaluated using the following relation

$$\varepsilon_{i,j}^{\delta u} = \frac{\delta u_{i,j}}{\Delta s_i} = \frac{\sin \beta_i}{R_i} \frac{\delta u_{i,j}}{\Delta \varphi_i} \quad (17)$$

Where s is a coordinate embedded in the wire axis and $\Delta \varphi_i = 2\pi/npc_{i,i-1}$. Using (16) and (17) with (13), we obtain the tension within a segment of wire in micro-slipping

$$T_{i,j}^{elast} = T_{0i} + E_i A_i \cos^2(\beta_i) \left[R_i \sin(\varphi_{i,j}) \kappa_c - \frac{\tan(\beta_i)}{R_i \cos(\beta_i)} \frac{\delta u_{i,j}}{\Delta \varphi_i} \right] \quad (18)$$

The effective bending stiffness of the cable can now be computed from (11)

$$EI(\kappa_c) = \sum_{i,j=stick} EI_{i,j}^{elast} + \sum_{all \text{ wires}} EI_{i,j}^{wire} \quad (19)$$

$$EI_{i,j}^{elast}(\kappa_c) = T_{i,j}^{elast} R_i \sin(\varphi_{i,j}) \cos(\beta_i) / \kappa_c \quad ; \quad EI_{i,j}^{wire} = E_i \pi \frac{d_i^4}{64} \cos \beta_i$$

NUMERICAL RESULTS

Impact of the tangential compliance on the computed behavior of overhead line cables

As explained in detail in [2], when a conductor segment (with friction) is bent to a constant curvature κ_c , an additional tension arises in the wires. This tension varies with the distance $R_i \sin(\varphi_{i,j})$ from the cable neutral axis. In a *stick-slip* system, the tension differential across a contact ellipse increases with the imposed curvature κ_c , without any movement, until the tangential traction force Q reaches the maximum friction force μP , as shown in Figure 3.

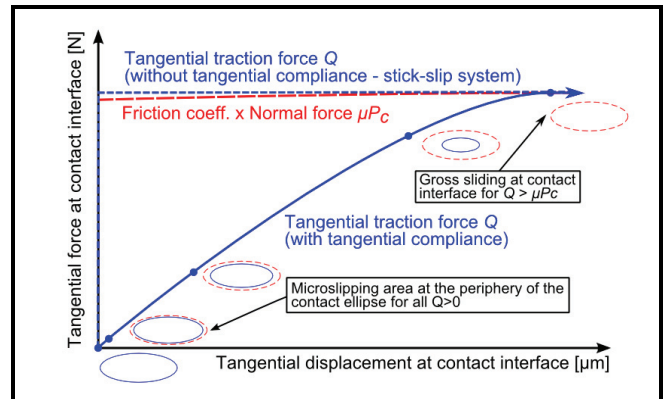


FIGURE 3 – Force displacement relation at contact ellipse

In reality, because the normal force P tends towards zero at the edge of the contact ellipse [11], micro-slipping starts at the periphery of the contact patches for any $Q > 0$. This translates into a reduced value of EI and a non-linear behavior, even before the onset of gross slipping. This behavior leads to reduced EI values at low curvature, as shown in Figure 4 (TC curves). The EI calculations presented for a *stick-slip* system (StS) have been performed according to [2] and [7], using a wire slip criterion.

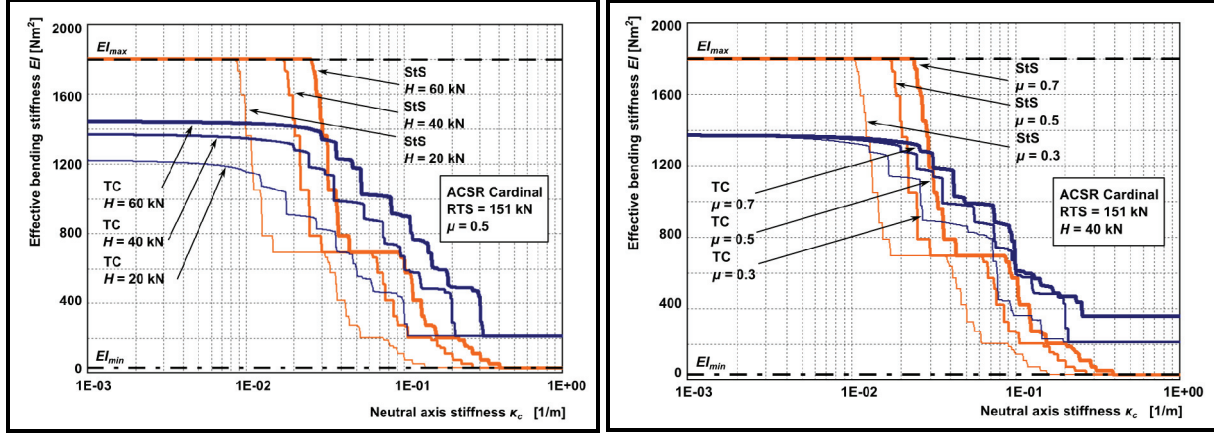


FIGURE 4 - Computed EI values taking (TC) and not taking (StS) into account the tangential compliance of contact interfaces. Influence of tension H (left); influence of friction coefficient μ (right).

We observe on Figure 4 that the bending stiffness computed with the new model (TC) varies with the cable tension H , even for low curvature values. Furthermore, we notice that the tangent stiffness never reaches the theoretical value EI_{min} , a direct consequence of the use of the slip criterion described in [8]. For the ACSR Cardinal conductor modeled in this paper (see properties in Table 1), with $\kappa_c \rightarrow 0$ and a tension H of 13%, 26% and 40% RTS (Rated Tensile Strength), the initial EI values computed with the new model (TC) varies between $0.68EI_{max}$ and $0.80EI_{max}$.

Figure 5 shows the computed bending stiffness for two additional cables, a 32 mm steel cable and a AAC Petunia all-aluminum conductor. While the steel cable shows initial EI values of $0.75EI_{max}$ and $0.82EI_{max}$ for tensions of 17% RTS and 34% RTS, the inclusion of the tangential compliance in the model appears to have a more important impact on the computed bending stiffness of the Petunia conductor, with initial EI values between $0.55EI_{max}$ and $0.61EI_{max}$, for H values between 18% RTS and 28% RTS.

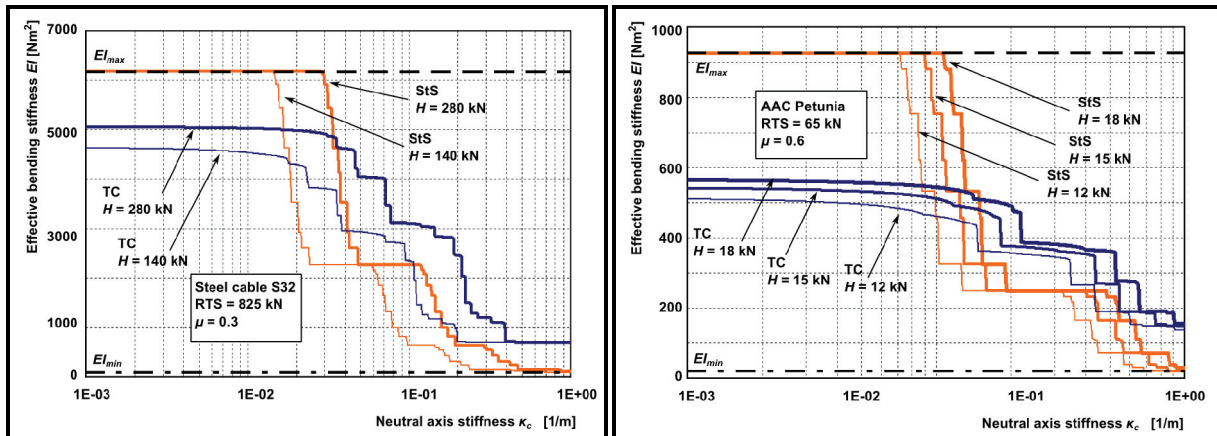


FIGURE 5 - Computed EI values taking (TC) and not taking (StS) into account the tangential compliance of contact interfaces, for a steel cable and an all-aluminum conductor. Influence of tension H (left); influence of friction coefficient μ (right).

TABLE 1 – Properties of cables used for analysis

ACSR CARDINAL					STEEL CABLE S32					AAC PETUNIA				
Layer	Nb. wires	wire diam.	Young	lay ang.	Layer	Nb. wires	wire diam.	Young	lay angle	Layer	Nb. wires	wire diam.	Young	lay ang.
		mm		°			mm		°			mm		°
core	1	3.34	210	--	core	1	3.72	200	--	core	1	3.62	69	--
1	6	3.34	180	6.06	1	6	3.54	180	14.2	1	6	3.62	69	6.0
2	12	3.32	65	11.99	2	12	3.54	180	13.7	2	12	3.62	69	15.0
3	18	3.32	65	11.80	3	18	3.54	180	14.0	3	18	3.62	69	15.0
4	24	3.32	65	13.10	4	24	3.54	180	14.0	--	--	--	--	--

Comparison with experimental data

In this section, numerical results generated with the new model are compared to experimental results presented in the literature. The results used for this validation are those obtained through quasi-static tests presented in [2]. These experiments are reproduced numerically through the use of a finite-element program written in the Matlab environment.

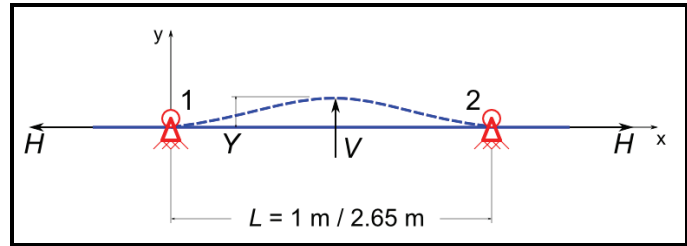


FIGURE 6 - Outline of the test arrangement used in [2]

The numerical results obtained through Papailiou's *stick-slip* model are also included for comparison, which makes for an easy evaluation of the effect of the tangential compliance on the computed behavior of the cable in free bending conditions. Figure 6 presents the test arrangement used for the validation of the new model.

In Figure 7, the load-deflection diagrams computed with the new model are compared against those presented in [2] for the Cardinal conductor. As expected, the curves presented in this paper show a lower average slope and reduced hysteresis area, a consequence of the lower initial EI values and higher minimum EI values computed. The curves obtained numerically are in fairly good agreement with the experimental results obtained through quasi-static tests.

Figure 8 shows the cable deflection at the point of application of the transversal load, compared with the results from [2]. Again, the numerical results are in good agreement with the experimental data.

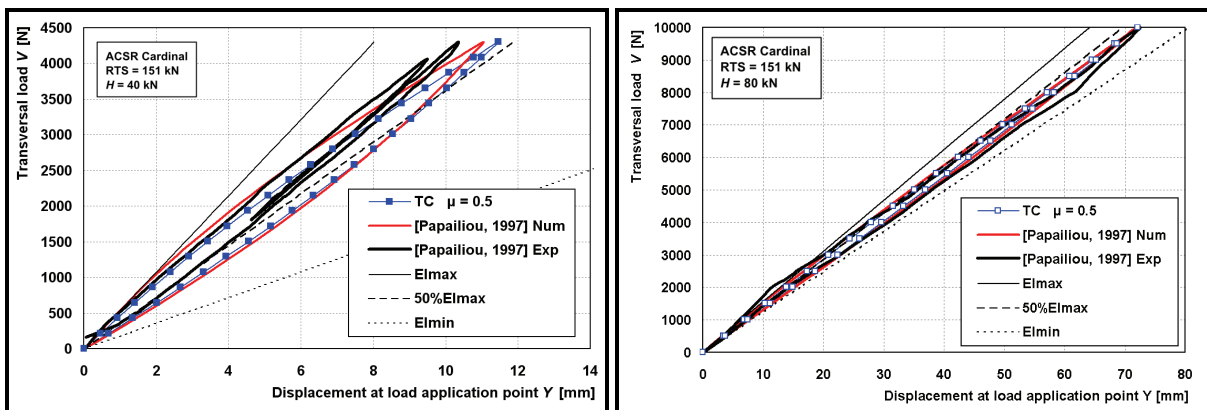


FIGURE 7 - Load-deflection diagram computed with the new model (TC), compared with the results presented in [2].
 $L = 1$ m (left) ; $L = 2.65$ m (right)

CONCLUSION

A bending model of a multilayer cable taking into account the tangential compliance of interlayer contact interfaces has been presented in this paper. It has been shown that the consideration of tangential compliance has the effect of reducing the initial EI values ($\kappa_c \rightarrow 0$) predicted and these values become dependant on the tension H applied to the cable. The model has been implemented into a finite-element program to validate its ability to reproduce the free bending behavior a multilayer simple straight strand. These first results obtained with the new model, although partial, appear promising.

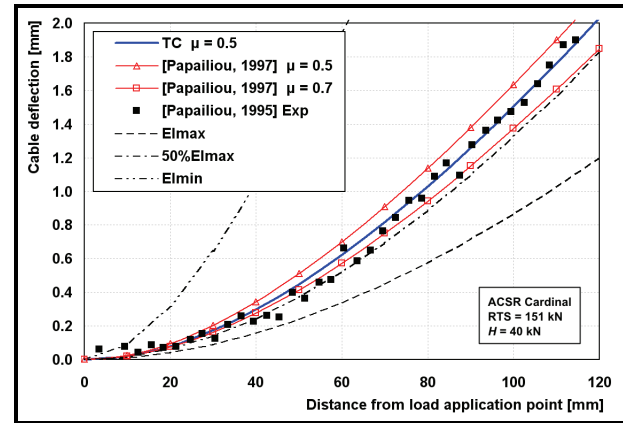


FIGURE 8 - Deflection curve at the fixed end of the cable computed with the new model (TC) compared with results presented in [2]

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