

Elastic-Plastic Fracture Mechanics. Lecture 1

Decouple elastic deformation of the body and inelastic process of separation. Up to this point we have been dealing with the following situation. When a load causes a crack to extend in a body, a large part of the body is elastic, and the inelastic process of separation occurs in a zone around the front of the crack. Inelastic process of separation includes, for example, breaking of atomic bonds, growth of voids, and hysteresis in deformation.

So long as the inelastic zone is much smaller than the body, we mentally decouple the elastic deformation of the body and the inelastic process of separation. The elastic deformation of the body is used to define the driving force for the extension of the crack, the energy release rate. The inelastic process of separation is left to experimental measurement. We have visited several landmarks:

- For glass, Griffith (1921) modeled the body by the linear elastic theory, and represented the process of separation by surface energy.
- For a metal, Irwin and Orowan (1950s) modeled the body by the linear elastic theory, and lumped the process of separation into a single parameter, the fracture energy, to be measured experimentally.
- For rubber, Rivlin and Thomas (1953) modeled the body by the nonlinear elastic theory, and lumped the process of separation into a single parameter, the fracture energy, to be measured experimentally.

The small-scale yielding condition is difficult to apply to ductile metals. When a load causes a crack to extend in a ductile metal, the size of the plastic zone often exceeds 1 cm. To satisfy the small-scale yielding condition would require a specimen of the size of a file cabinet. While such specimens have indeed been used, they are often impractical. Furthermore, even if the fracture energy is measured for such a ductile metal, the small-scale yielding condition limits the utility of the fracture energy to large structures containing large cracks.

Elastic-Plastic Fracture Mechanics in a nutshell. The phenomenon of plastic deformation is readily demonstrated by simple experiments, such as bending a paper clip. But the theory of plasticity is rather intricate, which you will have to learn properly in a separate course. Fortunately, in formulating the elastic-plastic fracture mechanics, we need only a very simple and small part of the theory of plasticity:

When a body is subject to proportional loading, the stress-strain behavior of plastic deformation is indistinguishable from that of nonlinear elastic deformation.

For a crack extending in a body, one can separately consider inelasticity of two types:

- *Messy inelasticity.* Growing voids. Breaking bonds. Hysteretic deformation. Lump everything that you don't want to deal with into the fracture process zone.
- *Tidy inelasticity.* The kind of inelasticity whose stress-strain behavior is indistinguishable from nonlinear elasticity. Model the body as a fictitious nonlinear elastic body.

So long as the zone of messy inelasticity (i.e., the fracture process zone) is much smaller than the specimen, one can extend the approach of Griffith to the fictitious nonlinear elastic solids.

This idea was initially developed in several theoretical works. The J integral was developed for the fictitious nonlinear elastic solids (Rice, 1968). The crack-tip field, the HRR field, was obtained for elastic-plastic solids (Hutchinson, 1968; Rice and Rosengren, 1968).

These theoretical works inspired the first experimental demonstration of elastic-plastic fracture mechanics for ductile metals (Begley and Landes, 1972).

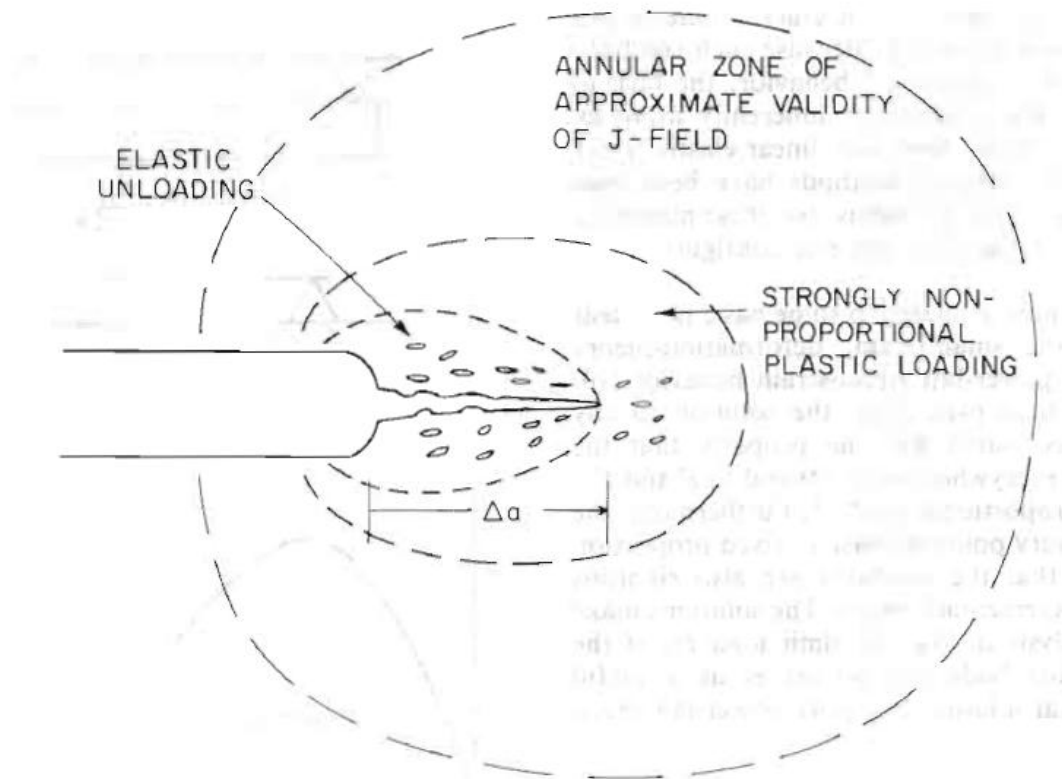


Fig. 8 Schematic of near-tip behavior under J -controlled growth conditions

This figure is taken from J.W. Hutchinson, Fundamentals of the phenomenological theory of nonlinear fracture mechanics. Journal of Applied Mechanics 50, 1042-1051 (1983).

Elements of metal plasticity. We will be mostly concerned with macroscopic plastic deformation.

- Sketch a stress-strain curve of a metal rod subject to uniaxial tension. Mark Young's modulus, yield strength, strain hardening, plastic loading, elastic unloading.
- Hysteresis. Deformation depends on the history of loading.
- When the rod is subject to an increasing tensile load, so far as the stress-strain behavior is concerned, plasticity is indistinguishable from nonlinear elasticity.
- Plastic deformation is incompressible.
- Superposing a hydrostatic stress does not affect plastic deformation.

- Consider a thin-walled tube subject to a combination of an axial force and a torque.
- Represent a material particle by a cube. Subject to forces on its six faces, the cube deforms to some other shape.
- The components of the stress form a space. A point in the space represents a state of stress, and a path in the space represents a history of loading.
- The components of the strain form another space. A point in the space represents a state of deformation, and a path in the space represents a history of deformation.
- Deformation depends on the history of loading.
- A radial path in the stress space represents proportional loading.
- When a body is subject to proportional loading, so far as the stress-strain behavior is concerned, plasticity is indistinguishable from nonlinear elasticity. In particular, the stress-strain behavior is derivable from energy density as a function of strain.

Energy release rate. Consider a crack in an elastic body subject to a load. The elastic energy U stored in the body is a function of two independent variables: the displacement Δ of the load, and the area A of the crack. That is,

$$U = U(\Delta, A).$$

When the displacement of the load varies by $\delta\Delta$ and the area of the crack varies by δA , the elastic energy stored in the body varies by

$$\delta U = \frac{\partial U(\Delta, A)}{\partial \Delta} \delta \Delta + \frac{\partial U(\Delta, A)}{\partial A} \delta A.$$

One partial derivative defines the load:

$$P = \frac{\partial U(\Delta, A)}{\partial \Delta}.$$

The other partial derivative defines the energy release rate:

$$G = -\frac{\partial U(\Delta, A)}{\partial A}.$$

Using the above definitions, we write

$$\delta U = P \delta \Delta - G \delta A.$$

Define the potential energy of the body as

$$\Pi = U - \Delta P.$$

This definition, together with $\delta U = P \delta \Delta - G \delta A$, leads to

$$\delta \Pi = -\Delta \delta P - G \delta A.$$

Consequently, the energy release rate is also defined as

$$G = -\frac{\partial \Pi(P, A)}{\partial A}.$$

For an elastic-plastic solid, U is still the area under the load-displacement curve, while the area of the crack is fixed. That is, U is the work done by the load. This work done by the load, however, is no longer stored as elastic energy in the body. Much of the work dissipates as heat. Calling G the energy release rate may cause semantic confusion. The J integral is almost exclusively used in the literature. In this lecture I'll use G and J interchangeably. One way or another you have to confront the issue of regarding the elastic-plastic solid as a fictitious elastic solid. Replacing G by J does not resolve the issue.

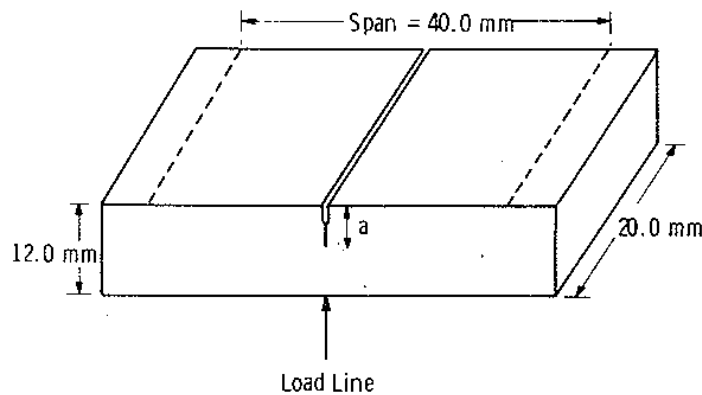


FIG. 3—Double thickness bend bar.

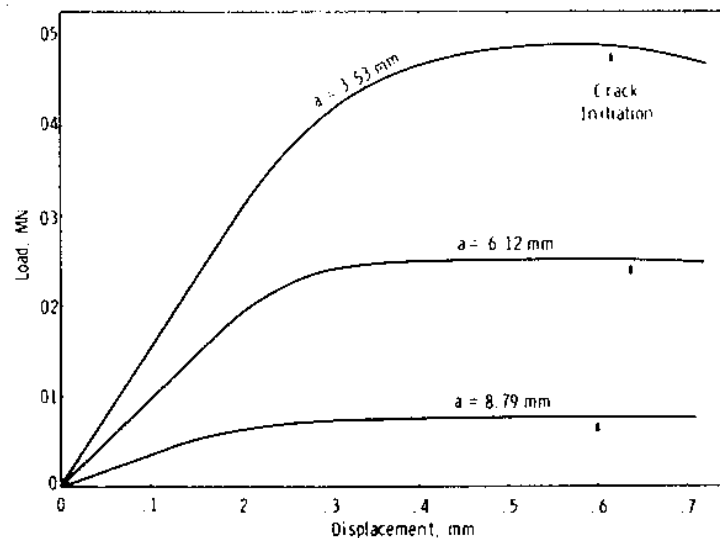


FIG. 5—Load-displacement curves for double thickness bend bars.

The Begley-Landes (1972) experiment. The object of this experiment is to show that G works under the large-scale yielding condition.

- Measure the load-displacement curves for similarly loaded bodies containing cracks of different lengths.
- Obtain the critical energy release rate for the onset of crack extension.
- Show that the critical energy release rate is a material property, independent of the length of the crack.
- Show that the critical energy release rate is a material property, independent of the type of specimen. Two types were used: center cracked panels and bend bars.

- Show that the critical energy release rate measured under the large-scale yielding condition is the same as that measured under the small-scale yielding condition.

Although Begley and Landis motivated their work in terms of the J integral and the HRR field, their experiments assumed no field theory, and their experimental method invoked no J integral. In fact, their experimental method is identical to that of Rivlin and Thomas (1953) for rubber.

What do we gain by specifying a field theory? Nothing, if all we want is to define the energy release rate as the driving force for the extension of a crack, and then measure the critical energy release rate. As demonstrated by Begley and Landes (1972), these objects can be achieved without specifying any field theory. All we need to know is that the specimen can be modeled as a fictitious nonlinear elastic body.

But we want more. By specifying a field theory, we can gain the following.

- Relate the energy release rate to the load and geometry by solving a boundary-value problem. Only a single specimen need be tested. By contrast, the Begley-Landes approach requires testing multiple specimens containing cracks of different lengths.
- Calculate the energy release rate by using the J integral.
- Solve the crack-tip field.
- Discuss the condition for the small-scale fracture process zone.

We next recall the main results for linear elastic theory, and then describe the consequences for nonlinear elastic theories.

Linear elastic theory. By the linear elastic theory we mean that

- the deformation is infinitesimal, and
- the stress-strain relation is linear.

Relate the energy release rate to load and geometry. When the linear elastic theory is used to model a body containing a crack and subject to a load, the field in the body is linear in the load. The form of the energy release rate is determined from the following considerations.

- Within the linear elastic theory, the energy release rate is quadratic in the load.
- The energy release rate has the dimension of energy per area.
- The linear elastic theory has no intrinsic length.

The energy release rate takes the following form:

$$G = Z \frac{a \sigma_{\text{appl}}^2}{E},$$

where σ_{appl} is a stress representative of the load, a is a length, E is Young's modulus, and Z a dimensionless factor. The factor Z depends on dimensionless numbers in the problem, and is in general determined by solving the boundary-value problem.

Crack-tip field. We have previously determined the crack-tip field by solving an eigenvalue problem. The essential features of the crack-tip field, however, can be determined by qualitative considerations. (These qualitative considerations may not be rigorous.)

Let r be the distance from the tip of the crack. When r is small compared to all lengths specifying the geometry of the body, the external boundary

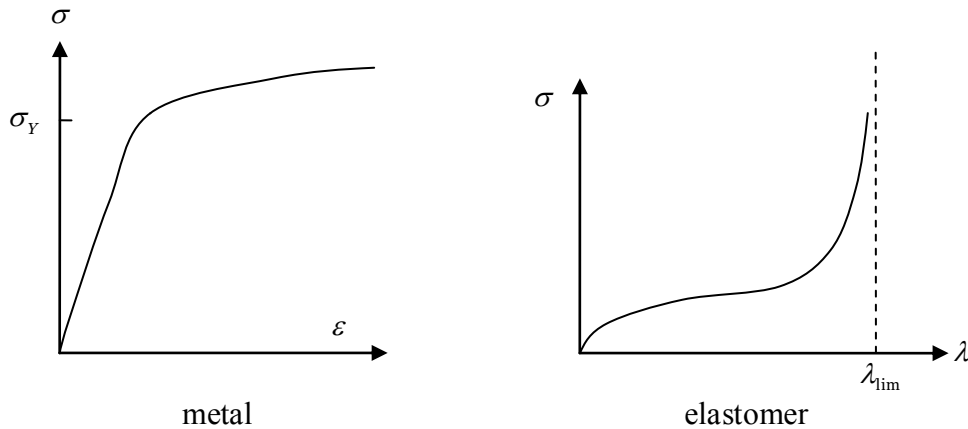
conditions affect the field at r through the energy release rate alone. In addition to r , the problem has only one other length, G/E . The two lengths form a dimensionless ratio $G/(Er)$. The stress field is linear in the applied load, but the energy release rate is quadratic in the applied load. Consequently, the crack-tip field takes the form:

$$\sigma_{ij}(r, \theta) = E \sqrt{\frac{G}{Er}} f_{ij}(\theta).$$

Note that the square-root singularity results from the above qualitative considerations.

Size of plastic zone. The crack-tip field can be used to estimate the size of the plastic zone. In the above, the material is assumed to be linearly elastic. We now assume that the material yields when the stress is on the order of the yield strength σ_Y . Thus, the size of the plastic zone scales as

$$r_p \sim \frac{GE}{\sigma_Y^2}.$$



Nonlinear elastic theory of finite deformation. This theory assumes that

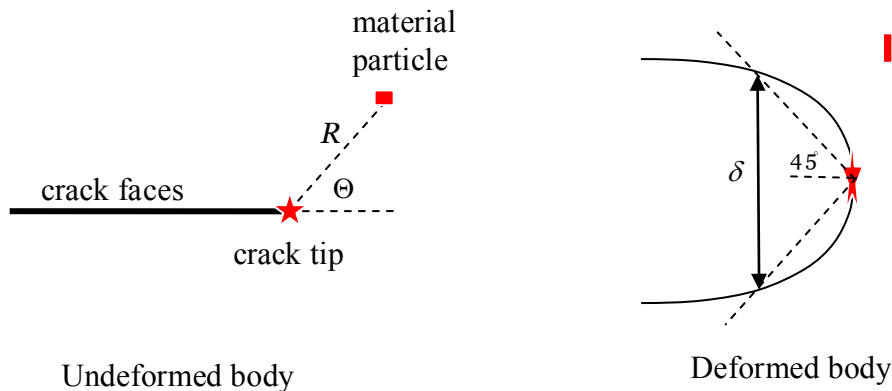
- the deformation is finite, and
- the stress-strain relation is nonlinear.

The theory is of the same kind as that used to study fracture of elastomers. But the stress-strain relations for metals and elastomers differ markedly at large deformation. A metal deforms indefinitely (i.e., flows) with a small increase in stress. By contrast, an elastomer approaches a limiting stretch with great increase in stress.

Crack-tip field. This problem was analyzed by Rice and Johnson (1970) and by McMeeking (1977). In an undeformed body, a crack pre-exists and is sharp. Centered at the tip of the crack in the undeformed body are the polar coordinates (R, Θ) . Use the polar coordinates to name material particles.

Subject to a load, the body deforms, and the crack is blunted. Represent the load by the energy release rate G . A metal flows when the stress exceeds the yield strength σ_Y . At the tip of the crack, the strain is typically much larger than

the elastic strain. Consequently, Young's modulus has no effect on the form of the crack-tip field. In addition to the distance R , the only other length is G/σ_Y .



To quantify the degree of blunting, define the crack opening displacement δ by the convention shown in the figure. Because G/σ_Y is the only length scale that determines the crack opening displacement, we expect the scaling relation

$$\delta \sim \frac{G}{\sigma_Y}.$$

That is, the crack opening displacement is linear in the energy release rate, and is inversely proportional to the yield strength. Of course, the crack opening displacement also depends on dimensionless parameters in the material model, such as the hardening exponent.

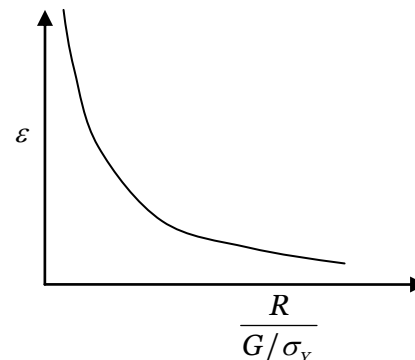
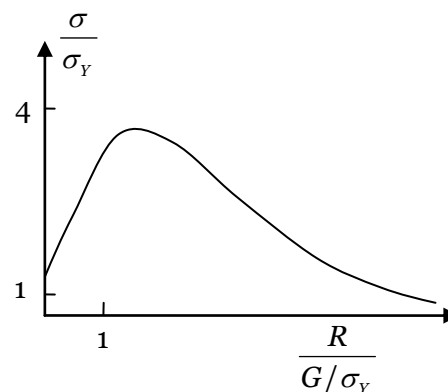
The crack-tip field takes the form:

$$\frac{\sigma_{ij}}{\sigma_Y} = f_{ij}\left(\frac{R}{G/\sigma_Y}, \Theta\right).$$

The dimensionless function also depends on other dimensionless parameters of the metal, such as the hardening exponent.

The main features of the field are understood as follows:

- At the tip of the crack, the metal is under biaxial stresses. The plastic flow is easy. Consequently, the stress is around the level of the yield strength, but the strain is high.
- Inside the material, at a distance comparable to G/σ_Y , the material is under triaxial stresses. Plastic flow is constrained. The stress is can be several times of the yield strength.
- Inside the material, at a distance far beyond G/σ_Y , both the stress and strain drops with the distance increases.



Deformation theory vs. flow theory.

Relate energy release rate to load and length of a specimen. To solve a boundary-value problem, one has to specify a material model. Boundary-value problems for cracks in nonlinear elastic body have been solved in many cases. Here we just mention one class of problems.

Recall that Rivlin and Thomas (1953) obtained expressions of the energy release rate for a few types of specimens without specifying the form of the energy-density function. Here is an example suitable for testing metals (Rice, Paris and Merkle, 1973).

$$M = b^2 f(\Omega)$$

$$U(b, \Omega) = \int_0^{\Omega} b^2 f(\Omega) d\Omega$$

$$G = \frac{2}{b} \int_0^{\Omega} M d\Omega$$

Review

J.W. Hutchinson, Fundamentals of the phenomenological theory of nonlinear fracture mechanics. *Journal of Applied Mechanics* **50**, 1042-1051 (1983).
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