

Introduction

I started this project with the idea of simply catching up on the progress made in fundamental concepts for interpreting crack growth using LEFM. I had little time to seriously think about these basic concepts after 1970 when my paper appeared in ASTM STP 482. At that time my technical direction had shifted to broader issues related to aircraft structural integrity. My conclusion from the review is that there must be a better way to approach the interpretation than those currently in use as current concepts have obvious limitations. The two technical notes I am posting in this Blog are the result of my re thinking of our basic assumptions. In retrospect, I wish that I had been able to form and present these notes before the current concepts became entrenched as the concepts contained lead to questions that are possibly more important than those being addressed by most of the current research.

I welcome your comments and more importantly, your participation in testing and extending the concepts developed in these notes. If there is sufficient interest other notes will follow. I have selected iMechanica as my venue as I have no compulsion to publish another formal paper and the potentials of an open and free exchange of ideas, example applications (including those that seem contradictory) and, related data are appealing. If you find these notes useful, all I ask is that appropriate credit be given and that you share these note with others.

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Adapting Neuber's Rule to Fracture Mechanics and its implications

This note stems from a long-standing curiosity about the similarities between Irwin's strain energy term K^2/E and Neuber's rule in the form of $K_t^2/E = \sigma \epsilon$. In this note the similarities are explored and the implications of the resulting analogy to Neuber's rule in terms of fracture mechanics relatable quantities are examined.

A brief summary of Neuber's rule applied to notch fatigue.

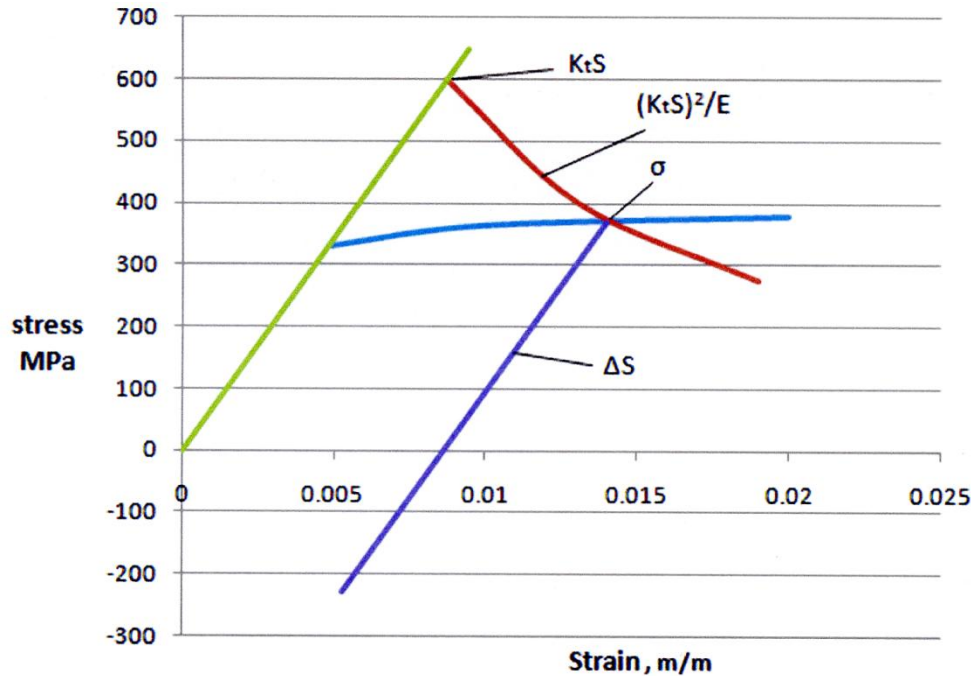
Neuber's rule [1] can be expressed as $K_t = (K_\sigma K_\epsilon)^{1/2}$, where K_t is the elastic stress concentration, K_σ the true notch stress concentration and K_ϵ the true notch strain concentration. Neuber's rule was derived for the case of pure shear and applies to any arbitrary stress strain relationship. Subsequent studies have shown that it also applies to other stress states [2, 3].

Topper, Wetzell and Morrow [3] adapted Neuber's rule to notch fatigue by assuming that $K_\sigma = \sigma/S$ and $K_\epsilon = \epsilon E/S$. Thus,

$$(K_t S)^2 / E = \sigma \epsilon \quad \text{Equation 1}$$

Where K_t is the elastic stress concentration factor, E is Young's modulus, S is the stress applied remote from the notch, σ is the true notch stress and, ϵ is the true notch strain. The application of equation 1 is best explained using Figure 1

Figure 1 Application of Neuber's rule to notch fatigue.



The uniaxial tension stress-strain curve for 2024 T351 is assumed to apply. The elastic slope is extended beyond yield. A curve is drawn representing stress and strain combinations that satisfy equation 1. For this example the $K_t S$ for a zero-to-tension applied cyclic loading is 600 MPa. For large radius notches the intersection of the selected stress-strain curve and the curve representing solutions to equation 1 determines the estimated maximum true stress and true strain at the completion of the first $\frac{1}{2}$ cycle of applied stress and, the origin for applying Neuber's rule for the next $\frac{1}{2}$ cycle. Subsequent $\frac{1}{2}$ cycles are estimated by shifting the origin to the end point of the previous $\frac{1}{2}$ cycle. The stress strain curve for each $\frac{1}{2}$ cycle must reflect the effects of prior strain history.

For constant amplitude cyclic loading and, materials and stresses of primary interest, after the first loading to maximum applied stress, subsequent cycles (up to about $2 \times \sigma_{\max}$) will be assumed elastic and therefore subsequent cycles are the full elastic range. No further significant change in peak stress is assumed to occur. The complexities of cyclic strain hardening or strain softening will not be considered at this time. With these simplifying assumptions, the cycles following the first $\frac{1}{2}$ cycle are essentially elastic, the peaks and valleys of subsequent cycles can be determined without actually drawing the Neuber curves.

When applying the Neuber's rule to fatigue it is necessary to relate the notch stress given by the Neuber model to fatigue-life data. For relatively sharp notches, this relationship is established by replacing K_t with a fatigue notch factor K_f that accounts for the differences between the theoretical K_t and the actual fatigue inducing conditions at the notch. These conditions include stress-strain gradient, and other differences.

The above background and simplifying assumptions are adequate as a preface to initially exploring an adaptation of Neuber's rule for estimating a new maximum stress intensity factor (k_{\max}) that is reduced from the elastic value as a result of plastic deformations near a crack tip.

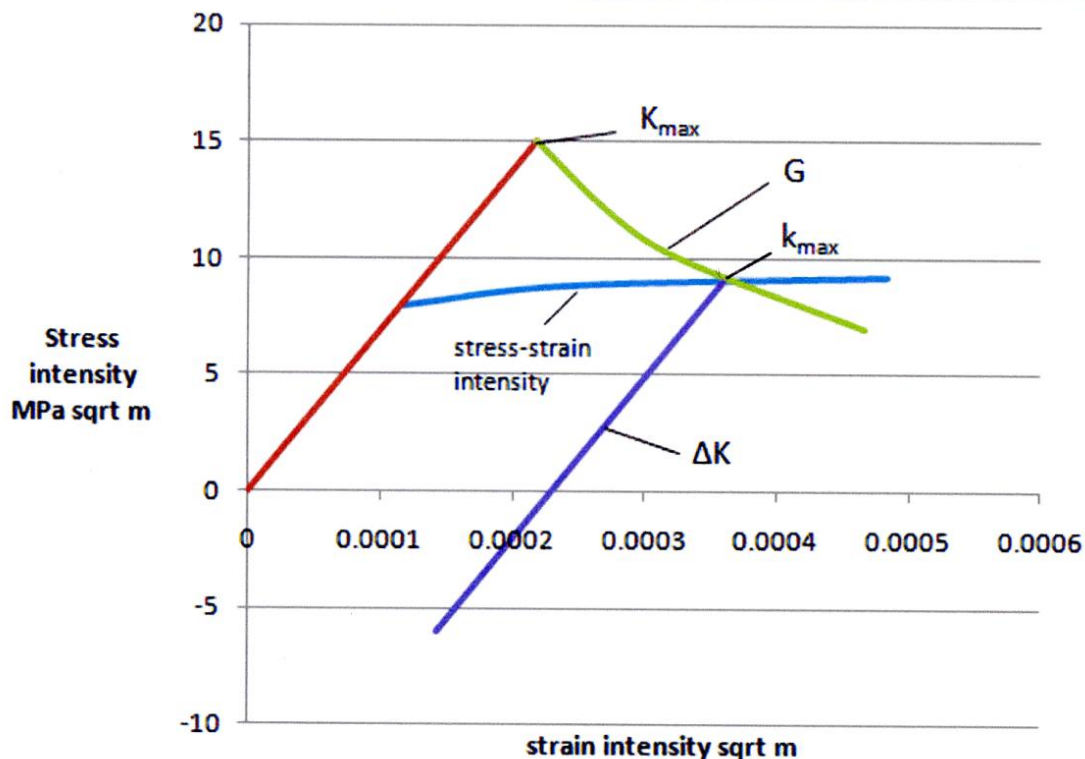
Stress-strain Intensity model

The facts that Neuber's rule is shown to apply to stress states other than pure shear where it could be mathematically proven and it applies to any stress-strain law, allow speculation that it will apply in the domain of stress intensity. Recognizing that the fracture mechanics $(\pi a)^{1/2}$ is analogous to K_I , Equation 1 can be transposed and written as $\pi a/E = C_\sigma C_\epsilon$. Where a is $1/2$ the length of an imbedded crack or the length of an edge crack and $C_\sigma C_\epsilon$ is the equivalent to $K_\sigma K_\epsilon$. Both C_σ and C_ϵ have dimension square root of the selected dimensional units: Following the analogy to fatigue, C_σ is defined as k_σ/S and C_ϵ is defined as $k_\epsilon E/S$ where k_σ is the "true" stress intensity and k_ϵ is the "true" strain intensity. Then it follows:

$$S^2 \pi a / E = k_\sigma k_\epsilon = K^2 / E = G \quad \text{Equation 2}$$

Where K is the elastic stress intensity and G is a measure of strain energy. At fracture G becomes the strain energy release rate. Equation 2 is the fracture mechanics equivalent to equation 1 and figure 2 is the equivalent to figure 1

Figure2 Adaptation of Neuber's rule to stress intensity



In figure 2 the applied stress is cycling 0 to tension. The elastic stress intensity factor at the peak load and elastic stress intensity factor range ΔK are $15 \text{ MPa m}^{1/2}$. The G curve is developed, using equation 2, by selecting values of stress intensity and computing the corresponding values of strain intensity. Figure 2 also shows a curve analogous to the stress-strain curve that relates crack tip stress intensity and strain intensity (in this example the stress-strain intensity curve is assumed to be simply a scaled down stress strain curve). The primary assumption for the model is that an appropriate stress-strain intensity curve does exist. Technical Note 2 will show that this curve can sometimes be deduced from da/dn data. However, it should be possible to develop this curve directly from COD measurements.

The curve is treated similar to a stress strain curve. Its intersection with the G curve identifies the stress-strain intensities for the region near the crack tip. The assumption is that k_{\max} has a unique relationship to the actual stress distribution near the crack tip and that it is the same unique distribution related to K_{\max} .

The initial "elastic" slope of this stress-strain intensity curve is defined by E as used in equation 2. The assumption that E applies requires that, at least in the elastic range, that the crack tip stress-strain intensity

curve be proportional to the stress-strain curve. While it would be convenient to assume that the post yield portion is also proportional, this possibility needs further study. However, we do know that k_{\max} and k_e are single valued proxies for the distribution of stress-strain in the region near the crack tip and thus a curve representing these quantities should exhibit the general characteristic of the material's stress-strain curve.

The stress-intensity range ΔK shown on figure 2 is shown as the full elastic range that would be true for an open notch (up to about $2 \times k_{\max}$). There are a wide variety of opinions as to how or if this range is affected by closure. What we do know is that whatever these closure and reversed plasticity affects are that result in Δk (the "true" stress intensity range) differing from the applied ΔK , the reasons for the difference are not the same as those resulting in the differences between k_{\max} and K_{\max} .

Contributions of maximum and stress range to crack Growth

When the stress-strain intensity curve is in the initial linear range, equation 1 will be used to approximate the contributions of max stress and stress range to crack growth. The basis for this equation is developed in reference 2

$$\Delta K_{\text{effective}} = \Delta k^m K_{\max}^{1-m} \quad \text{Equation 1}$$

Once beyond this range, the equation will be modified to

$$\Delta K_{\text{effective}} = \Delta k^m k_{\max}^{1-m} \quad \text{Equation 2}$$

This modification becomes necessary once we recognize that the relationships between k_{\max} and K_{\max} and between Δk and ΔK are not the same beyond the initial linear range.

Discussion

On figure 2, both K_{\max} (through the G curve) and k_{\max} have the same relationship to the stress-strain state near the crack tip. This confirms that for fracture toughness (a single parameter), relating toughness to K is a valid approach. Considering that the stress-strain intensity curves for plane strain and plane stress will differ, use of the Neuber's rule adaptation might add to our understanding of the observed differences between plain strain and plane stress toughness. With some additional thought, this line of reasoning might be extendable to address the R curve as well. My work to-date has not addressed the later stages of crack growth that logically merges with stable tear or fracture.

Crack growth involves two stress parameters, k_{\max} and Δk that are single valued proxies having unique relationships to the actual stress strain distribution near the crack tip. Each of these parameters relates to its elastic counterpart by different ground rules. Thus attempting to explain crack growth in terms of K_{\max} and ΔK in a two parameter equation or concept or, ΔK alone, can have only limited success. The direct approach of estimating the stress and strain distribution near the crack tip has promise but is cumbersome to apply as an engineering tool. It also has the disadvantage of requiring a fatigue data base related to stress and/or strain.

The adaptation of Neuber's rule developed in this note and the use of the resulting k_{\max} to determine an effective stress appears to provide a relatively simple approach to crack growth that addresses recognizable faults in current fracture mechanics based approaches. For future reference this approach

will simply be referred to as the “stress-strain intensity” approach (unless someone has a better idea)
Subsequent notes in this series will address its applications to gain a better understanding of da/dn curves.

References

1. Neuber H., Theory of Stress Concentration for Shear Strained Prismatic Bodies with Arbitrary Nonlinear Stress-Strain Law, Transactions .of the ASME, December 1961, p 544.
2. Walker K., The Effect of Stress Ratio during the Crack Propagation and Fatigue for 2024-T3 and 7075-T6 Aluminum, ASTM STP 462, January 1970, p1-14
3. Topper, T.H., Wetzel, R. M., and Morrow, JoDean, Neuber’s Notch Rule Applied to Fatigue of Notched Specimens. ASTM Journal of Materials, Vol. 4. No. 1, March1969, p. 200.