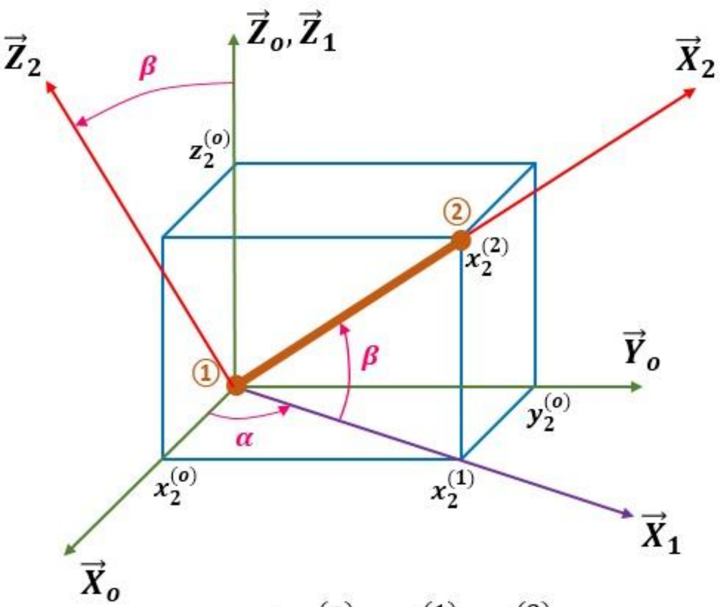
The transformation matrix of a two-nodes beam element is obtained with the combination of two rotations α and β using local and reference coordinates systems:

$$[\lambda]_{(3\times3)} = [P_{o-2}] = [P_{o-1}][P_{1-2}]$$

Where: $$[P_{o-1}] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And: $$[P_{1-2}] = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

Then: $$[P_{o-2}] = \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha & \cos\alpha\sin\beta \\ -\sin\alpha\cos\beta & \cos\alpha & -\sin\alpha\sin\beta \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$



$case\ of\ x_1^{(o)} = x_1^{(1)} = x_1^{(2)} = 0$

The computations use the element nodes coordinates such that:

$$\cos\alpha = \frac{x_2^{(o)} - x_1^{(o)}}{x_2^{(1)} - x_1^{(1)}}$$

$$\sin\alpha = \frac{y_2^{(o)} - y_1^{(o)}}{x_2^{(1)} - x_1^{(1)}}$$

$$\cos\beta = \frac{x_2^{(1)} - x_1^{(1)}}{L}$$

$$\sin\beta = \frac{z_2^{(o)} - z_1^{(o)}}{L}$$

The coordinates of any element are known in the global coordinates system.

Where: $$L = \sqrt{\left(x_2^{(o)} - x_1^{(o)}\right)^2 + \left(y_2^{(o)} - y_1^{(o)}\right)^2 + \left(z_2^{(o)} - z_1^{(o)}\right)^2}$$

And: $$x_2^{(1)} = \sqrt{\left(x_2^{(o)}\right)^2 + \left(y_2^{(o)}\right)^2}$$

$$x_1^{(1)} = \sqrt{\left(x_1^{(o)}\right)^2 + \left(y_1^{(o)}\right)^2}$$

If the element has six degrees of freedom:

$$\lfloor d \rfloor = \lfloor u_1\ v_1\ w_1\ \theta_{x1}\ \theta_{y1}\ \theta_{z1}\ u_2\ v_2\ w_2\ \theta_{x2}\ \theta_{y2}\ \theta_{z2} \rfloor$$

The transformation matrix becomes:

$$[T]_{(12\times12)} = \begin{bmatrix} [\lambda] & [0] & [0] & [0] \\ [0] & [\lambda] & [0] & [0] \\ [0] & [0] & [\lambda] & [0] \\ [0] & [0] & [0] & [\lambda] \end{bmatrix}$$