Traction Prediction of a Smooth Rigid Wheel in Soil using Coupled Eulerian-Lagrangian Analysis

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Traction is an important performance requirement for a tire and is the force that propels a vehicle forward. Accurate traction prediction has value in reducing development cycle time and improving mechanistic understanding of tire traction performance. On a rigid surface like a paved road, traction can be predicted using an appropriate tire-road friction model in a simulation of tire performance. However, traction on a deformable medium like soil, snow, water, etc. is a more complex phenomenon. Available traction on soil is the sum of the friction at the tire-soil interface and the amount of soil strength extracted by the tire to propel itself.

Soil is a highly plastic material and flows like a liquid as the material approaches its shear strength. The Lagrangian formulation of the balance laws cannot be used satisfactorily to simulate soil because of the excessive soil deformation under even a small load. We present here an approach where the balance laws of continuum mechanics are solved in Eulerian formulation in a subset of the analysis domain (e.g. soil) and in Lagrangian formulation in the remainder of the domain (e.g. tire) thereby allowing very large deformations to occur in soil. This approach has been implemented as the Coupled Eulerian-Lagrangian (CEL) technique in ABAQUS/Explicit.

The CEL capability of ABAQUS/Explicit is used to predict traction during rolling of a rigid cylindrical wheel on soil. The soil medium is modeled using modified Drucker-Prager/Cap constitutive equations available in ABAQUS and the material properties are determined from soil data published in the literature. The approach to modeling soil is validated by comparing predicted traction of a rolling rigid wheel to measured traction test data available in the *literature. Comparison of the measured and predicted traction force shows that this approach is reasonable for predicting traction in soil.*

Keywords*: Tires, Plasticity, Soil, CEL, Experimental Verification, Geomechanics*

1. Introduction

Traction and soil compaction are important tire performance requirements for an agricultural tire. While soil traction is the force driving farming machinery, reducing soil compaction aids root growth and water drainage in farming applications. Accurately predicting traction and soil compaction helps reduce time, cost and resources for agricultural tire development and also in improving mechanistic understanding of tire traction performance. On a rigid surface like a paved road, traction performance can be predicted in a simulation using an appropriate tire-road friction law. However, on a flexible terrain like soil or snow, traction is a more complex phenomenon due to the complex mechanical behavior of the terrain. Soil is an elasto-viscoplastic material that can undergo excessive deformation under the slightest of forces. In addition soil can experience a material instability with relative ease, i.e., undergo catastrophic deformation, without any additional application of forces (e.g. landslides).

Several efforts have been made to predict traction and rut depth during rolling of a tire. The earliest ones are empirical and semi-empirical in nature (Upadhyaya, 1989; Brixius, 1987). However, the mechanics of tire and soil are not thoroughly considered in these studies such as the effect of tire geometry, lug geometry, inflation pressure, slip rate, and the complex deformation and mechanics of soil. Hence, it is difficult to apply such approaches to accurately predict or compare traction performance of modern agricultural tires.

One approach to numerically simulate soil is to consider individual soil particles as rigid spheres and solve the equations of motion for each of the particles along with the contact constraints. This approach has been used for dry soil with some success (Ting, 1989 and Nakashima, 2004). However, all soil particles are assumed to be spherical and the size of individual rigid bodies has to be fictitiously increased many times to keep the computational costs reasonable. Additionally, the effect of moisture content is modeled approximately. Agricultural soils are sometimes wet and the large size of the soil bin to obtain a steady state solution makes this approach impractical with current computational resources.

Another approach is to approximate soil as a continuum and solve the balance laws of continuum mechanics using a numerical technique such as the finite element method. Effectiveness of this approach depends on whether soil can be approximated as a continuum; the approximation being reasonable only when the size of the smallest finite element is much larger than the largest soil particle. Hence, validity of this approach depends on the size of soil particles (soil type) and application. The effect of individual particles and moisture content is averaged over a finite element in this approach.

Several researchers (Chiroux, 2005; Shoop, 2008) have used the continuum approach with moderate success. For example, Chiroux simulated rolling of a rigid wheel on soil under various vertical loads and slip rates using ABAQUS/Explicit. Predicted and measured rut depth compared well but predicted stresses under the soil did not compare well with measurements. To the best of the authors' knowledge all prior studies considered soil motion in a Lagrangian description of motion which is not well suited for similar analyses as finite elements in the soil domain undergo excessive deformation causing numerical errors and inaccuracies in the solution.

Here we present an improvement to the continuum approach by formulating the balance laws in the soil domain in an Eulerian description of motion. This formulation allows excessive deformation of soil without numerical errors and is the formulation typically used for fluid dynamics simulations. However, the boundary of the soil domain is not tracked exactly (see Benson, 1992 for details). The aforementioned approach is evaluated by comparing results from simulations to experimental data reported in literature. Block (Block, 1991) conducted traction measurements on a rigid cylindrical wheel with rubber coating on its surface and rolling on soil. He measured traction using a custom developed traction control and measurement system Traction Research Vehicle (Burt, 1980; Lyne, 1983) at the USDA-ARS National Soil Dynamics Lab at Auburn, AL.

The problem statement is formulated in Section 2, the solution approach is described in Section 3 and the approach to determine the material properties is briefly reviewed in Section 4. Section 5 describes the mechanics of traction in soil, Section 6 presents results of the simulations and validation of predicted traction and Section 7 summarizes the conclusions of this work.

2. Problem Definition

A schematic sketch of one-half of the problem in the undeformed configuration involving a rigid cylindrical wheel of radius R and thickness $2h$ rolling on a rectangular bin of soil is depicted in Figure 1. The origin of the coordinate system is indicated by a reference dot and a plane of symmetry is assumed on the plane $X_2 = 0$. A vertical force F, an angular velocity ω, and a translational velocity v are applied to the wheel spindle such that a predefined slip rate ξ is reached. The problem described here corresponds to the experimental test setup by Block, 1991.

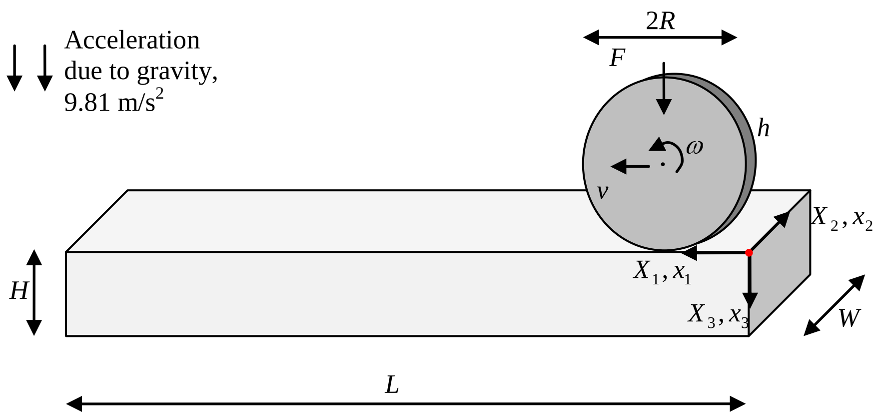

Figure 1. A schematic diagram of the undeformed configuration of a rigid wheel rolling on soil.

The balance laws of continuum mechanics: balances of mass, of linear momentum, and of moment of momentum are solved in the soil domain to obtain the deformation and stress field. The balance of linear momentum includes a body force per unit mass equivalent to acceleration due to gravity of 9.81 m/s^2. The following boundary conditions are also imposed on the soil domain: Reflective symmetry conditions on the plane $x_2 = 0$, traction free condition on the top surface $x_3 = 0$ except where the wheel is in contact with the soil, and fixed boundary conditions on all other faces of the soil domain. Deformation and traction within the region where soil is in contact with the wheel is limited by contact conditions. The prescribed boundary conditions on the wheel are increased slowly using a smooth function to ensure that the motion of the wheel is smooth and continuous. The weight of the wheel and the load on the wheel are both included in the applied force F at the wheel spindle, since the gravitational body force is only applied in the soil domain. The slip rate of the wheel is defined by Equation 1 which is the Society of Automotive Engineers (SAE) convention.

$$\xi = \frac{\omega R - v}{v} \tag{1}$$

Soil and the rigid wheel are initially assumed to be at rest, stress free, and barely touching each other.

Constitutive equations accurately describing the mechanical behavior of soil are critical for accurately predicting tire traction on soil using the approach outlined in this work. Soil is a composite of small granular particles of different sizes, air, water and some organic matter. The relative composition of each of these individual components and size distribution of solid particulates together define the elasto-viscoplastic response of soil. We use the modified Drucker-Prager/Cap constitutive equations to describe the mechanical behavior of soil and details of these equations are presented in the ABAQUS theory manual. These constitutive equations capture many important features of soil mechanics – shear failure, soil compaction, and excessive plastic deformation. Viscosity/Creep of soil is ignored here as the time scale for rolling of an agricultural tire is too short compared to time scales of soil viscosity. The material parameters for this constitutive equation can be determined using triaxial testing of soil.

3. Solution Approach

We solve the above problem using ABAQUS/Explicit 6.11-2, a commercial Finite Element Analysis software. The soil domain is discretized into 8-noded brick elements with one integration point. The portion of the void region above the top surface of the soil needs to be discretized to track the boundary of soil as it gets pushed up as the wheel rolls on the soil. The amount of soil inside a finite element and the boundary of the soil surface are tracked internally by ABAQUS/Explicit through a variable that is equivalent to the volume fraction of soil inside a finite element. The entire Eulerian domain was discretized into 235 × 36 × 36 elements and the region near the contact with the wheel was made finer as depicted in Figure 2. The rigid wheel was modeled using 4-noded shell elements. Mass scaling and time scaling were not used to accelerate the solution process.

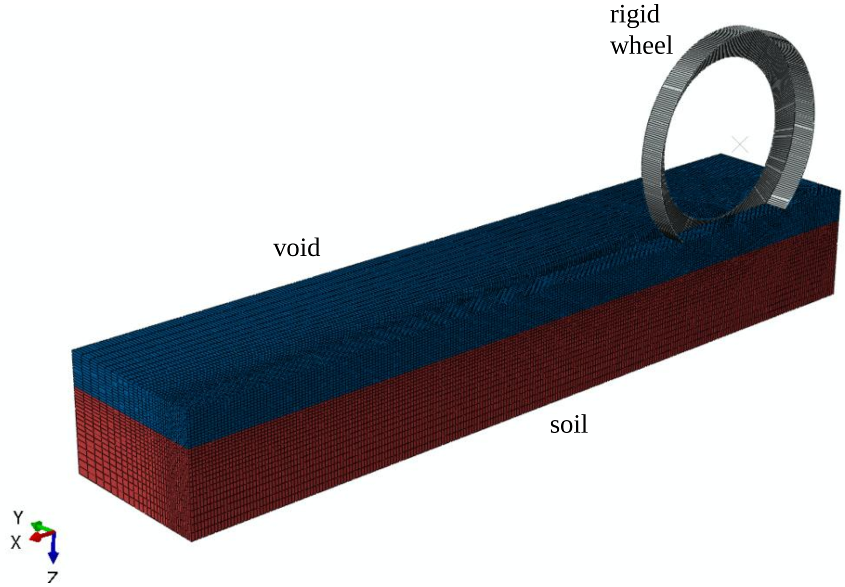

Figure 2. Finite element discretization used for simulating rolling of wheel on soil. Finer discretization is used near the region of contact with wheel

4. Determination of material parameters

Bailey and Johnson (Bailey, 1989) measured yield surfaces of four soils using a custom-built Triaxial test rig and proposed a USDA-ARS National Soil Dynamics Lab – Auburn University

(NSDL-AU) soil model, a unified model for the yield surface of a soil. Values of material parameters for the NSDL-AU soil model were used to determine values of material parameters in the modified Drucker-Prager/Cap constitutive equations for Norfolk Sandy Loam (72% sand, 17%silt, and 11% clay) and Decatur Clay Loam (27% sand, 43% silt, 30% clay) and are listed in Table 1. Cohesion *c* is assumed to be nearly zero for loose agricultural soil, angle of friction of the material and the normal consolidation curve can be determined directly from the NSDL-AU soil parameters. Values of parameters of the transition surface were chosen so as to make the transition surface small compared to the cap and shear failure surface. Values of the coefficient of friction measured by Plyler (Plyler, 2002) were used in this analysis. Plyler used an instrumented split rigid wheel and a pressure bar to measure the local normal and tangential stress along the wheel circumference to determine an average coefficient of friction. We used a constant coefficient of friction as the measured coefficient of friction was reported to not significantly depend on sliding velocity or normal pressure.

Table 1. Values of material parameters in the modified Drucker-Prager/Cap constitutive equations for Norfolk Sandy Loam and Decatur Clay Loam. See ABAQUS Theory manual for description of these parameters.

	Norfolk Sandy Loam	**Decatur Clay Loam**
Angle of Friction ϕ [deg]	59.41	70.18
Coefficient of friction between rubber and soil μ	0.4	0.38
Young's modulus E [kPa]	326.0	199.0
Poisson's Ratio ν	0.0	0.0
Mass density ρ [kg/m^3]	1,255.0	1,096.0

5. Mechanics of Soil Traction

A schematic diagram of the forces acting at the interface of a wheel rolling on soil is exhibited in Figure 3. The net traction force on soil is the force parallel to the direction of travel, developed by the wheel (ASABE, 2011), and is the sum of the frictional force at the wheel-soil interface and the amount of soil strength extracted by the tire with the mechanics of each of these components being quite different. The frictional component of net traction at each node in the soil wheel interface is the 1-component of the tangential force vector due to soil-wheel friction at the node. The total frictional contribution to net traction is the sum of the 1-component of the force vector for all nodes within the wheel-soil interface. The source of tangential contact force is the interface friction; however, the tangential contact force can change depending on the load and slip rate. Furthermore, a portion of the frictional force can assist or oppose motion of the wheel depending on the location of the slip plane.

Similarly, the soil strength contribution to net traction can be determined from the 1-component of the normal contact force vectors due to the interface contact pressure on the wheel surface. Normal contact force vectors can oppose or assist wheel motion depending on the direction of the 1-component of the contact force vector from contact pressure. Contributions to normal contact force vectors come from the elastic response of compacted soil and from the soil shear strength. However, due to weak elastic properties and almost all soil deformation being plastic (Johnson, 2000) the contribution to contact pressure from elastic response of soil is likely very small. The phenomenon of soil shear strength contributing to traction is singularly complex and depends on the local stress state and compaction of soil in a large region underneath the wheel. Maximum traction is attained when slip planes form in soil and the value of traction is linked to the shear strength of soil along these slip planes. However, slip planes are local instabilities in soil and several factors together determine their formation – the compaction and stress state of soil and also the confinement offered by soil in the neighborhood.

We use the following convention in the rest of this document. Sum of 1-component of normal and tangential contact force vectors opposing motion of the wheel is referred to as 'motion resistance'. The sum of 1-component of normal contact force vectors is referred to as 'soil-strength-driving', while the sum of 1-component of tangential contact force vectors is referred to as 'friction-driving'. Tangential and normal contact force vectors at a contact interface, respectively, are conveniently output as CSHEARF and CNORMF variables by ABAQUS/Explicit.

6. Results

We present here results for the test setup used by Block (Block, 1991) by setting R = 686 mm, h = 152.4 mm, and choosing Norfolk Sandy Loam soil. Dimensions of the soil bin used by Block are 57.3 m long × 6.1 m wide × 1.8 m deep; however, to reduce computational cost we simulate only a small region of interest close to the wheel within the soil bin. We simulate a 4 m long × 1.0 m wide × 0.5 m deep soil bin which corresponds to half of the soil domain. Furthermore, a 0.2 m high void region above the top surface of the soil was also simulated. The wheel moves longitudinally at a speed of v_1 = 0.15 m/s irrespective of the slip rate; the rotational speed is chosen to ensure the desired slip rate from Equation 1.

Figure 4 depicts a contour plot of variable PEEQV and PEQC2 in the soil domain for a vertical load of F = 5.8 kN and slip rate ξ = 23% after the wheel has traveled 1.5 m. The PEEQV variable is the position of the cap surface on the hydrostatic pressure axis and is a measure of soil compaction and the volumetric plastic strain. A contour plot of PEEQV shows that soil under the wheel is in a compacted state and that there is some relaxation of soil after the wheel rolls over it. Soil beside the wheel sidewall is looser than the rest of the domain after rolling of the wheel. Slip planes can also be been seen in the soil in front of the wheel where the soil is in a looser state than in the rest of the domain. PEQC2 is the plastic strain on the cap surface and is a monotonically increasing variable that is a measure of the total deformation on the cap surface. Note that the maximum contour value of PEQC2 is 1,109% indicating that the material has deformed excessively and we use logarithmic distribution of contour levels so that even moderate strains of 50 to 70% are distinguishable. A contour plot of PEQC2 also indicates a slip plane where the material on the left has not deformed at all (strains less than 1%) while the material on the right has deformed significantly (strain is greater than 40%). These contour plots indicate that the simulations using a continuum approach can predict presence of slip planes in soil.

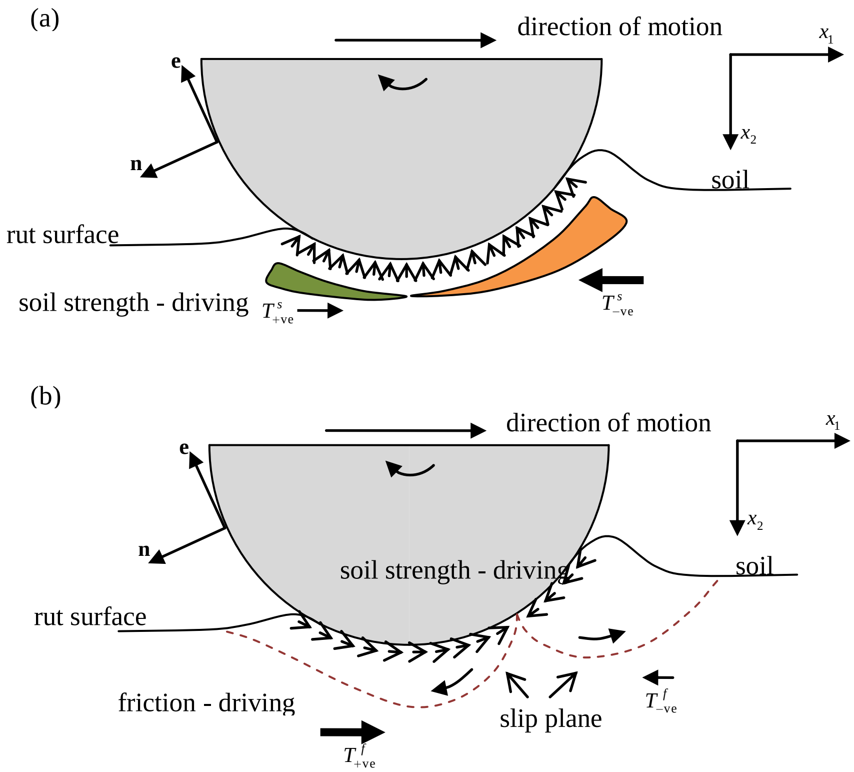

Figure 3. A schematic diagram describing forces contributing to net traction on a rigid wheel rolling on soil. Contribution due to (a) soil shear strength and (b) friction are illustrated.

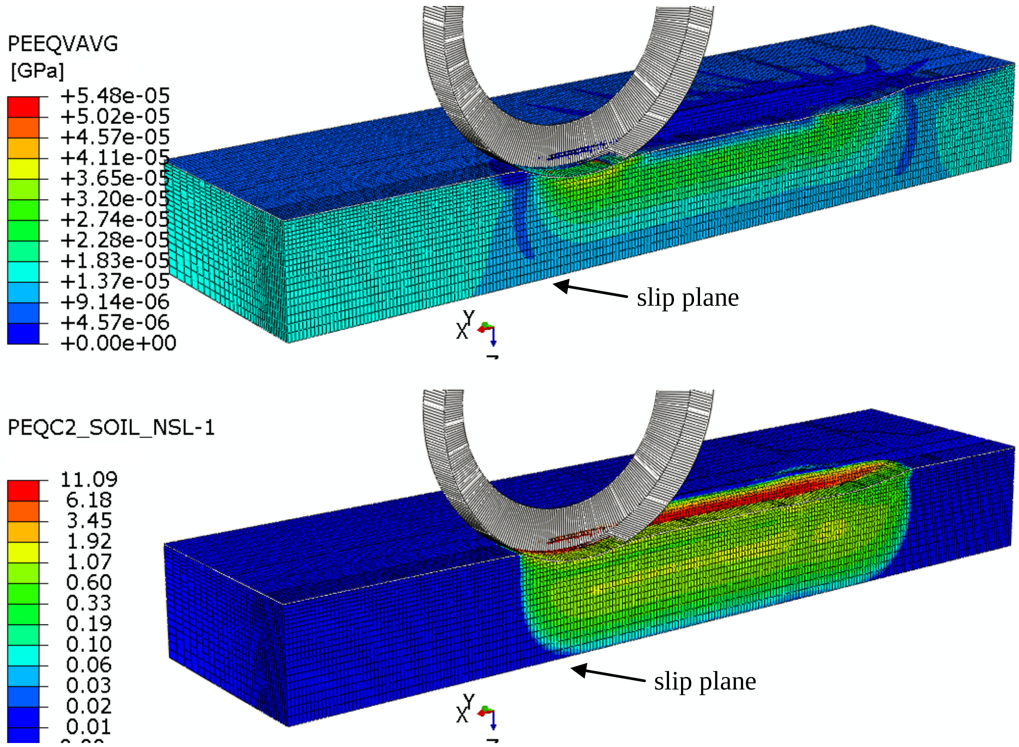

Figure 4. Contour plot of PEEQV in GPa (top) which is the position of the cap surface and of PEQC2 (bottom) which is the plastic strain on the cap surface at each material point.

The view along the X_2 axis of the deformed surface showing the rut surface is exhibited in Figure 5. The surface of soil directly underneath the wheel is below the rut surface indicating some relaxation of soil after the wheel has passed over soil. Rut depths after the wheel rolled over the soil were not reported by Block and hence experimental data was not available for comparing with this prediction.

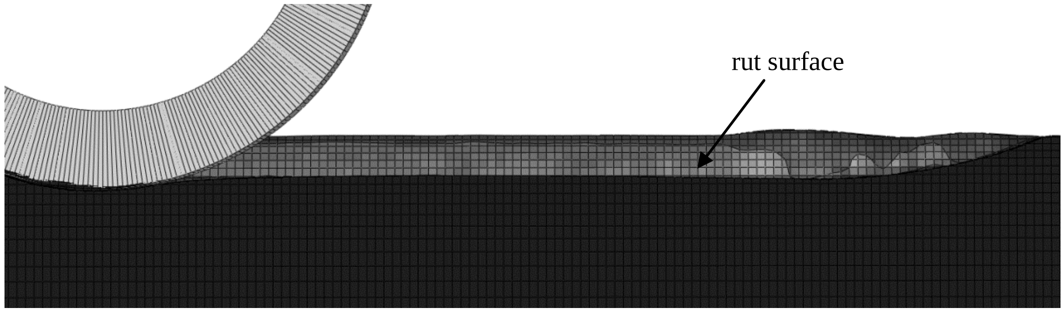

Figure 5. The view along the X_2 axis of the deformed soil surface

The time history of soil traction and its contributions from friction and shear strength of soil depicted in Figure 6 show that the solution reached steady state after 1 m translation of the wheel. There is some numerical noise in the solution. In the rest of this work, predicted net traction at a particular loading is computed as its average value during the last 1.3 m of wheel translation. It can be seen that motion resistance is opposing motion of the wheel while soil shear strength and friction are aiding motion of the wheel. Net traction is the sum of these individual components and was equal to 0.85 kN for one-half of the rigid wheel.

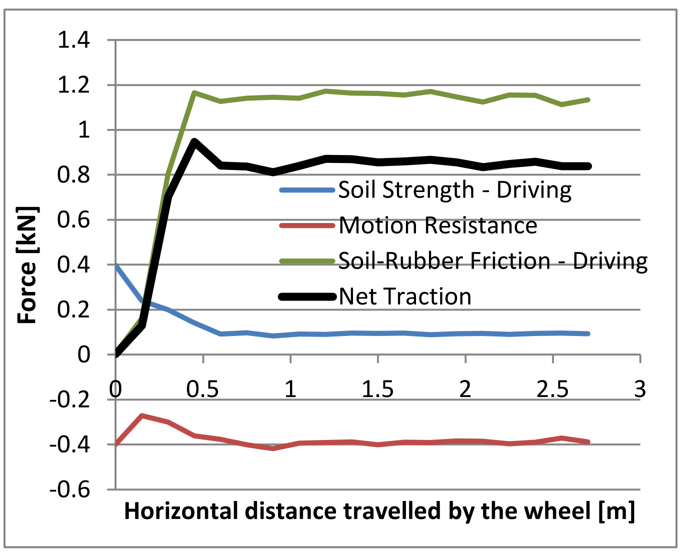

Figure 6. Time history of net traction and contributing components during rolling of one-half of the rigid wheel over soil

Distributions of the contribution of frictional force and contact pressure to net traction along the wheel circumference are exhibited in Figure 7 at three instances during the simulation: a) after vertical loading, b) after the wheel moved 1m, and c) after the wheel moved 2.7m. After loading, the distribution of pressure and friction is such that there is not net traction from both components. Due to a high slip rate of 23.0% friction is driving the motion over the entire footprint during wheel motion as opposed to the schematic description in Figure 3. However, contact pressure in front of the wheel is resisting motion (motion resistance) and contact pressure behind the wheel is aiding motion of the wheel. This behavior is reasonable and is the expected response during a wheel rolling on soil. The solutions of contact pressure and friction are not smooth but general trends are reasonable.

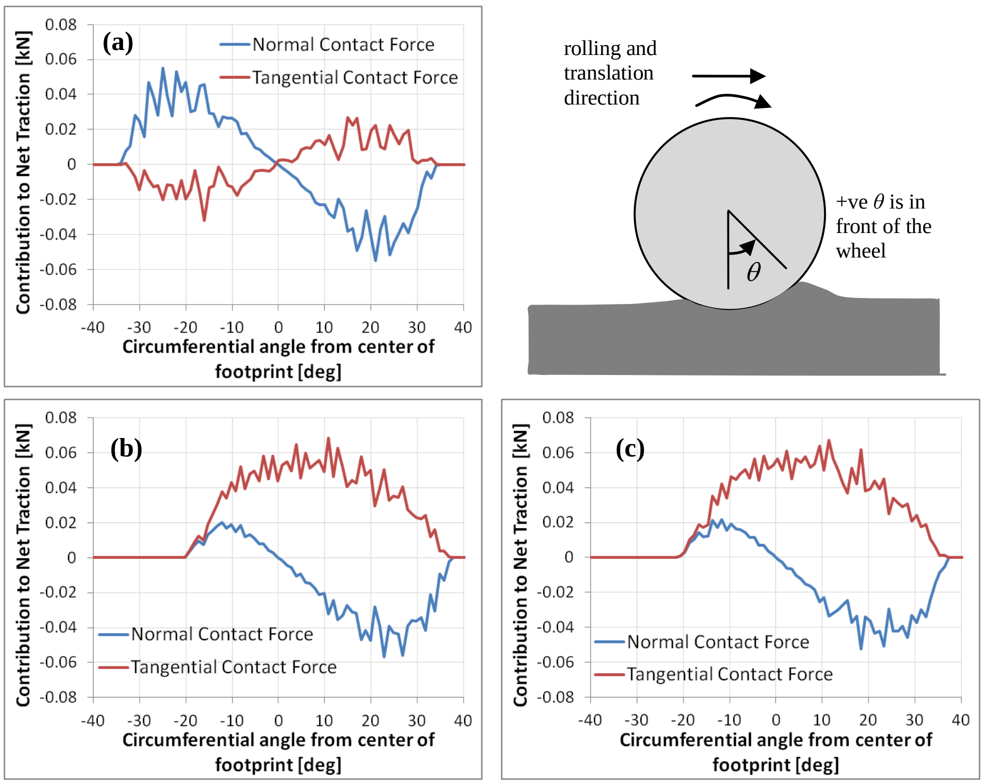

Figure 7. Distribution along the wheel circumference of contribution of friction and soil strength to net traction of rigid wheel at different times: a) after loading, b) after wheel moved 1 m, and c) after wheel translated 2.7 m. Zero value of circumferential angle implies centerline of the wheel and positive angle is in front of the wheel

Table 2. List of vertical loads and slip rates tested by Block

Condition Label	Vertical Load [kN]	Slip Rate [%]
A1	2.9	11.1
A2	2.9	23.0
B1	5.8	11.1
B2	5.8	23.0
C1	8.7	11.1
C2	8.7	23.0
D1	11.6	11.1
D2	11.6	23.0

6.1 Validation of Net Traction on Soil

Validation of predicted traction is presented here for the loading conditions tested by Block listed in Table 2. Block used a different definition of slip rate but we ensure that the applied rotational and translational speeds in our simulations (see Equation 1) are consistent with Block's study. Predicted and measured net traction for the rigid wheel rolling on soil are compared in Figure 10 and 11, respectively, for Norfolk Sandy Loam and Decatur Clay Loam for the loading conditions listed in Table 2. Predicted net traction corresponds to the whole rigid wheel and hence is twice the value from the half wheel simulation. Measured net traction is the average of four repetitions tested by Block. Slope and R^2 of fitted predicted vs. measured line are also reported in Figures 10 and 11.

Results indicate good correlation between predicted and measured net traction as R^2 value of 0.97 is high for both Norfolk Sandy Loam and Decatur Clay Loam. Values of slope and intercept indicate better prediction of net traction for Norfolk Sandy Loam compared to Decatur Clay Loam.

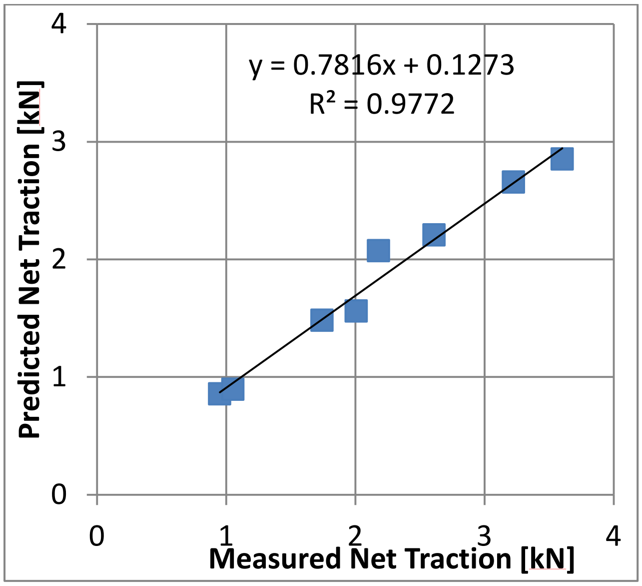

Figure 8. Comparison of predicted and measured traction for Norfolk Sandy Loam for loading conditions listed in Table 2

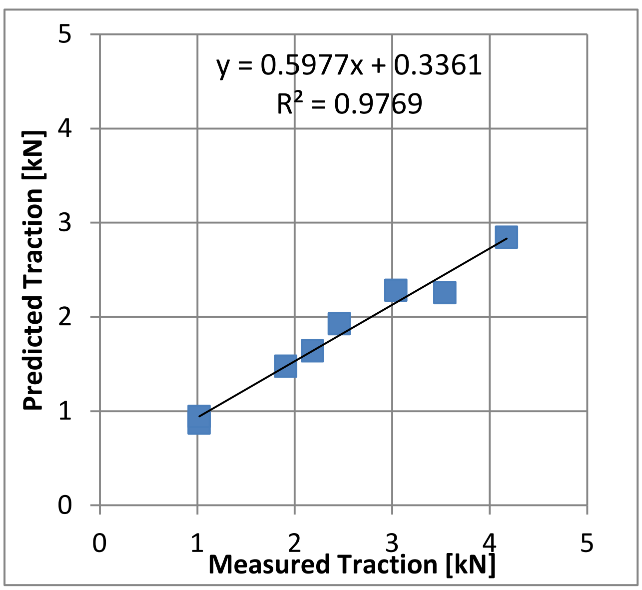

Figure 9. Comparison of predicted and measured traction for Decatur Clay Loam for loading conditions listed in Table 2

7. Summary

We present here an approach to predict traction of a rigid wheel rolling on soil, where the soil material is assumed to be a continuum. ABAQUS/Explicit was used to solve balance laws of continuum mechanics for the soil domain formulated in an Eulerian description of motion to effectively account for excessive deformation of the material. Modified Drucker-Prager/Cap constitutive equations were used to describe the mechanical behavior of soil. The resulting partial differential equations were solved by the Finite Element Method using ABAQUS/Explicit. Values of material parameters of Drucker-Prager/Cap constitutive equations were determined from soil data in the literature.

The approach to predict traction is validated by comparing measured and predicted net traction during various vertical loads and slip rates on two soil types: Norfolk Sandy Loam and Decatur Clay Loam. Measured and predicted net traction correlated well with each other for both soil types and the correlation was better for the sandy loam soil than the clay loam soil.

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