Consider a 3-dimensional Cosserat type continuum. If the configuration space of a Cosserat material point is $\mathbb{R}^3 \times SO(3)$, then the deformation of the material point is given by $(\varphi, \Lambda) : B \subset \mathbb{R}^3 \times SO(3) \to \mathbb{R}^3 \times SO(3)$ where B is the reference configuration of the body. An equivalent description of the configuration would be the posion of the material point and the rigid directors attached to it [1].

In Lagrangian description the equations of motion are

$$\text{Div}\mathbf{T} + f = \rho \dot{v} \tag{1}$$

$$\text{Div}\mathbf{M} - \text{ax}(\mathbf{FT} - \mathbf{T}^{\text{T}}\mathbf{F}^{\text{T}}) + m = \mathbf{J}\dot{\omega} \tag{2}$$

where $\text{Div}(.)$ and $\text{ax}(.)$ respectively refer to the divergence in reference configuration and the axial vector of a skew symmetric tensor. $\mathbf{T}$ and $\mathbf{M}$ are the stress and couple stress tensors which are similar to the first Piola-Kichhoff stress used in classical continuum description. $\mathbf{F}$ is the deformation gradient $(\nabla\varphi)$. f and m are body force and body couple densities in reference configuration. $v = \dot{\varphi}$ is the velocity expressed in reference configuration. $\rho, \mathbf{J}$ and ω are the reference mass density, the micro-inertia tensor and 'angular velocity field'.

We are trying to find the work conjugate strain (and wryness) measures to $\mathbf{T}$ and $\mathbf{M}$ and we know that in classical theory the conjugate strain measure is identified using the stress power, which appears in the power balance equation derived taking the inner product of the equation of motion with velocity.

1. Given the equations of motion Eqs.(1) and (2), how do we extend the procedure of calculating the stress power for Cosserat media?

This boiles down to the question of finding the time rate (material derivative) of the deformation map. Time rate of φ is understood to be v in Eq. (1). But how do we define the vector field ω in Eq. (2) from Λ? Moreover, what would be the appropriate inner product to derive the power balance equation?

2. It is common to refer the relative velocity between two material points x and y as the difference between velocity vectors at x and y i.e. $v(x) - v(y)$. We want a similar notion for relative ω (or relative material time rate of Λ), i.e. how can we define a quantity like $\omega(x) - \omega(y)$?

References

[1] W Pietraszkiewicz and VA Eremeyev. On natural strain measures of the non-linear micropolar continuum. *International Journal of Solids and Structures*, 46(3):774–787, 2009.