Introduction
This set of HT 60 steel da/dn curves is selected because it is a relatively simple set and the application is relatively intuitive. This allows focus on the application of the stress-strain intensity approach and the reinterpretation of LEFM parameters that are affected by the stress-strain intensity concept. I have tried to provide background in my rationales for these reinterpretations as the rethinking process has been, at times, difficult.

Technical Note 2 November 2011
Application of "stress-strain intensity "approach to HT60 steel da/dn curves

The following is an excerpt from the conclusions of refence 1

The D6AC steel data presents a material where the crack growth rate is independent of stress ratio in the Paris regime, but displays significant "fanning" near threshold. The lack of a strong dependence of crack growth rate on stress ratio in the Paris regime implies there is little closure in high strength steels. Contrarily, the D6AC steel near-threshold crack growth rate data shows a considerable dependence on stress ratio. There is no clear explanation to why closure would exist near-threshold and nearly vanish in the Paris regime."

Figure 1 is taken from reference 2. The characteristics observed in D6AC steel are clearly present in these data. The data is from reference 3 Steel D [1] is described as 16 mm thick HT60 steel [3]. The mechanical properties of HT60 steel are approximately: 2% yield 510 MPa; Ultimate strength 600 MPa and, elongation 31%. The data of Figure 1 include both negative and positive stress ratios.

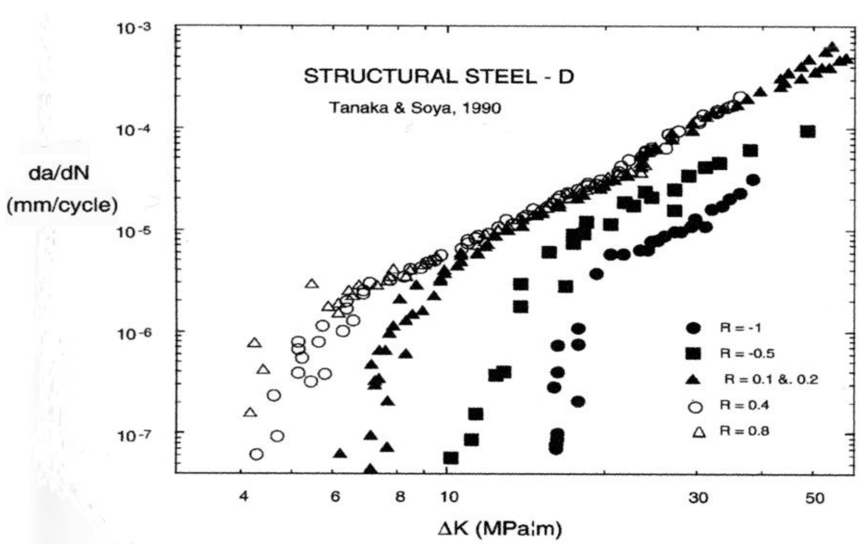

Figure 1 da/dn curves for HT 60 Steel

In This note we will build a model for steel D based on the concepts developed in Technical Note 1. Where these concepts impact related equations and/or assumptions, suitable adaptations or modifications are made and the rational for these adaptations or modifications are presented. The process of rethinking is addressed as sequential steps to aid in explanation. However, these steps were often iterative and overlapping as understanding is gained.

Step 1

From Note 1. When the stress-strain intensity curve is in the initial linear range, equation 1 will be used to approximate the contributions of max stress intensity and stress intensity range to crack growth. The basis for this equation is developed in reference 4

ΔK effective $=\Delta k^m K_{max}^{1-m}$ Equation 1

Once beyond this range, the equation will be modified to

ΔK effective $=\Delta k^m k_{max}^{1-m}$ Equation 2

From the above we can assume

- there will be a change in the slope of a da/dn curve at the end of the range for equation 1
- If there are several da/dn curves representing different stress ratios in the same data set, and thus the same stress-strain intensity curve, similar features of the changing slopes will be at the same k_{max} and, through G, the same K_{max}.
- Considering that the stress strain intensity curve has characteristics of a stress-strain curve and that the da/dn curve is influenced by the shape of the stress-strain intensity curve, we can expect that there is a loose similarity between all of these curves.

To examine these assumptions we first examine whether the onset of slope changes in figure 1 occur at a single value of K_{max}. The K max selected is 10.2 MPa $m^{1/2}$.This selection is made while considering all of the slope changes The values of ΔK at each R ratio where K_{max} is 10.2 MPa $m^{1/2}$ is determined by

$\Delta K = K\ max\ (1-R)$ Equation 3

The resulting values of ΔK are used to construct the vertical lines on figure 2. As this is an informal technical note a detailed log scale is shown below the figure to aid in locating the ΔK values computed. Also shown is a horizontal line to illustrate that for R = 0, R = -.5 and R = -1, the vertical lines not only intersect the data at what can be assumed as the same K max and breaks in slopes but also occur at the same cracking rate. Considering the above equations and assumptions, if da/dn is the same and K max is the same, then Δk must be the same. Thus the applied negative stress ratios appear to result in the same Δk as at R = 0. This will occur if the applied negative stress is transferred across a closed crack. This assumption has been suggested by others and is not new.

If we accept the idea that only positive applied stress effects Δk, and observe that the da/dn curves at negative R ratios are parallel to R = 0, then in using Δk as the independent variable, the R = 0, R = -.5 and R = -1 data will superimpose. In this example, further consideration of the negative R ratios is not necessary.

The da/dn curve for R = .8, by the above arguments, should have a break in slope at about ΔK = 2 MPa $m^{1/2}$. The possible reason that this does not occur and that the data appears scattered and inconclusive below ΔK of about 6 MPa $m^{1/2}$ may be due to plasticity resulting in the crack not satisfying the requirements for LEFM. For LEFM to apply in the conventional form, the crack length must be long enough so that local features such as plastic zone size, crack front geometry and microstructure can be ignored (long crack). At $\Delta K = 6$ MPa $m^{1/2}$, K max at R = .8 is approximately 3 times that of R = .4. G is thus about 9 times larger than at R = .4. As K max is the same, k_ε is also 9 times larger. Considering the practicalities of testing, the crack size for R = .8 is probably smaller. While the probability that at R = .8 is too small relative to the plastic zone is only conjecture, we will note that above $\Delta K = 6$ MPa $m^{1/2}$ the crack growth curve is coincident with that of R = .4 and proceed.

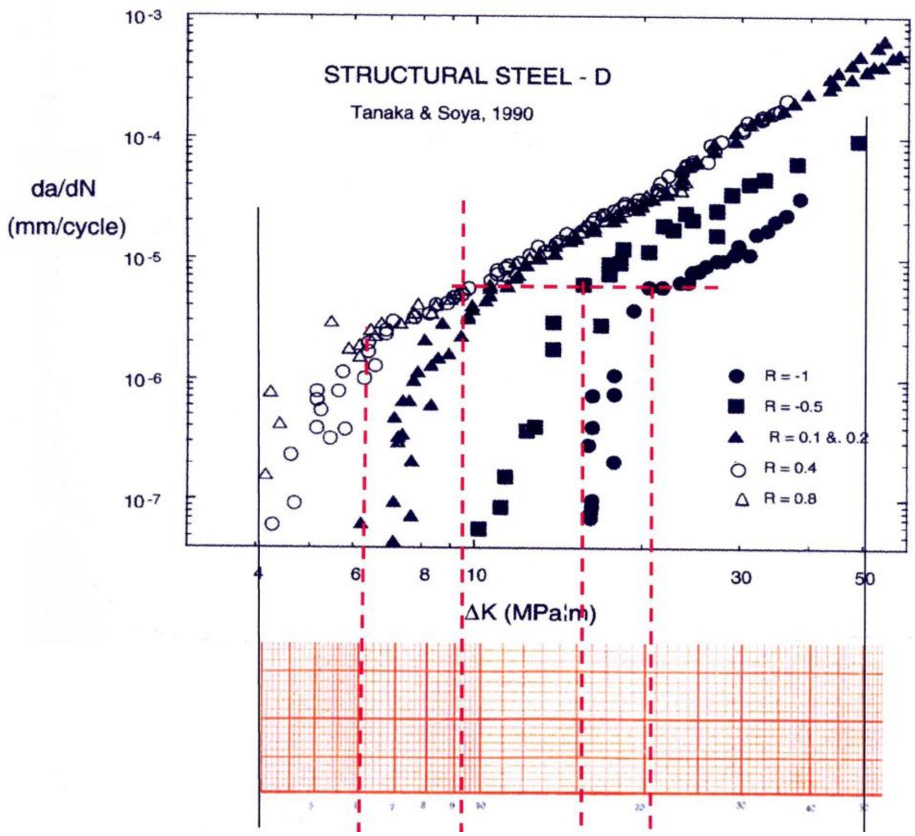

Figure 2 locating the same maximum K on the da/dn curves

Step 2

The stress-strain curve is assumed to have an initial linear elastic slope defined by Young's modulus E. For some materials assuming a linear elastic initial slope is a convenience rather than a reality (Yield defined by 2% offset). The linear assumption allows linear elastic analysis including the derivation of the strain energy term $G = K^2 / E$ and the Neuber rule adaptation to fatigue $(K_t S)^2 / E = \sigma \varepsilon$. We have (again for convenience) assumed that the stress-strain intensity curve also has an initial linear slope defined by E so that it is compatible with the derivation of G. We also have assumed that for this portion of the curve LEFM applies and thus the effective stress can be determined by equation 1.

With this background we are going to assume that there is (at least in theory) a linear portion of a da/dn curve that can be modeled by

Da/dn =C (ΔK effective)^n Equation 4

This equation is that of a straight line in log-log space and it appears identical in form to the Paris equation used to model the region where equation 2 is needed in the stress-strain intensity approach. Thus the constant C and the slope n will differ substantially from those of the Paris equation.

This linear portion requires a long crack region prior to the beginning of significant slope change. In any particular set of data, the existence or useful extent of this region depends upon the crack sizes used, the thickness of the material and, material properties. For steel D these conditions appear to be satisfied although the actual slope is difficult to determine. The slopes shown were obtained by iterations between steps 2, 3, 4.

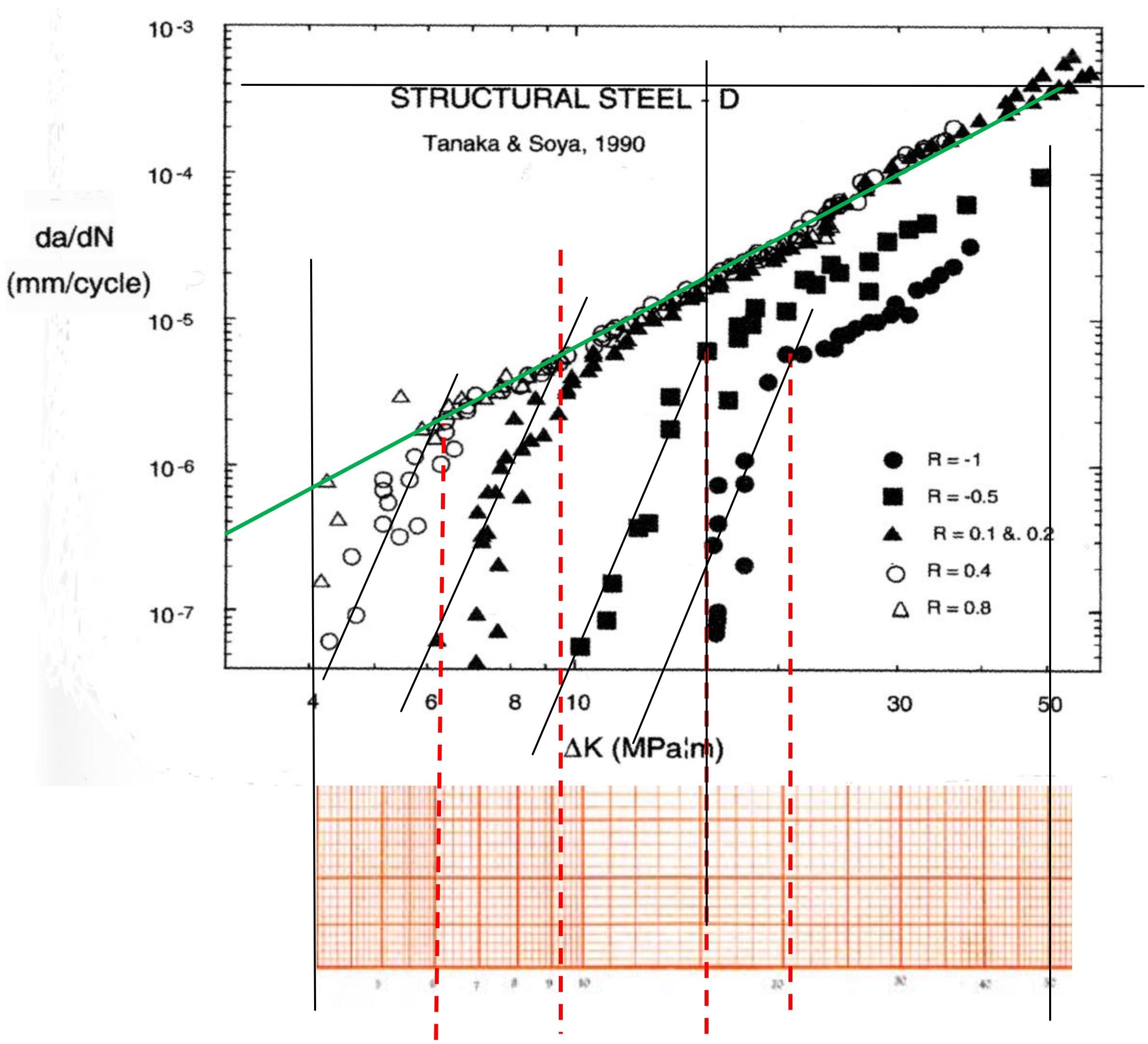

Figure 3 shows the estimate of linear slope.

Step 3

We can write ΔK effective as $\Delta K/(1\text{-}R)^{(1\text{-}m)}$ by replacing K max with its equivalent ΔK/(1-R) from equation 3. For a given da/dn, equation 4 for any two stress ratios in the LEFM portion of the curve becomes

$$\Delta K_2/(1-R_2)^{(1-m)} = \Delta K_1/(1-R_1)^{(1-m)} \text{ and thus } \Delta K_1/\Delta K_2 = [\,(1-R_2)/(1-R_1)\,]^{(1-m)} \quad \text{Equation 5}$$

The value of m is selected so that equation 5 is true. In this case, selecting ΔK at the break in slope on the R = .4 curve and ΔK on the R = .1 curve at the same da/dn

$$8.4/6.1 = (.9/.6)\,]^{(1-m)}$$

The value of (1 – m) is estimated to be .75 and m as .25

Step 4

An examination of the data for steel D shows that when the data is plotted with ΔK as the independent variable, data for all stress ratios fall on a common sloping straight line after $K_{max} =$ 10.2 MPa $m^{1/2}$. This can only occur if k_{max} is the same for all values of K_{max}. Thus the stress-strain intensity curve must look like a stress-strain curve for a mild steel. And appear as in figure 4.

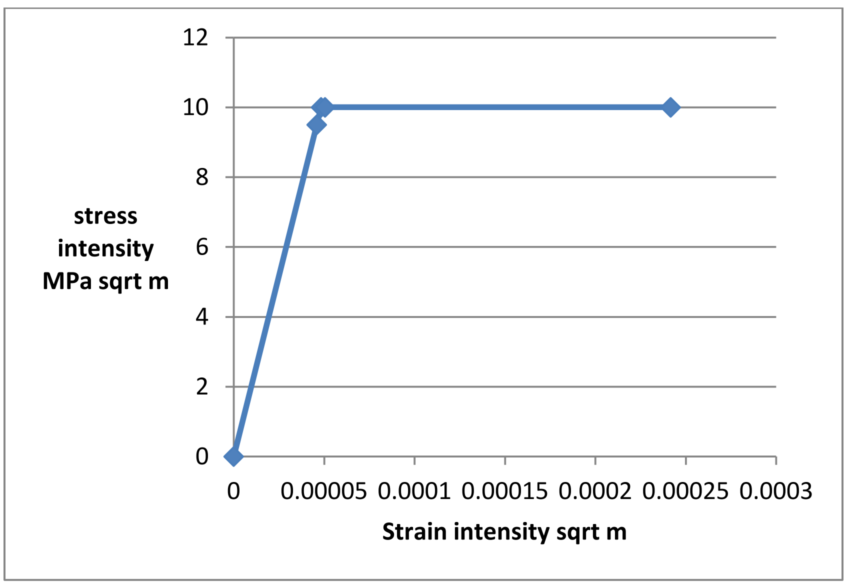

Figure 4 Stress-strain intensity curve for Steel D

Step 5

Table 1 shows the values of ΔK effective calculated using equation 2 , the value of m estimated above (.25), the stress-strain intensity curve of figure 4 and, and the ΔK coordinates of representative da/dn taken from the common da/dn curve.

Also (as will be demonstrated) extending the elastic slopes will also provide an estimate of ΔK effective at any da/dn including where equation 2 is required. This is a very important "discovery."

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Figure 5 shows the extended elastic slope for R = 0 determined by simply drawing a line parallel to the set of lines of figure 3 that intersects the data at ΔK = 10.2 MPa $m^{1/2}$. The location of this line can also be determined with equation 5. The R = 0 line is convenient as the values of ΔK are the values of ΔK effective and also K max.

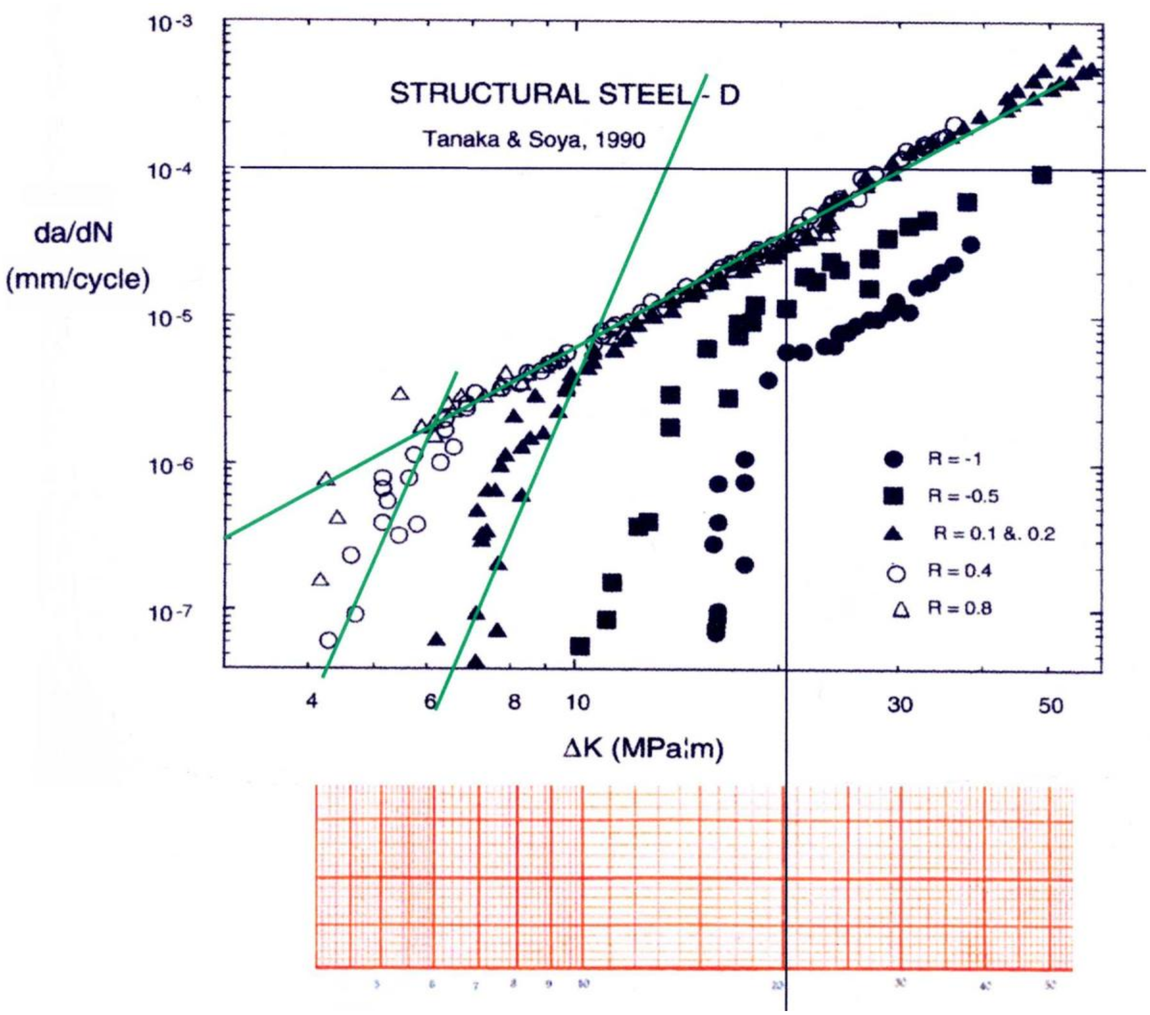

Figure 5 R = 0 elastic slope

Step 6

The last column of Table 1 shows values of ΔK effective estimated using da/dn = C ΔK^n with values of C = 3.468E-20 and n = 13.4 obtained by selecting two points on the elastic slope for R = 0. The values of ΔK effective shown are extremely sensitive to C and n. While not shown, ΔK values scaled directly from the R = 0 elastic slope are almost identical to those calculated using m and ΔK scaled from the data in equation 2.

	m		n		C	
	0.25		10.36		1.53E-16	
da/dn	del K data	del K^m	K max	K max ^(1-m)	Del K eff calculated	Del K eff elastic slope
1.00E-05	12	1.861209718	10.2	5.70755531	10.62295741	11.06493084
5.00E-05	24	2.213363839	10.2	5.70755531	12.63289654	12.92459225
1.00E-04	29	2.320595787	10.2	5.70755531	13.24492881	13.81891037
3.00E-04	50	2.659147948	10.2	5.70755531	15.17723399	15.36483763

Table 1 Calculations for ΔK effective

Closing remarks

The sloping straight line that represents the data runs from about 6 MPa $m^{1/2}$ to about 50 MPa $m^{1/2}$ with only a small apparent alteration at slightly above ΔK = 20 MPa $m^{1/2}$. The assumption that ΔK is the full elastic range is possibly reasonable up to about 20 MPa $m^{1/2}$ while the straight line representation appears reasonable to 50 MPa $m^{1/2}$. If we visualize a second application of the stress-strain intensity diagram with origin at k_{max} and a stress strain intensity curve with the influence of prior load history so that yield will occur at -10.2 MPa $m^{1/2}$ and, a G curve where the strain energy range is given by $\Delta K^2 / E = \Delta G$ then it becomes apparent that the strain energy range will be the same regardless of whether ΔK is or is not fully elastic. Thus if we consider the full elastic ΔK as representative of ΔG and the k_{max} representative of the maximum G of the fatigue cycle, and, ΔK effective as representative of ΔG effective, the lack of any appreciable change in data trend beyond ΔK of 20 MPa $m^{1/2}$ begins to make sense. This might also help explain why the extended elastic slope yields ΔK effective. Extending this reasoning further, the importance of the elastic slope to the stress-strain intensity diagram and to the da/dn curves may be more than coincidental.

Scattered throughout this note are new ideas and reinterpretations of old ideas that stem from the adaptation of Neuber’s rule and the concept of stress-strain intensity. It is obvious that working alone without access to a laboratory and with limitations on available time, it will not be possible to take these concepts and reinterpretations much further. A forum where these ideas can be examined and discussed from different perspectives and backgrounds might be worthwhile. I am well aware that the concepts in this note will be controversial. I do however hope that the notes stimulate positive thought and a fresh look at the entrenched concepts now the subject of most research.

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References

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