A MATERIAL MODEL FOR FLEXURAL CRACK SIMULATION IN REINFORCED CONCRETE ELEMENTS USING ABAQUS

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Abstract: This paper presents a material model to simulate load induced cracking in Reinforced Concrete (RC) elements in ABAQUS finite element package. Two numerical material models are used and combined to simulate complete stress-strain behaviour of concrete under compression and tension including damage properties. Both numerical techniques used in the present material model are capable of developing the stress-strain curves including strain softening regimes only using ultimate compressive strength of concrete, which is easily and practically obtainable for many of the existing RC structures or those to be built. Therefore, the method proposed in this paper is valuable in assessing existing RC structures in the absence of more detailed test results. The numerical models are slightly modified from the original versions to be comparable with the damaged plasticity model used in ABAQUS. The model is validated using different experiment results for RC beam elements presented in the literature. The results indicate a good agreement with load vs. displacement curve and observed crack patterns.

Key words: Reinforced concrete, crack, tension stiffening, compressive stress-strain relationship.

1 INTRODUCTION

Reinforced concrete is a complicated material to be modelled within finite element packages. A proper material model in finite element model should inevitably be capable of representing both elastic and plastic behaviour of concrete in compression and tension. The complete compressive behaviour should include both elastic and inelastic behaviour of concrete including strain softening regimes. Simulation of proper behaviour under tension should include tension softening, tension stiffening and local bond effects in reinforced concrete elements. Therefore, the development of a finite element model (FEM) may need intensive material testings to incorporate into the material model in any of the finite element [FE] packages available. There are quite large numbers of numerical material models available in the literature with potential to develop complete stress-strain curves of concrete for compression and tension separately based on experiment results. However, these methods are not directly applicable with the input format required for the finite element packages. Therefore, this paper presents and modifies two numerical models that can be easily adopted with minor changes to the ABAQUS FE package (Dassault Systèmes Simulia Corp. [SIMULIA], 2008). Modifications for two stress-strain curves under compression and tension are suggested to be used with the damaged plasticity model in ABAQUS. This material model presented in the paper is capable of representing the formation of cracks and post-cracking behaviour of reinforced concrete elements.

Two numerical stress-strain relations and the damaged plasticity model in ABAQUS are briefly discussed in the first section. This is followed by the suggested modifications and comparison with experiment results on RC beam elements in the following sections.

2 DEVELOPING THE MATERIAL MODEL

2.1 **ABAQUS Damaged Plasticity Model**

ABAQUS software (SIMULIA, 2008) provides the capability of simulating the damage using either of the three crack models for reinforced concrete elements: (1) Smeared crack concrete model, (2) Brittle crack concrete model, and (3) Concrete damaged plasticity model. Out of the three concrete crack models, the concrete damaged plasticity model is selected in the present study as this technique has the potential to represent complete inelastic behaviour of concrete both in tension and compression including damage characteristics. Further, this is the only model which can be used both in ABAQUS/Standard and ABAQUS/Explicit and thus enable the transfer of results between the two. Therefore, development of a proper damage simulation model using the concrete damaged plasticity model will be useful for the analysis of reinforced concrete structures under any loading combinations including both static and dynamic loading ("Abaqus Analysis User Manual – Abaqus Version 6.8" [Abaqus Manual], 2008).

The concrete damaged plasticity model assumes that the two main failure mechanisms in concrete are the tensile cracking and the compressive crushing. In this model, the uniaxial tensile and compressive behaviour is characterized by damaged plasticity.

2.1.1 Tension Stiffening Relationship

In order to simulate the complete tensile behaviour of reinforced concrete in ABAQUS, a post failure stress-strain relationship for concrete subjected to tension (similar to Fig. 01) is used which accounts for tension stiffening, strain-softening, and reinforcement (RF) interaction with concrete. To develop this model, user should input young's modulus (E_0), stress (σ_t), cracking strain ($\tilde{\varepsilon}_t^{ck}$) values and the damage parameter values (d_t) for the relevant grade of concrete. The cracking strain ($\tilde{\varepsilon}_t^{ck}$) should be calculated from the total strain using (1) below:

$$\tilde{\varepsilon}_t^{ck} = \varepsilon_t - \varepsilon_{ot}^{el} \quad (1)$$

Where, $\varepsilon_{ot}^{el} = \sigma_t / E_0$, the Elastic strain corresponding to the undamaged material, ε_t = total tensile strain.

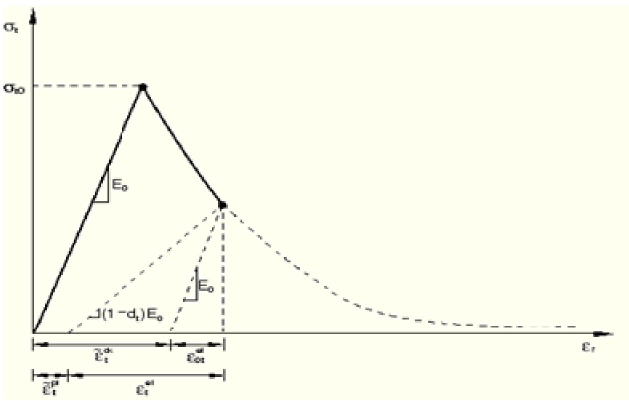


FIGURE 01: Terms for Tension Stiffening Model (Abaqus Manual, 2008)

ABAQUS (SIMULIA, 2008) checks the accuracy of damage curve using the plastic strain values ($\tilde{\varepsilon}_t^{pl}$) calculated as in (2) below. Negative and/or decreasing tensile plastic strain values are indicative of incorrect damage curves which may lead to generate error massage before the analysis is performed (Abaqus Manual, 2008).

$$\tilde{\varepsilon}_t^{pl} = \tilde{\varepsilon}_t^{ck} - \frac{d_t}{(1-d_t)}\frac{\sigma_t}{E_0} \tag{2}$$

2.1.2 Compressive Stress-Strain Relationship

To define the stress-strain relation of concrete, user needs to enter the stresses (σ_c), inelastic strains ($\tilde{\varepsilon}_c^{in}$) corresponds to stress values, and damage properties (d_c) with inelastic strains in tabular format. Therefore, total strain values should be converted to the inelastic strains using (3):

$$\tilde{\varepsilon}_c^{in} = \varepsilon_c - \varepsilon_{oc}^{el} \tag{3}$$

where, $\varepsilon_{oc}^{el} = \sigma_c / E_0$, ε_{oc}^{el} = Elastic strain corresponding to the undamaged material and ε_c = Total tensile strain. Further, corrective measures should be taken to ensure that the plastic strain values ($\tilde{\varepsilon}_c^{pl}$) calculated using (4) are neither negative nor decreasing with increased stresses. (Abaqus Manual, 2008)

$$\tilde{\varepsilon}_c^{pl} = \tilde{\varepsilon}_c^{in} - \frac{d_c}{(1-d_c)}\frac{\sigma_c}{E_0} \tag{4}$$

Typical compressive stress-strain relationship with damage properties and terms are illustrated in Fig. 02.

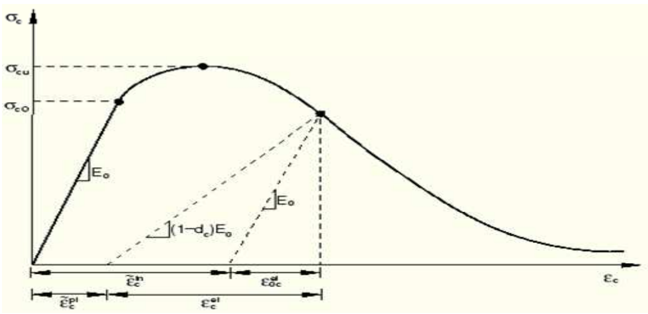


FIGURE 02: Terms for Compressive Stress-Strain Relationship (Abaqus Manual, 2008)

2.2 **Numerical Model for Tensile Behaviour of Concrete**

There are different forms of tension stiffening models presented in the literature as reviewed in the paper by Nayal and Rasheed (2006). The model developed by Nayal and Rasheed (2006) is selected for the present study as it is applicable for both reinforced and fibre reinforced concrete with only minor changes. Also, this method indicates similarity to the tension stiffening model that is needed for ABAQUS concrete damaged plasticity model. This tension stiffening model was originally based on the homogenized stress-strain relationship developed by Gilbert and Warner (1978) which accounts for tension stiffening, tension softening and local bond slip effects. Two descending portions of the tensile stress-strain graph has accurately captured the response caused by primary and secondary cracking phenomena (Gilbert & Warner, 1978). The layered tension stiffening parameters used by Gilbert and Warner (1978) is replaced with a single set of stiffening parameters applicable to the entire tensile zone by the Nayal and Rasheed to avoid the discontinuity in global response (Nayal & Rasheed, 2006). The Nayal and Rasheeds' tension stiffening model for reinforced concrete elements is shown in Fig. 03 and the modified tension stiffening model for the present study is presented in Fig. 04.

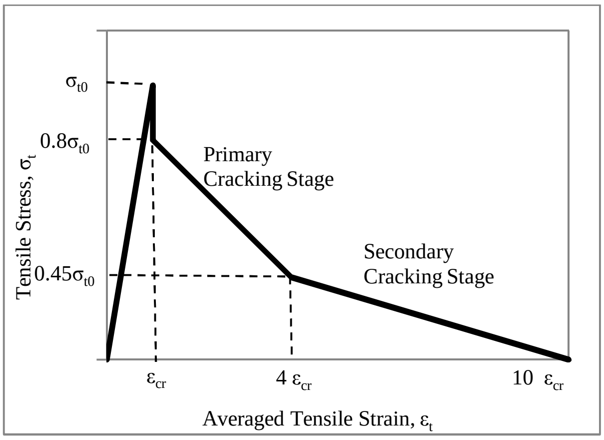


FIGURE 03: Nayal and Rasheed's (2006) Tension Stiffening Model

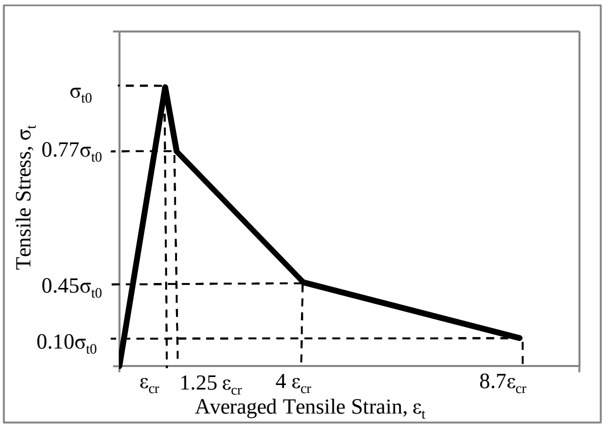


FIGURE 04: Modified Tension Stiffening Model for ABAQUS

The sudden drop at critical tensile strain ε_{cr} from maximum tensile stress σ_{t0} to 0.8 σ_{t0} as used by Nayal and Rasheed (2006) and Gilbert and Warner (1978) is slanted from (ε_{cr}, σ_{t0}) to (1.25 ε_{cr}, 0.77 σ_{t0}) to avoid run time errors in ABAQUS material model. After this point, the stress-strain curve follows exactly the Nayal and Rasheed (2006) tension stiffening model in both primary and secondary cracking regions but stopped at (8.7 ε_{cr}, 0.10 σ_{t0}) to avoid ABAQUS run time errors.

2.3 **Numerical Model for Stress-Strain Curve in Compression**

The complete stress-strain curve for concrete under compression is derived using the experimentally verified numerical method by Hsu and Hsu (1994). This model can be used to develop the stress-strain relationship under uni-axial compression up to 0.3 σ_{cu} of stress in the descending portion only using the maximum compressive strength (σ_{cu}). This method is briefly presented in the below section for the concrete with maximum compressive strength up to 62 MPa. For other concrete grades, modifications should be made by referring to the original paper by Hsu and Hsu (1994, p.308). Fig. 05 defines the ultimate compressive stress (σ_{cu}), strain at σ_{cu} (ε_0) and the strain corresponding to the stress at 0.3 σ_{cu} in the descending portion (ε_d). A linear stress-strain relationship which obey Hooke's law is assumed up to 50% of the ultimate compressive strength (σ_{cu}) in the ascending portion. The numerical model by Hsu and Hsu (1994) is used only to calculate the compressive stress values (σ_c) between the yield point (at 0.5 σ_{cu}) and the 0.3 σ_{cu} in the descending portion using (5):

$$\sigma_c = \left(\frac{\beta(\varepsilon_c/\varepsilon_0)}{\beta - 1 + (\varepsilon_c/\varepsilon_0)^{\beta}}\right)\sigma_{cu} \tag{5}$$

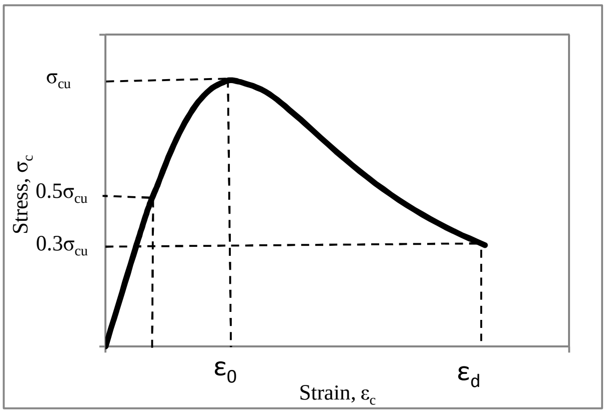


FIGURE 05: Compressive Stress-Strain Relationship for ABAQUS

where, the parameter β which depends on the shape of the stress-strain diagram is derived from (6) and the strain at peak stress ε_0 is given by (7):

$$\beta = \frac{1}{1 - [\sigma_{cu} / (\varepsilon_0 E_0)]} \tag{6}$$

$$\varepsilon_0 = 8.9 \times 10^{-5} \sigma_{cu} + 2.114 \times 10^{-3} \tag{7}$$

The initial tangential modulus, E_0 is given by:

$$E_0 = 1.2431 \times 10^2 \sigma_{cu} + 3.28312 \times 10^3 \tag{8}$$

Note: in the above equations, σ_c, σ_{cu} and E_0 are in kip/in^2 (Conversion factor: 1 MPa = 0.145037743 kip/in^2).

The ε_d (strain at the 0.3 σ_{cu} in the descending portion) is iteratively calculated using (5) when σ_c =0.8 σ_{cu}.

2.4 Other Material Properties

2.4.1 Damage Parameters

The tensile damage parameter, d_t is defined as the ratio of the cracking strain to the total strain. Similarly, the compressive damage parameter, d_c is defined as the ratio between the inelastic strain and total strain. If damage parameters are not specified, the model behaves as a plasticity model.

2.4.2 Stiffness Recovery

Default values for stiffness recovery factors are assumed in the present study. Therefore, the compressive stiffness recovery factor, w_c = 1 is used assuming that compressive stiffness is fully recovered upon crack closure as the load changes from tension to compression and the tensile stiffness recovery factor, w_t = 0 is used assuming that tensile stiffness is not recovered as the load changes from compression to tension once crushing of concrete is initiated.

2.4.3 Rate dependant data

No attention is paid to the rate dependant data which accounts for the strain rate effect. Therefore, in order to simulate accurate response for impact loadings, the parameters should be defined where relevant.

All other material properties are set to default values in ABAQUS (SIMULIA, 2008).

3 APPLICATION ON FLEXURAL CRACKS

3.1 Three Point Bending Test of a RC Beam

Peeters et al. (1996) and Maeck (2003) have presented the experiment results of a RC beam tested under three point bending. The 6m long RC beam was supported at 1.2m distance from either of the edges and loaded at centre. The experiment load vs. deflection curve at different loading and unloading steps are presented by Peeters et al. (1996). Crack patterns at different end of different loading steps are presented in Peeters et al. (1996) and Maeck (2003). This experimental work is simulated in ABAQUS FE package (SIMULIA, 2008) using the above material model for the validation purposes. The maximum compressive strength of 51.2 MPa (Peeters et al., 1996) is used to develop stress-strain relations under compression and tension.

3.2 Finite Element Modelling of the RC Beam

To develop the FE model of RC beams, smeared reinforcement technique available in ABAQUS is used accounting for both flexural and shear RF. The experiment beam consists of continuous longitudinal reinforcement with 31mm cover to the centre of RF from both top and bottom outer surfaces to enhance the flexural strength. Shear links have been provided using 8mm bars with 15mm clear cover from each of the outer surfaces with 200mm spacing. The beam cross section was partitioned as in Fig. 06 to represent all the RF layers accurately using smeared RF technique. The surfaces 01 and 04 are assigned with RF layers of 603.2mm^2 area with 212mm spacing in the corresponding local direction. The surfaces 02, 03, 04 and 05 are assigned with RF layers of 50.27mm^2 area with 200mm spacing perpendicular to the direction mentioned above to represent the shear RF. All the RF layers are embedded within the concrete element using embedding technique available in ABAQUS (SIMULIA, 2008).

The FE model of RC beam thus consists of two types of materials (concrete and reinforcement). The material properties for concrete with 51.2 MPa ultimate compressive strength are derived using above relationship. Tab. 01 gives the compressive stress-strain the values including compressive damage properties whereas Tab. 02 gives the tensile stress-strain values with tensile damage properties. Material for all the reinforcement is treated as steel with young's modulus equal to 218 GPa.

The beam is meshed with approximate element size with 50mm x 50mm x 12.5mm in longitudinal, transverse and thickness direction respectively. Mesh convergence study carried out using displacement measurements depicts that the above mesh is finer enough to obtain a reliable result.

Load vs. displacement values measured at the centre of the beam from the experiment results (Peeters et al., 1996) and the present finite element modelling are shown in Fig. 07 & 08 respectively. The experimentally obtained crack patterns (Maeck, 2003) and FE crack patterns at two loading steps (at 8kN, and 24kN) are compared in Fig. 09 & 10. Results indicate that the FE results using above material model is well matched with the experiment results.

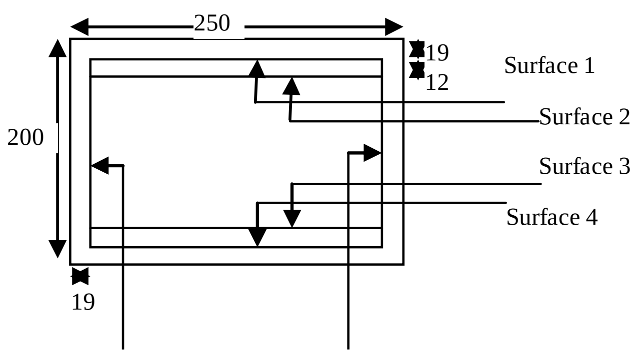


FIGURE 06: Cross Section of the RC Beam with RF Surfaces

TABLE 01: Compressive Stress-Strain Values for 51.2 MPa Concrete

Stress (σ_c) N/mm^2	Inelastic Strain ($\tilde{\varepsilon}_c^{in}$)	Damage (d_c)
2.56E+07	0.00E+00	0.00E+00
3.64E+07	1.00E-04	1.05E-02
4.49E+07	2.81E-04	2.95E-02
4.97E+07	5.87E-04	6.16E-02
5.12E+07	1.01E-03	1.06E-01
4.90E+07	1.76E-03	1.85E-01
4.43E+07	2.60E-03	2.73E-01
3.89E+07	3.46E-03	3.63E-01
3.37E+07	4.31E-03	4.53E-01
2.92E+07	5.14E-03	5.40E-01
2.54E+07	5.95E-03	6.25E-01
2.22E+07	6.74E-03	7.07E-01
1.95E+07	7.51E-03	7.88E-01

TABLE 02: Tensile Stress-Strain Values for 51.2 MPa Concrete

Stress (σ_t)N/mm^2	Cracking Strain ($\tilde{\varepsilon}_t^{ck}$)	Damage (d_t)
2.36E+06	0.00E+00	0.00E+00
1.89E+06	4.07E-05	3.85E-01
9.45E+05	2.93E-04	9.00E-01
2.13E+05	8.07E-04	9.91E-01

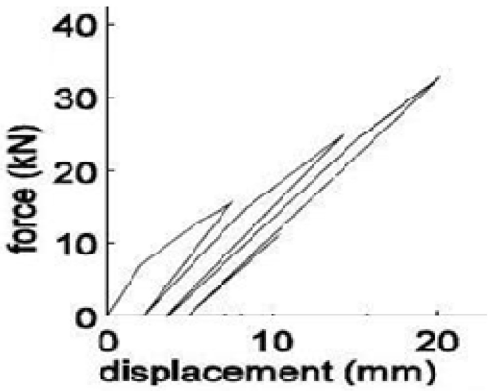


FIGURE 07: Experiment Load vs. Displacement (Peeters et al., 1996, p.1355)

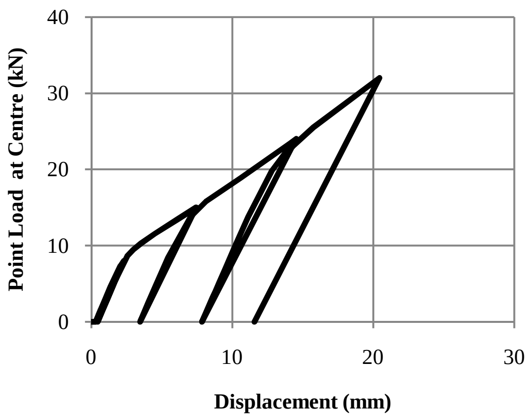
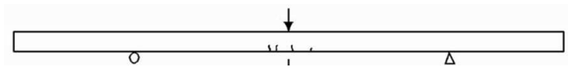
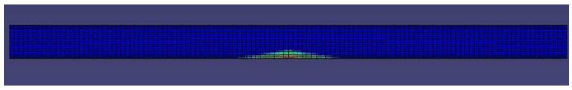


FIGURE 08: Finite Element Load vs. Displacement



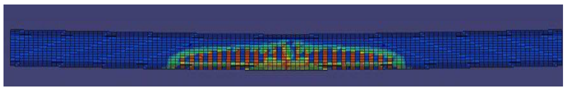
Experiment Crack Pattern at 8kN



FEM Crack Pattern at 8kN

FIGURE 09: Crack Patterns for 3 point bending test at 8kN

Experiment Crack Pattern at 32kN



FEM Crack Pattern at 32kN

FIGURE 10: Crack Patterns for 3 point bending test at 32kN

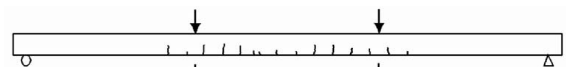
Displacement due to self weight is neglected in Fig. 08 to be comparable with the experiment results presented by Peeters et al. (1996) as in Fig. 07.

3.3 Four Point Bending Test

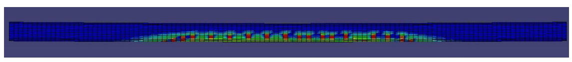
Fig. 11 compares the crack patterns between experiment and FEM results of a beam under four point bending test with material and cross section properties similar to the above beam. The beam is supported at 0.15m and 5.85m and loaded at 2.0m and 4.0m from the left end of the beam with 4kN point loads (Maeck, 2003).

3.4 Un-symmetric Support and Loading Arrangement

To check the validity of the FEM, un-symmetric support and loading arrangement used in the experiment work by Maeck (2003) is used. In this test arrangement, the beam is supported at 2.1m and 5.9m from the left edge and loaded with point load of 13kN at 4.0m from the left edge of the beam. The experiment and FEM crack patterns are compared in Fig. 12 below.

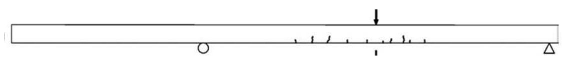


Experiment Crack Pattern at 4kN



FEM Crack Pattern at 4kN

FIGURE 11: Crack Patterns for 4 point bending test at 4kN



Experiment Crack Pattern at 13kN

FEM Crack Pattern at 13kN

FIGURE 12: Crack Patterns for at 4kN with un-symmetric supports

(Note: Experiment Load vs. displacement values were not reported for the four points bending test and un-symmetric support and loading arrangements. Therefore, only crack patterns were compared for above last two cases.)

3.5 Simply Supported Beams with Uniformly Distributed Loads

Experimentally measured initial and long term deflections for 15 RC beams are presented by Neville, Dilger and Brooks (1983) with different concrete grades, reinforcement ratios, and member sizes. This is taken as a valuable source for the present FEM validation. The moment values tabulated in the original document are converted to uniformly distributed load values and applied to the relevant FEMs. Out of 15 beams, first five beams were modelled in ABAQUS using the above material model. Tab. 03 shows the section properties including dimensions, effective depths, reinforcement areas, and ultimate compressive strengths recorded (Neville et al, 1983). Tab.04 compares the experimentally obtained results with present FE results for the initial deflection values. The long term deflection values measured in the experiment setup are neglected as it is out of scope in present study. The beam designations are shown as in the original document (Neville et al, 1983), and all other notations are explained at the end of Tab.04. The ratios between FEM and experiment displacement are indications of validity of the present material model for concrete in both tension and compression.

TABLE 03: First Five Beams (Neville et al, 1983)

Beam Designation	**A1-A4**	**B1-B4**	**C1-C4**	**D1-D4**	**E1-E4**
σ_{cu} (MPa)	28.1	23.6	22.7	24.3	25.2
L (m)	6.096	6.096	6.340	3.810	5.334
b (mm)	203	152	305	305	305
h (mm)	305	203	127	127	76
d=d' (mm)	48	46	25	25	17
M (kNm)	25.65	7.25	6.01	6.07	1.98
A_s (mm^2)	852	400	516	516	284
A_s' (mm^2)	852	400	516	516	284

TABLE 04: Comparison of Experiment and FEM Displacement Values

Beam Designation	**Experiment Displacement a_{exp} (mm)**	**FEM Displacement a_{FEM} (mm)**	**a_{FEM}/a_{exp}**
A1-A4	13.5	13.1	0.97
B1-B4	23.4	23.0	0.98
C1-C4	40.1	39.4	0.98
D1-D4	12.0	11.4	0.95
E1-E4	59.4	57.6	0.97

Notations used in Tab. 03 and 04:
σ_{cu} = Maximum compressive strength in concrete,
L = Span of the simply supported RC beam,
b = Width, h = Total depth,
d, d' = Effective depths to the compressive and tensile reinforcement,
M = Moment at mid span caused by uniformly distributed load,
A_s = Area of tensile reinforcement,
A_s' = Area of compressive reinforcement.

4 CONCLUSIONS

This paper presents a material model which can be used to simulate the non-linear behaviour of reinforced concrete elements. The material model needs only the maximum compressive strength of the concrete, and utilise two numerical techniques to derive stress-strain curves both at compression and tension including softening regimes. Necessary modifications are suggested to the above two numerical techniques to be applicable with ABAQUS damaged plasticity model to simulate damage in RC structures. This material model is validated using experiment results available in the literature with different material and structural arrangements. Result section indicates that both displacement and crack patterns obtained from FEM are well matched with the experiment results. Therefore, the above material model minimise the number of tests needed to develop an accurate material model in FE simulation.

The numerical technique used to develop the stress-strain relationship for tensile region is acceptable for both reinforced and fibre reinforced concrete as reported by Nayal and Rasheed (2006, p.835). Though, the present study focus only on reinforced concrete elements, the similarity in the tension stiffening model may enable to adopt the present material model with fibre reinforced concrete as well with minor parametric changes. The stress-strain relation is capable of accurately representing the strain softening regime as proven by Hsu and Hsu (1994, p.306), enabling to accurately simulate damage caused by concrete crushing. Thus the material model presented can be applied for both reinforced and fibre reinforced concrete elements to simulate or assess the damage due to both tensile cracking and concrete crushing. This paper verifies the accuracy of the proposed material model using experiment results for reinforced concrete elements subjected to flexural loading and tensile cracking.

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