

New Numerical algorithm

Currently I am developing an efficient numerical method (based on Newton-Raphson method) to find out the roots of real-valued functions in one variable. The preliminary results for some functions are given below and the refinement of this work is going on. The time achievement is also being examined.

Polynomial functions**1. Function: $x^3 - 3x^2 - 9x + 27 = 0$**

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
25000	3.000000000	296	-3.000000000	7	-3.000000000	7
15000	2.999999999	623	-3.000000000	7	-3.000000000	7
10000	2.999999999	458	-3.000000000	7	-3.000000000	7
7000	3.000000000	322	-3.000000000	7	-3.000000000	7
500	2.999999999	292	-3.000000000	7	-3.000000000	7
10	3.000000000	122	-3.000000000	7	-3.000000000	7
1	3.000000000	680	-----	-----	3.000000000	126,7
-1	-----	-----	-3.000000000	6,11	-----	-----
-5	-3.000000000	6	-3.000000000	7	-3.000000000	7,9
-600	-3.000000000	18	-3.000000000	7	-3.000000000	19,19
-9000	-3.000000000	25	-3.000000000	7	-3.000000000	26,26
-12500	-3.000000000	26	-3.000000000	7	-3.000000000	27,27
-26000	-3.000000000	27	-3.000000000	7	-3.000000000	28,28

* -Two roots can be obtained at the same time. In this case both are the same.

2. Function: $x^5 - 11x^4 + 46x^3 - 90x^2 + 81x - 27 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
98765	3.000009907	1475	1.000000000	2,2	1.000000000	635
3995	2.999989503	1251	1.000000000	2,2	1.000000000	635
987	-----	-----	1.000000000	2,2	1.000000000	635
195	2.999982054	1233	1.000000000	2,2	1.000000000	635
-95	0.999999999	547	1.000000000	2,2	1.000000000	198
-987	1.000000009	1522	1.000000000	2,2	0.999999999	70
-9865	0.999999999	368	1.000000000	2,2	1.000000013	326
-98765	1.000000012	95	1.000000000	2,2	1.000000020	94

* - Two roots can be obtained at the same time. In this case both are the same.

3. Function: $x^3 - x - 1.0 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
126779	1.324717957	33	1.324717957	5,24	1.324717957	6,17
87765	1.324717957	33	1.324717957	5,24	1.324717957	6,17
8725	1.324717957	28	1.324717957	5,24	1.324717957	6,17
675	1.324717957	26	1.324717957	5,24	1.324717957	6,17
65	1.324717957	15	1.324717957	5,24	1.324717957	6,17
1	1.324717957	6	1.324717957	5,24	1.324717957	6,17
-1	1.324717957	22	1.324717957	5,24	1.324717957	15,19
-107	1.324717957	21	1.324717957	5,24	1.324717957	37,42
-9008	1.324717957	48	1.324717957	5,24	1.324717957	42,50
-77864	1.324717957	77	1.324717957	5,24	1.324717957	62,69

* - Two roots can be obtained at the same time. In this case both are the same.

4. Function: $x^3 - 2x^2 - 11x - 12 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
987432	4.000000000	37	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
87546	4.000000000	31	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
576	4.000000000	19	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
61	4.000000000	13	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
1	1.000000000	1	4.000000000 1.000000000	1 10	-----	----
-1	4.000000000	2	-3.000000000 1.000000000	6 6	-3.000000000 1.000000000	19 5
-68	-3.000000000	13	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
-743	-3.000000000	19	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
-56732	-3.000000000	30	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5
-487472	-3.000000000	35	-3.000000000 1.000000000	8 5	-3.000000000 1.000000000	19 5

- - Two roots can be obtained at the same time.

5. Function $x^3 - 2x + 2 = 0$ (Special case)

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
874531	-1.769292354	67	-1.769292354	6,17	----	----
34561	-----	-----	-1.769292354	6,17		
562	-1.769292354	107	-1.769292354	6,17		
43	-----	---	-1.769292354	6,17		
2	-1.769292354	10	-1.769292354	6,17		
-3	-1.769292354	7	-1.769292354	6,17		
-71	-1.769292354	14	-1.769292354	6,17		
-8934	-1.769292354	27	-1.769292354	6,17		
-76987	-1.769292354	31	-1.769292354	6,17		

For the chosen initial values, the second method fails.

6. Function: $x^2 - 612.0 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
345698	24.7386337537	19	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
14567	24.7386337537	14	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
456	24.7386337537	9	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
5	24.7386337537	8	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
-7	-24.7386337537	7	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	9 9
-654	-24.7386337537	10	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	6 6
-19654	-24.7386337537	15	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	8 8
-786542	-24.7386337537	20	24.7386337537 -24.7386337537	2 2	24.7386337537 -24.7386337537	11 11

* - Two roots can be obtained at the same time.

7. Function: $(x - 1)^3 + 0.512 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
99998.5	1.800000012	34	1.800000012	9,13	1.800000012	17,10
12675.5	1.800000012	29	1.800000012	9,13	1.800000012	17,10
175.5	1.800000012	19	1.800000012	9,13	1.800000012	17,10
1.5	1.800000012	7	1.800000012	9,13	1.800000012	17,7
-0.5	1.800000012	8	1.800000012	5,9	1.800000012	12,20
-4.5	1.800000012	14	1.800000012	5,9	1.800000012	19,19
-453.5	1.800000012	21	1.800000012	5,9	1.800000012	23,25
-15673.5	1.800000012	34	1.800000012	5,9	1.800000012	35,35
-78432.5	1.800000012	38	1.800000012	5,9	1.800000012	39,39

* - Two roots can be obtained at the same time. In this case both are the same.

8. Function: $x^3 - x^2 - 10x - 8 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
876234	4.000000000	36	-1.000000000	6	-1.000000000	1
			-2.000000000	11	-2.000000000	8
34672	4.000000000	28	-1.000000000	6	-1.000000000	1
			-2.000000000	11	-2.000000000	8
764	4.000000000	19	-1.000000000	6	-1.000000000	1
			-2.000000000	11	-2.000000000	8
1	-1.000000000	1	-1.000000000	7	-1.000000000	1
			4.000000000	8	-2.000000000	8
-1	-1.000000000	1	-1.000000000	1	-1.000000000	1
			-2.000000000	7	-2.000000000	8
-467	-2.000000000	20	-1.000000000	6	-2.000000000	21,21
			-2.000000000	11		
-6542	-2.000000000	26	-1.000000000	6	-2.000000000	27,27
			-2.000000000	11		
-87321	-2.000000000	33	-1.000000000	6	-2.000000000	34,34
			-2.000000000	11		

- - Two roots can be obtained at the same time.

9. Function: $x^3 - 3x^2 + 4 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
500000	2.000000000	90	-1.000000000 2.000000000	6 28	-1.000000000 2.000000000	6 28
73256	2.000000000	55	-1.000000000 2.000000000	6 28	-1.000000000 2.000000000	7 28
235	2.000000000	41	-1.000000000 2.000000000	6 28	-1.000000000 2.000000000	7 28
5	2.000000000	76	-1.000000000 2.000000000	6 28	-1.000000000 2.000000000	7 28
-10	-1.000000000	10	-1.000000000 2.000000000	6 28	-1.000000000 -1.000000000	11 12
-563	-1.000000000	20	-1.000000000 2.000000000	6 28	-1.000000000 -1.000000000	11 12
-96342	-1.000000000	32	-1.000000000 2.000000000	6 28	-1.000000000 -1.000000000	33 33

* - Two roots can be obtained at the same time.

10. Function: $x^4 + 3x - 4 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
107890	1.000000000	45	1.000000000	2,8	1.000000000 -1.742959202	2 12
76543	1.000000000	45	1.000000000	2,7	1.000000000 -1.742959202	2 12
546	1.000000000	27	1.000000000	2,8	1.000000000 -1.742959202	2 12
3	1.000000000	9	1.000000000	2,8	1.000000000 -1.742959202	2 12
-2	-1.742959202	6	-1.742959202	5,32	-1.742959202	10,25
-4321	-1.742959202	32	-1.742959202	5,8	-1.742959202	34, >1500
-432678	-1.742959202	49	1.000000000	2,8	-1.742959202	50, >1500

* - Two roots can be obtained at the same time.

11. Function: $x^3 - 0.165x^2 + 3.993e - 04 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
100000	0.146359512	40	6.237758018E-02 -4.373708626E-02	5 6	0.146359512 -4.373708626E-02	11 12
35678	0.146359512	38	6.237758018E-02 -4.373708626E-02	5 6	0.146359512 -4.373708626E-02	11 12
453	0.146359512	27	6.237758018E-02 -4.373708626E-02	5 6	0.146359512 -4.373708626E-02	11 12
1.0	0.146359512	27	6.237758018E-02 -4.373708626E-02	5 6	0.146359512 -4.373708626E-02	11 12
-15	-4.373708626E-02	19	6.237758018E-02 -4.373708626E-02	5 6	-4.373708626E-02	23,23
-6743	-4.373708626E-02	34	6.237758018E-02 -4.373708626E-02	5 6	-4.373708626E-02	23,23
-98456	-4.373708626E-02	40	6.237758018E-02 -4.373708626E-02	5 6	-4.373708626E-02	35,35

* - Two roots can be obtained at the same time.

12. Function: $x^3 - 0.03x^2 + 2.4e - 04 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
100000	2.661081797E-02	45	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	16 36
67543	2.661081797E-02	44	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	16 36
723	2.661081797E-02	32	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	16 36
2	2.661081797E-02	18	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	16 36
-20	-7.952197683E-03	18	-7.952197642E-03	6,13	2.661081797E-02 -7.952197683E-03	36 20
-67845	-7.952197683E-03	43	-7.952197642E-03	6,13	-7.952197683E-03	44
-120007	-7.952197683E-03	43	-7.952197642E-03	6,13	-7.952197683E-03	44

* - Two roots can be obtained at the same time.

13. Function: $2x^3 + 3.5x^2 - 4.21x + 0.546 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
564327	0.700000007	38	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
5637	0.700000007	28	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
74	0.700000007	17	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
0	0.149999999	5	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
-13	-2.600000005	10	0.700000006 0.149999999	8 5	-2.600000005 0.149999999	8 6
-2451	-2.600000005	23	0.700000006 0.149999999	8 5	-2.600000005	24,25
-123867	-2.600000005	33	0.700000006 0.149999999	8 5	-2.600000005	24,25

* - Two roots can be obtained at the same time.

14. Function: $0.004x^3 - 0.2x^2 + 2.45x - 5.4 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
34562	32.363031067	20	32.363031067 2.814202526	12 5	2.814202526	6,8
6534	32.363031067	21	32.363031067 2.814202526	12 5	2.814202526	6,8
31	32.363031067	5	32.363031067 14.822764775	4 6	32.363031067	5,8
0	2.814202526	6	2.814202526 32.363031067	5 12	32.363031067	5,8
-10	2.814202526	8	2.814202526 32.363031067	5 12	2.814202526	8,9
-5678	2.814202526	21	2.814202526 32.363031067	5 12	2.814202526	8,9
-26524	2.814202526	21	2.814202526 32.363031067	5 12	2.814202526	8,9

* - Two roots can be obtained at the same time.

Transcendental functions

1. Function: $e^{-x} - x = 0$

	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
100000	0.567143290	6	0.567143290	6,7	0.567143290	6,7
8525	0.567143290	6	0.567143290	5,7	0.567143290	6,9
525	0.567143290	6	0.567143290	6,7	0.567143290	6,7
25	0.567143290	6	0.567143290	6,7	0.567143290	6,9
1	0.567143290	5	0.567143290	5,7	0.567143290	6,7
-1	0.567143290	6	0.567143290	5,7	0.567143290	7,10
-20	0.567143290	25	0.567143290	5,7	0.567143290	26,28
-105	0.567143290	110	0.567143290	5,7	0.567143290	111,113
-515	0.567143290	520	0.567143290	5,7	0.567143290	521,523

* -Two roots can be obtained at the same time. In this case both are the same.

2. Function: $xe^x - 3.0 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
500	1.049908899	500	1.049908899	4,16		
150	1.049908899	159	1.049908899	4,16		
11	1.049908899	18	1.049908899	4,16		
0	1.049908899	9	-----	----		
-1	----	-----	----	-----		
-10			1.049908899	4,16		
-135			1.049908899	4,16		
-550			1.049908899	4,16		

* - Two roots can be obtained at the same time.

3. Function: $e^x - 3x = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
500	1.512134551	506	0.619061286 1.512134551	5 10	0.619061286 1.512134551	7 7
200	1.512134551	206	0.619061286 1.512134551	5 10	0.619061286 1.512134551	7 7
53	1.512134551	59	0.619061286 1.512134551	5 10	0.619061286 1.512134551	7 7
0	1.512134551	6	0.619061286 1.512134551	5 10	0.619061286 1.512134551	7 7
-24	1.512134551	7	-----	-----	0.619061286 1.512134551	8 31
-2234	1.512134551	7	0.619061286 1.512134551	5 6	-----	-----
-67456	1.512134551	7	0.619061286 1.512134551	5 6	-----	-----

*-Two roots can be obtained at the same time.

4. Function: $e^x + x - 2 = 0$

Initial Value	Newton-Raphson Method		New Method-1		New Method-2	
	Converged Value	No. of Iterations	Converged Value	No. of Iterations*	Converged Value	No. of Iterations*
500	0.442854401	505	0.442854401	5,9	0.442854401	5,8
150	0.442854401	155	0.442854401	5,9	0.442854401	5,8
2	0.442854401	7	0.442854401	5,9	0.442854401	5,8
0	0.442854401	5	0.442854401	5,9	0.442854401	5,8
-75	0.442854401	8	0.442854401	6,7	0.442854401	5,8
-2350	0.442854401	8	0.442854401	5,9	0.442854401	9,9
-98754	0.442854401	8	0.442854401	5,9	0.442854401	9,9

* - Two roots can be obtained at the same time. In this case both are the same.

Conclusion

1. The first algorithm is independent of initial guess.
2. It converges very faster than the other two, that is, the Newton-Raphson method and New Method-2.
3. The modifications carried out in the present two algorithms are very simple and the mathematical expressions used are very short statements involving only arithmetic operations.
4. The performance of the first algorithm is far better than that of the other two and in some cases the performance of the second algorithm is similar to that of the Newton-Raphson method.
5. In some cases, two different roots are simultaneously obtained by these two algorithms. In the case of the NR method, it is not possible.

Future Work

The above work is extended to the analysis of a system of non linear equations in 'n' variables to examine the performance of the algorithm.