

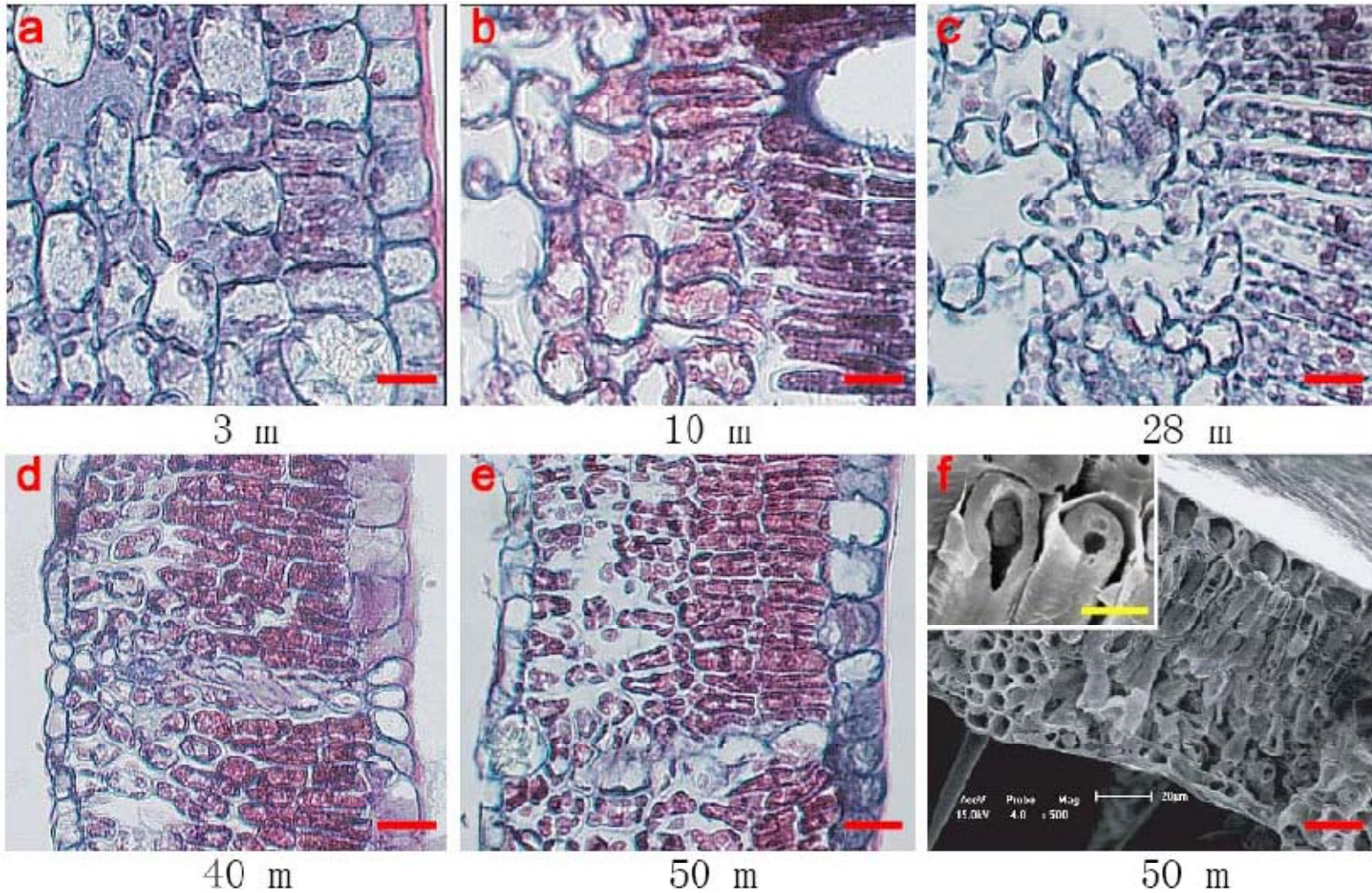
Why the mesophyll cell sizes
decrease with tree height?

---A mechanical view

Ming GUO

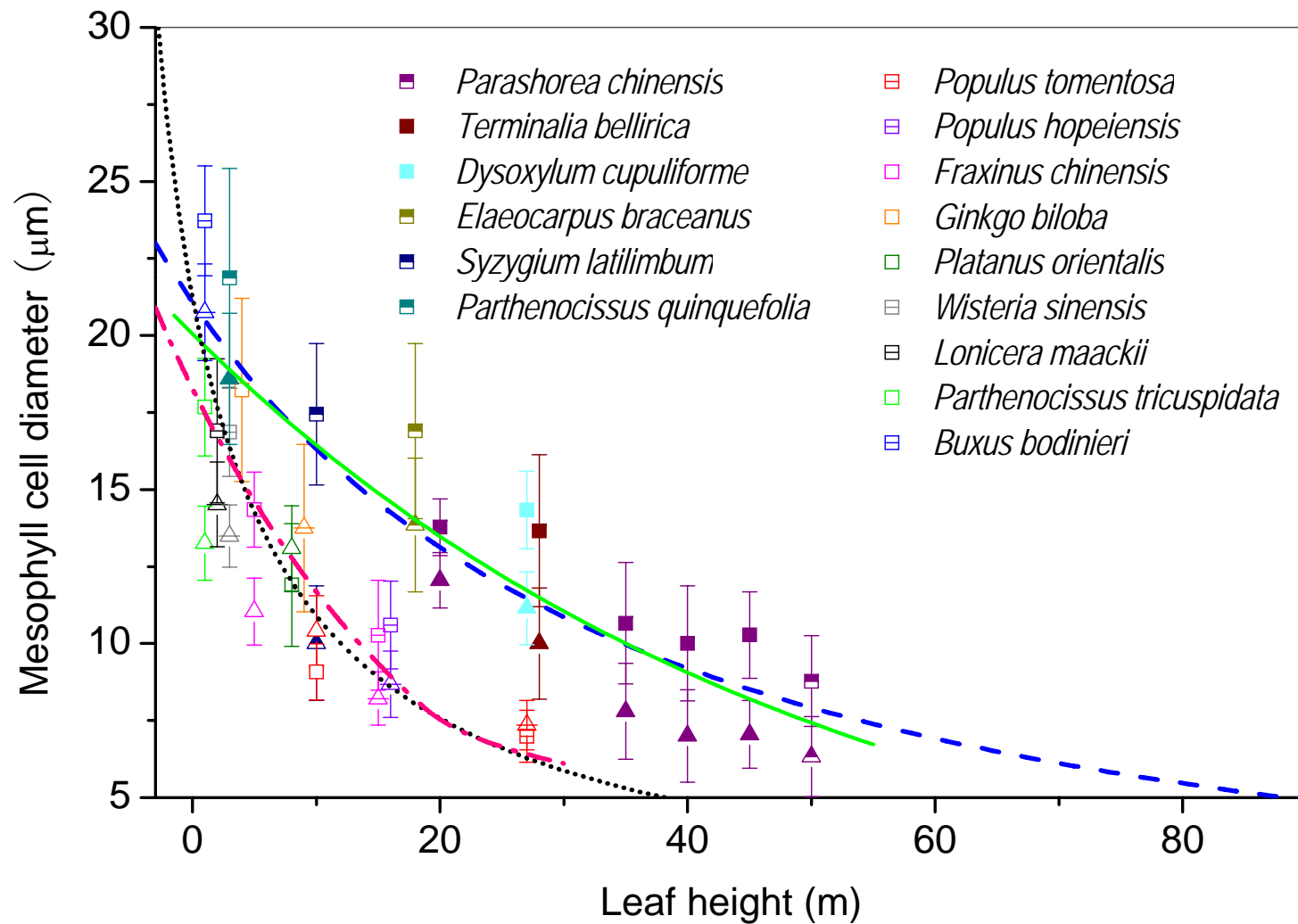
01/11/2008

Mesophyll cell sizes versus heights



Red Scale Bar = 20 μm

Mesophyll cell sizes versus heights

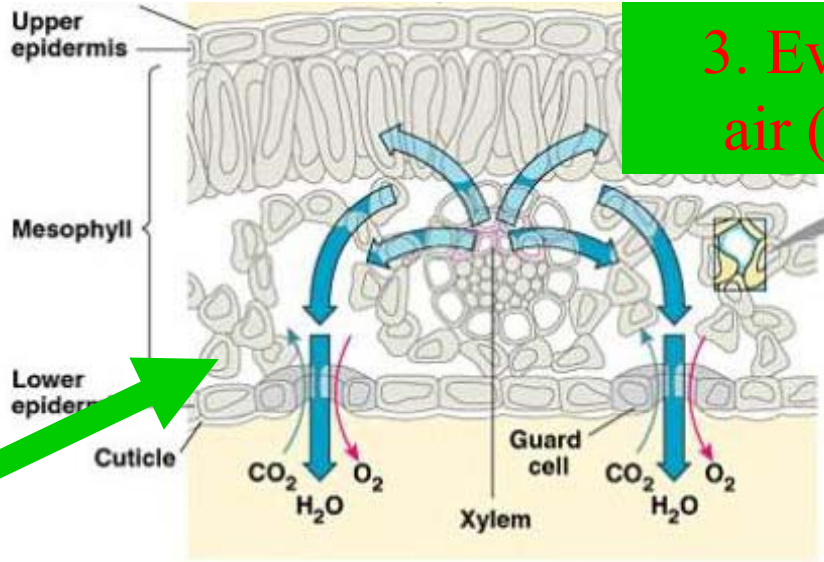


Summary on observations

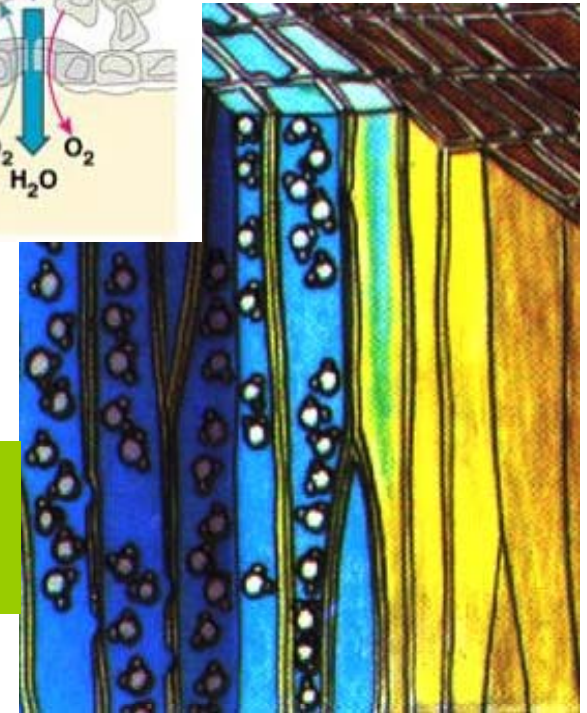
- Our microscopic observations of mesophyll cells reveal that the cell sizes are remarkably reduced with increasing heights of the leaves.
- The reduction rates depend strongly upon the growth regions.
- My previous explanation was based on a famous plant growth model.

Why the mesophyll cell sizes decrease with tree height, from the view of mechanics?

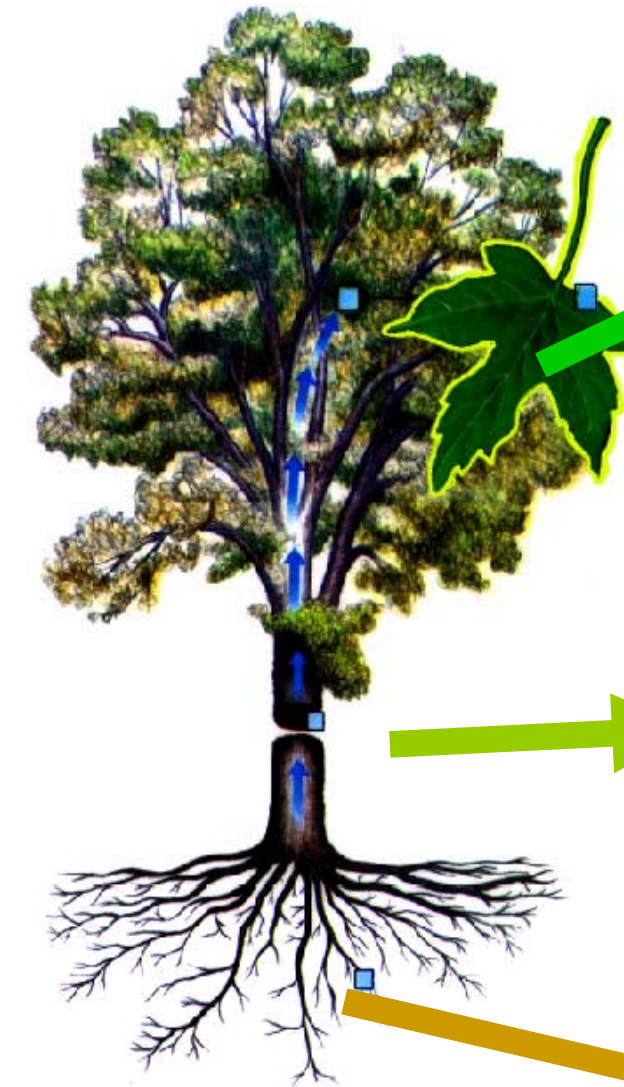
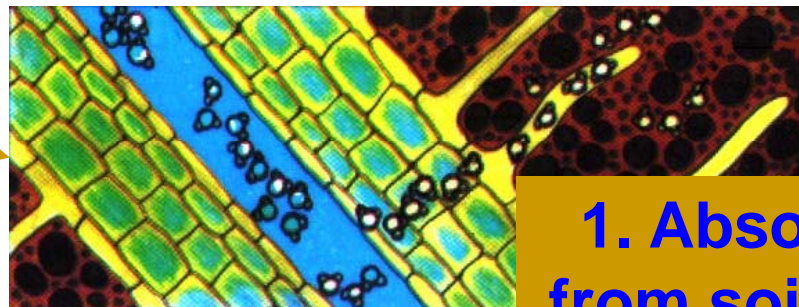
3. Evaporation into
air (transpiration)



2. Transport in
xylem vessels

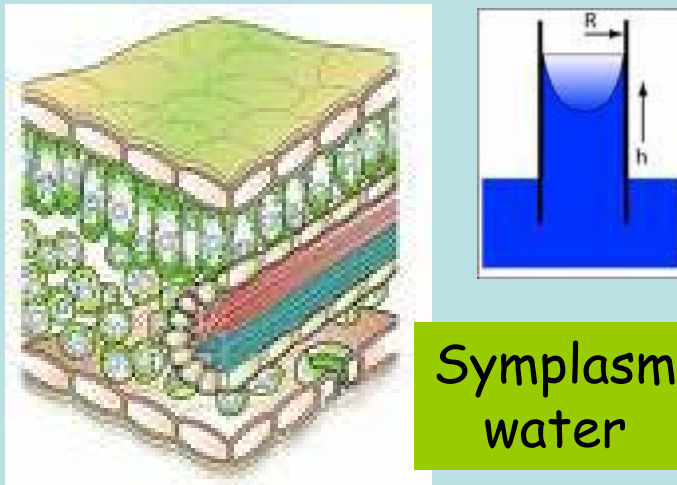


1. Absorb water
from soil by roots



Plant Physiology, 3rd ed. (2005)

Mesophyll cells - The Engines



Symplasm
water

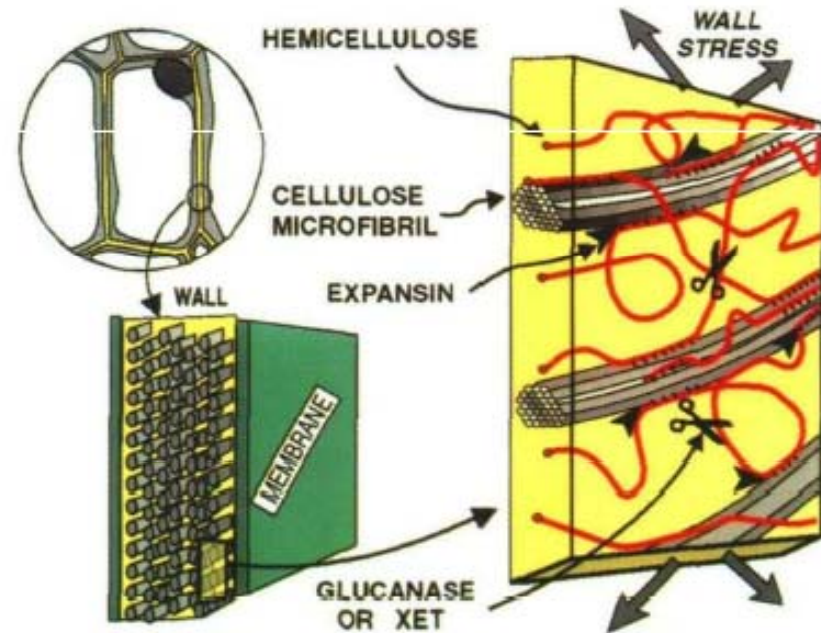
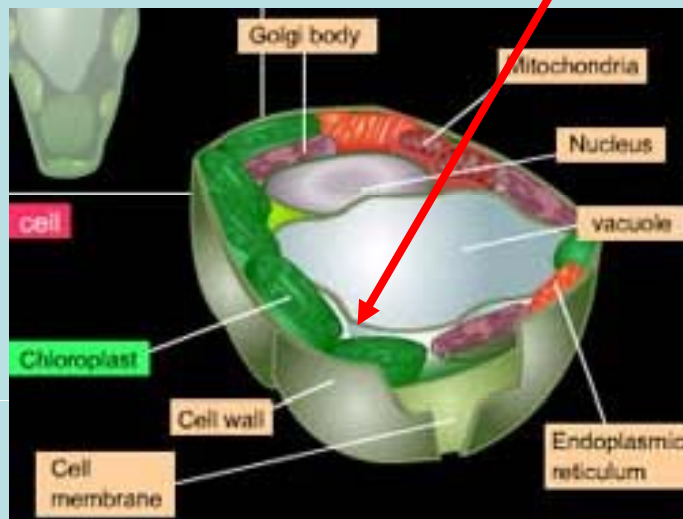
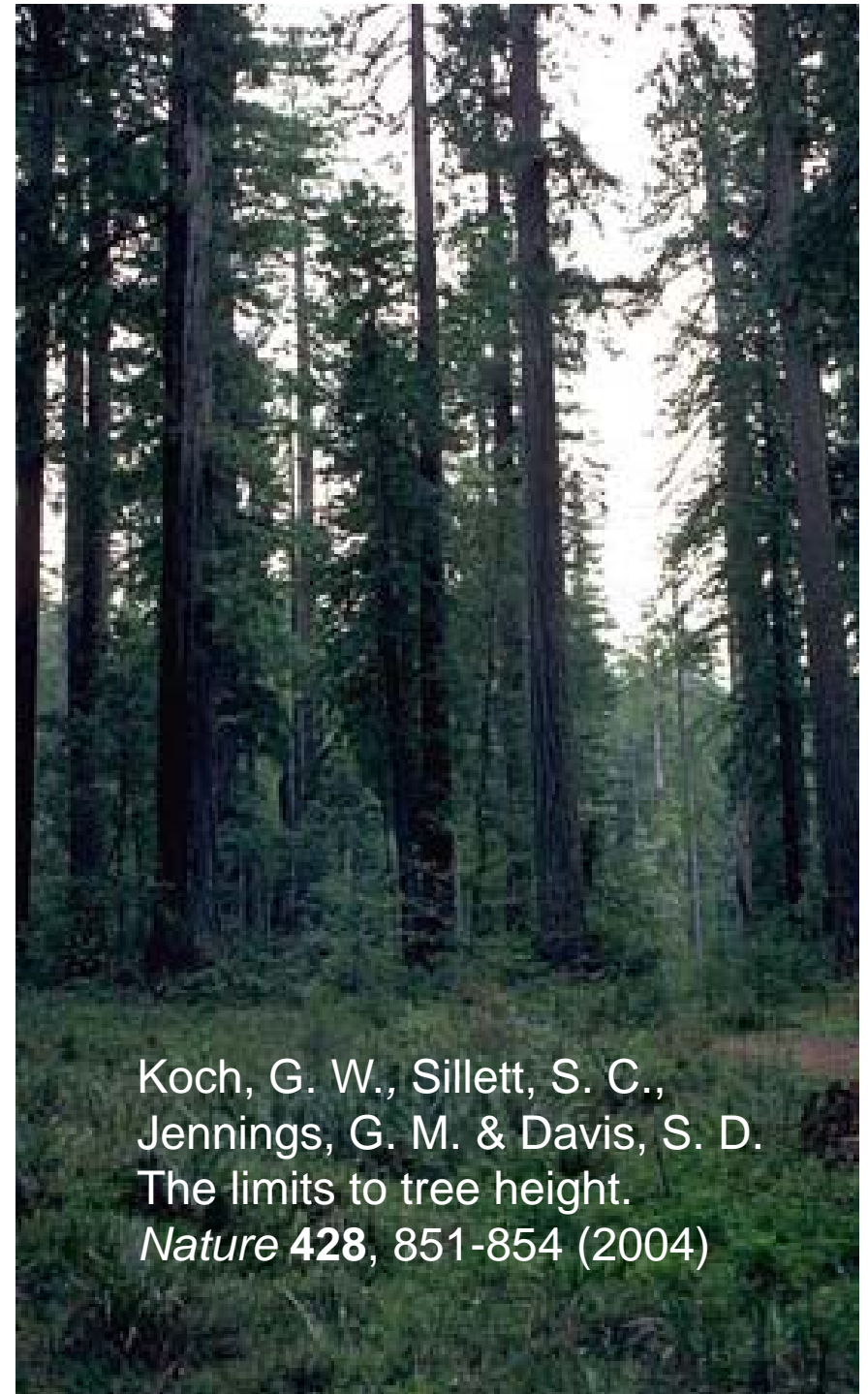
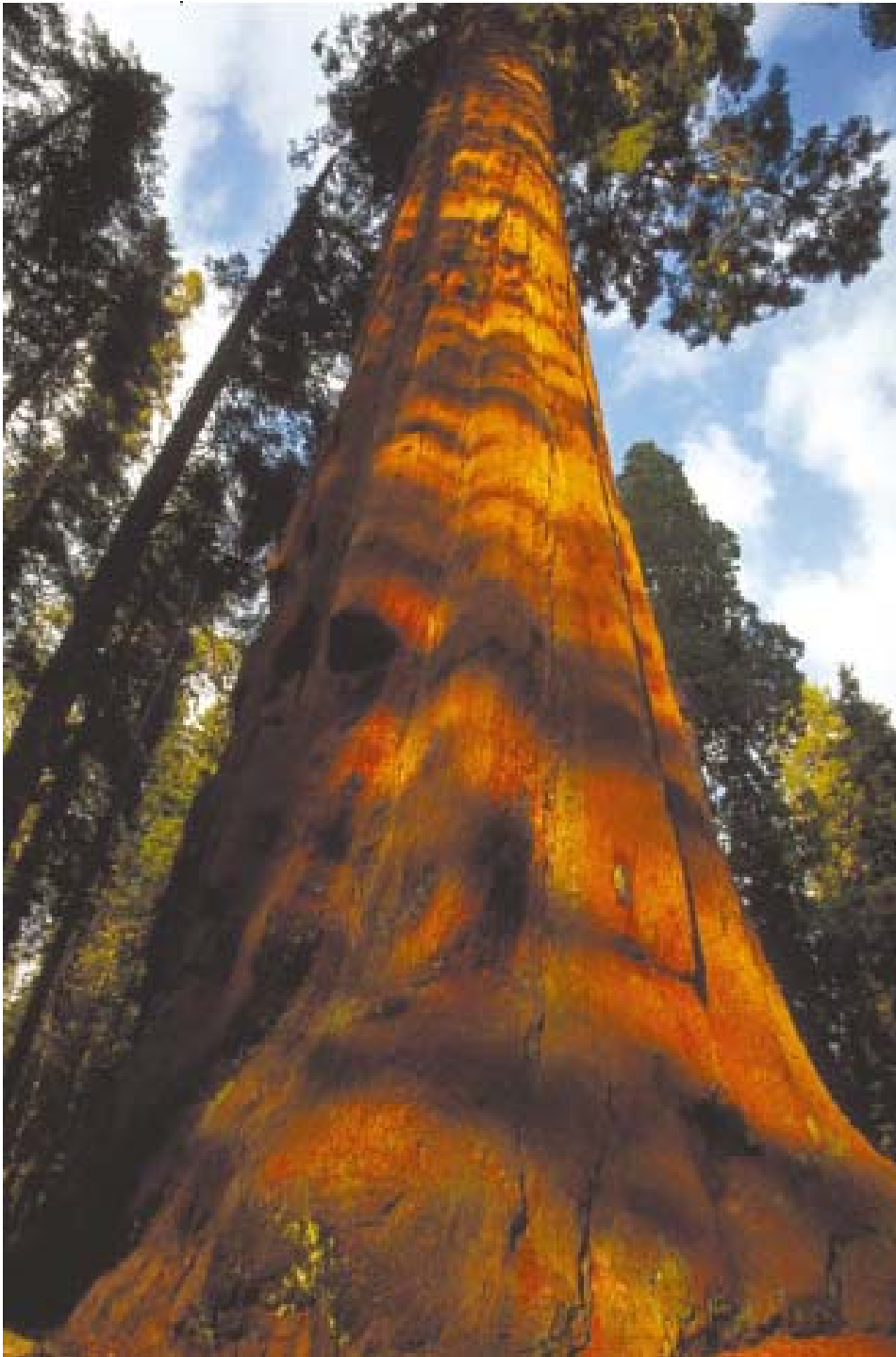


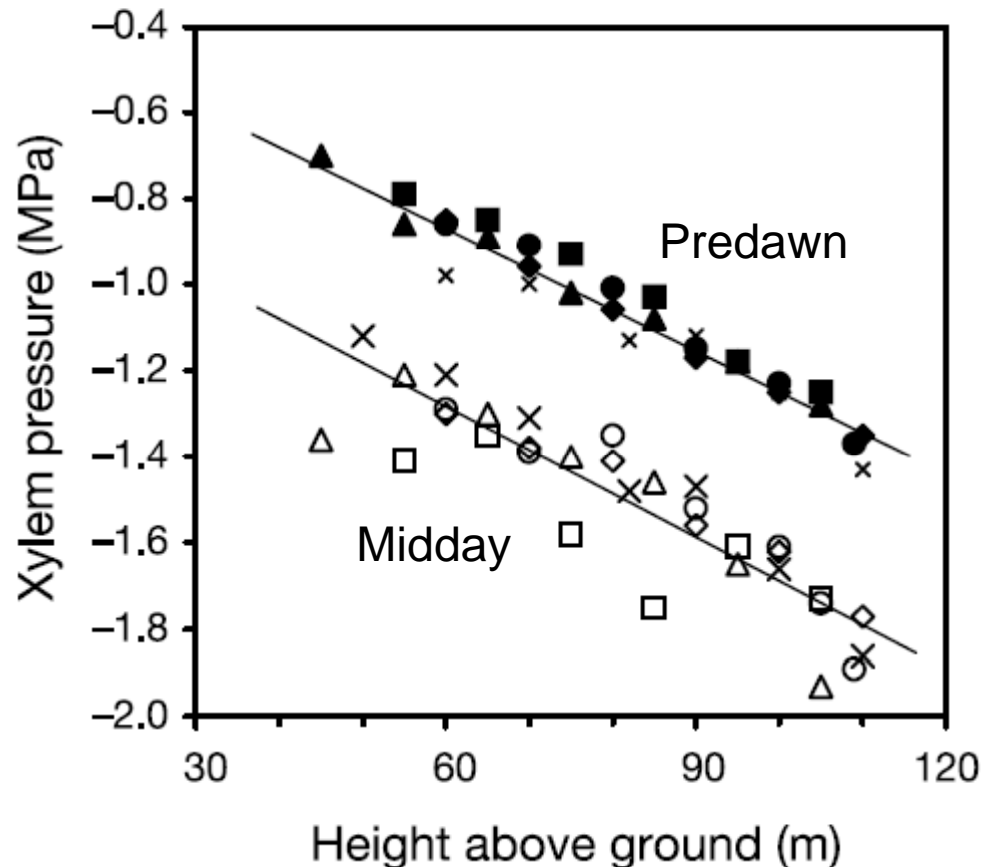
Figure 2. Primary Wall Architecture and Potential Mechanisms for Stress Relaxation and Creep of Cell Walls.

10~40 nm pores on outer surface
generate large negative pressure
~MPa



Koch, G. W., Sillett, S. C.,
Jennings, G. M. & Davis, S. D.
The limits to tree height.
Nature **428**, 851-854 (2004)

Relationship Between Xylem Pressure and Tree Height



Hydrostatic gradient before dawn is equal to the gradient due to gravity.

At midday, 2/3 of the xylem pressure due to gravity.

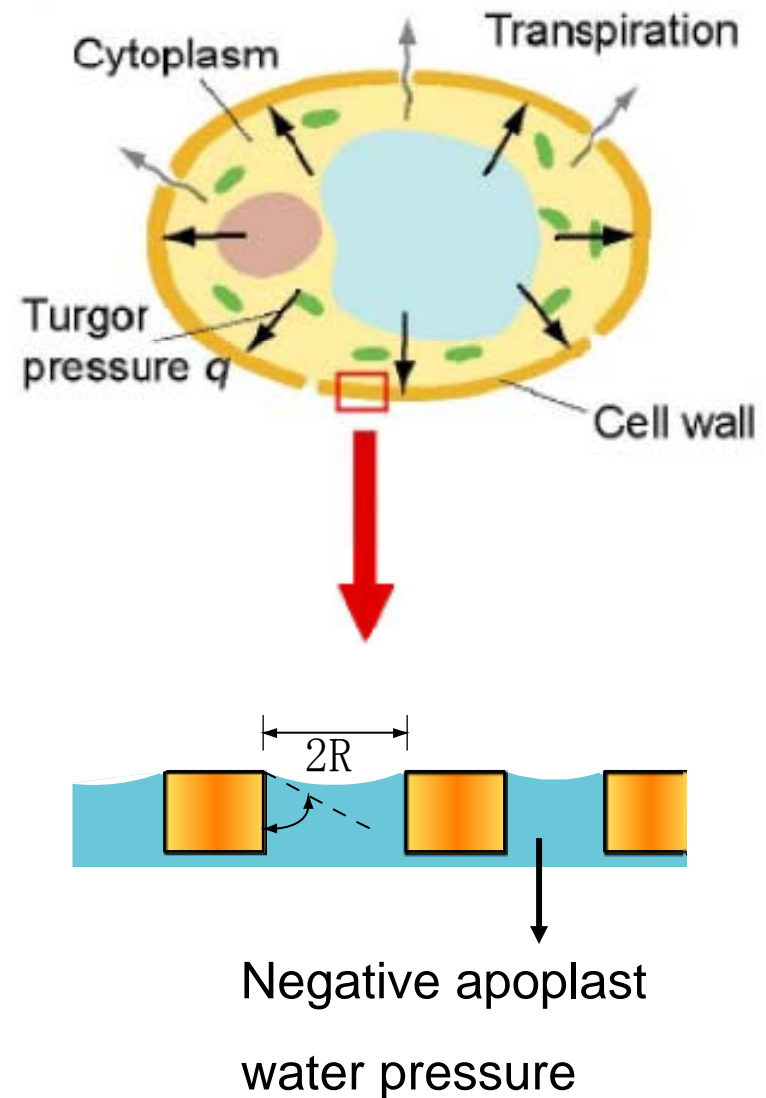
Guess negative water pressure is the reason.

- An important process in tree's life is the continuous water transport to tree leaves, which is mainly driven by the negative pressure inside the leaf mesophyll cells, generated by transpiration.
- However, the effects of these negative pressures on the mesophyll cells are mostly unknown.

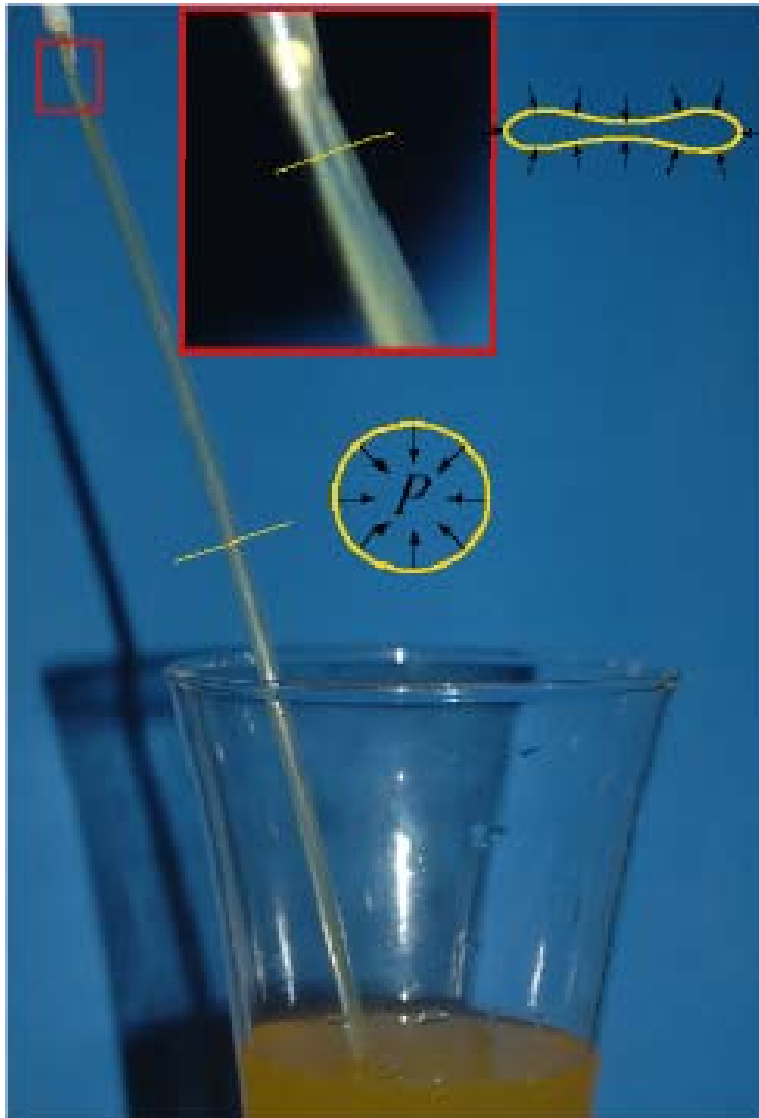
Focus on cells under transpiration!

Negative Pressures

- Transpiration on nanoscale wall-pores generate negative pressure in apoplast water.
- Xylem negative pressures become more negative with increasing height.
- What's the effects of negative pressure to the evolution and structure of mesophyll cells?

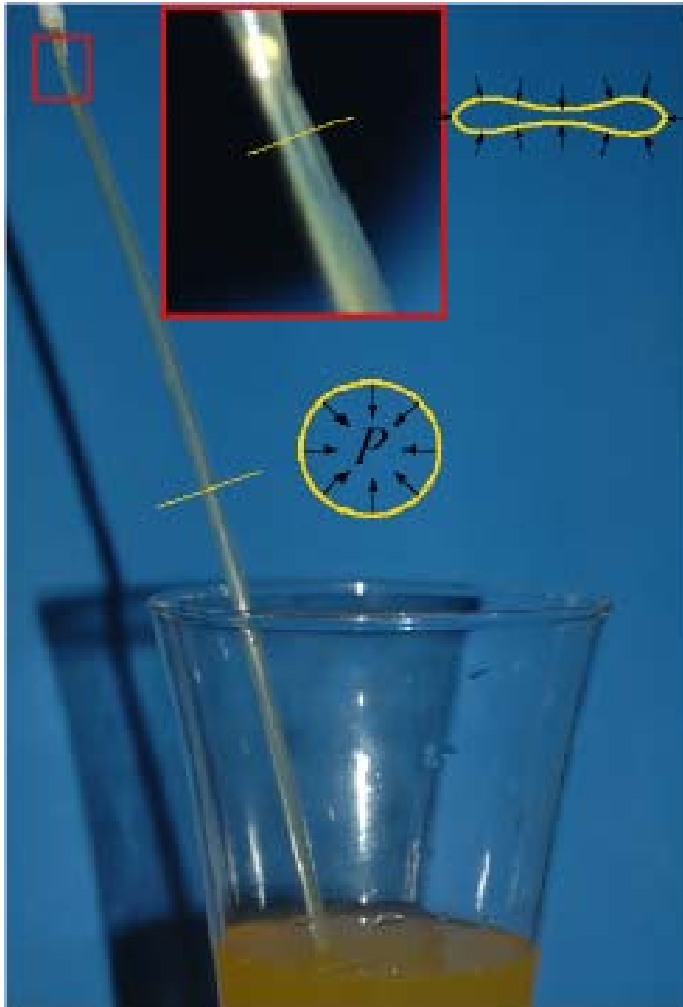


Straw Collapse Analogy



When drinking furiously through a straw, the straw may suddenly collapse into flat, due to the negative pressure inside.

Straw Collapse Analogy



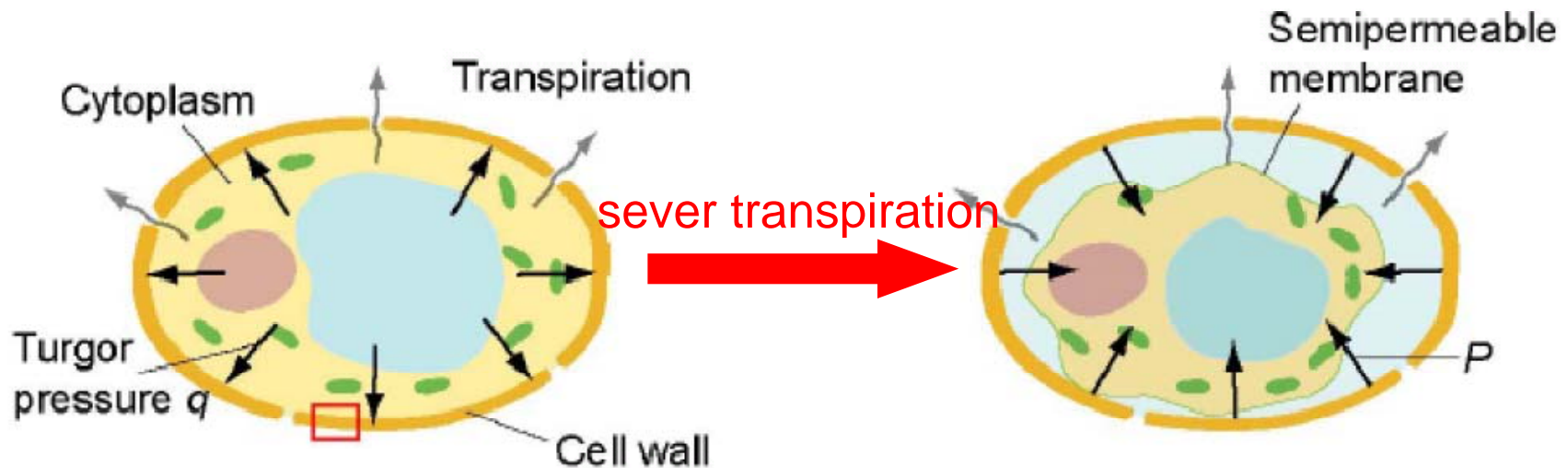
For cylindrical thin shell:

$$P_{cr} = \frac{2Et^3}{D^3(1-\nu^2)}$$

$$\text{so, } D_{cr} = \sqrt[3]{\frac{2Et^3}{(1-\nu^2)P}}$$

Timoshenko, Theory of elastic stability

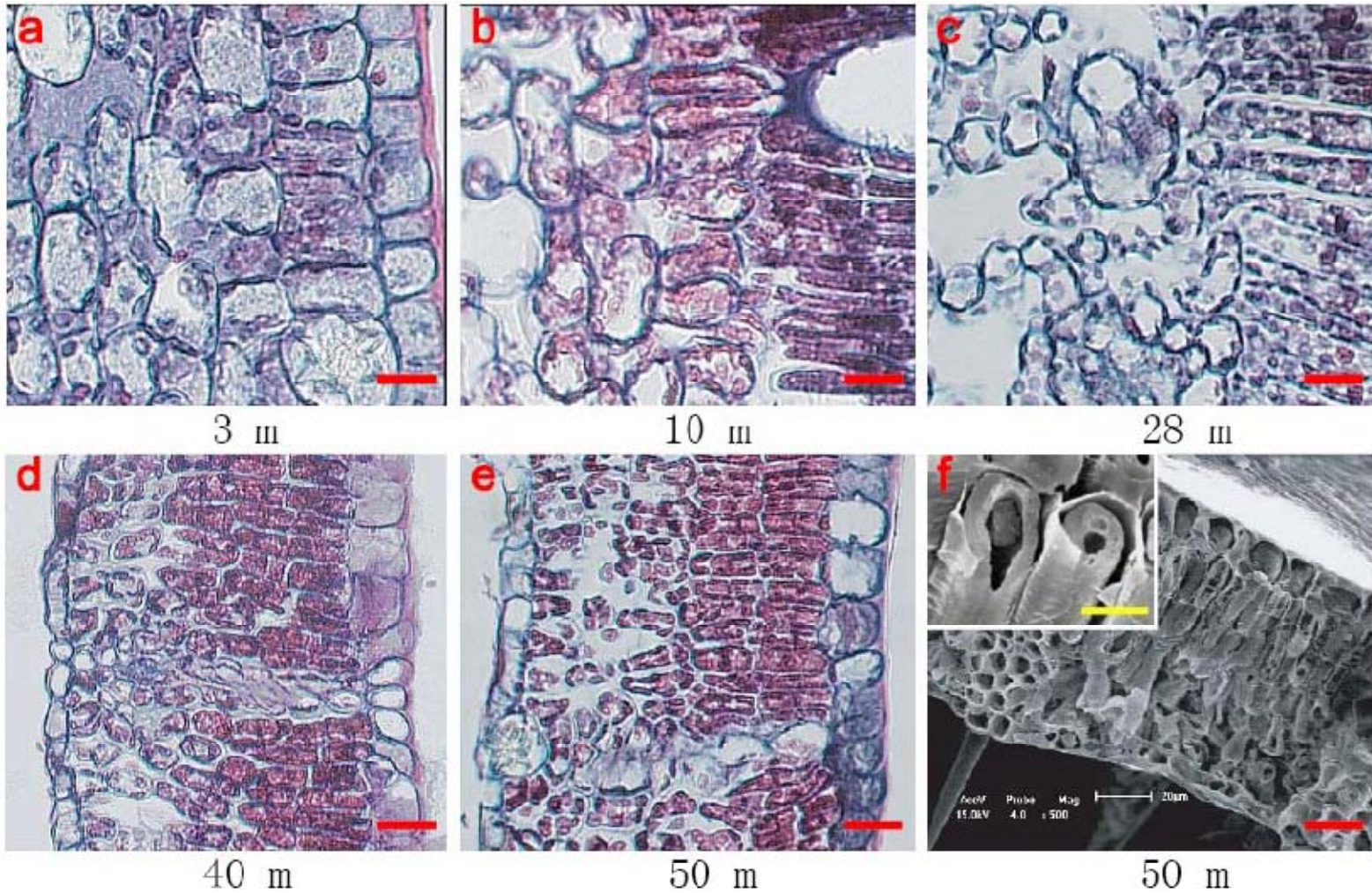
Elastic Instability Mechanism for large cells



cylindrical shell model:

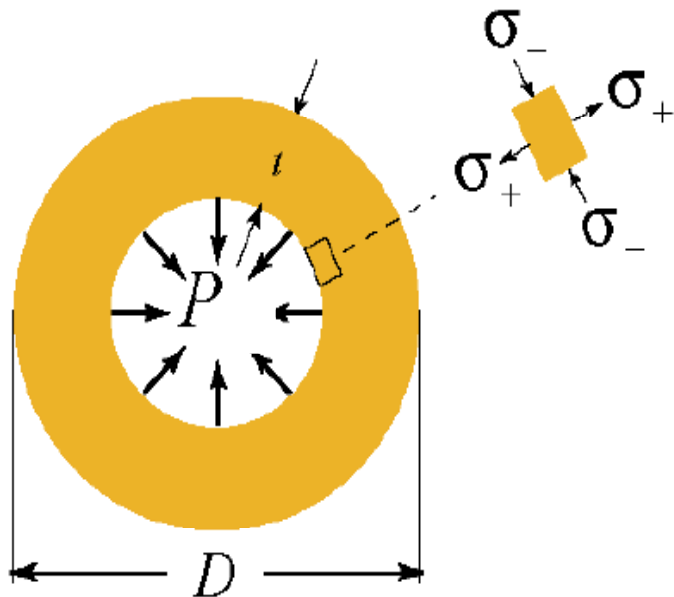
$$D < D_{\text{cr}} = \sqrt[3]{\frac{2Et^3}{(1-\nu^2)P}}$$

Mesophyll cell sizes versus heights



Red Scale Bar = 20 μm

Strength Failure Mechanism for small cells



Assume isotropic elastic material

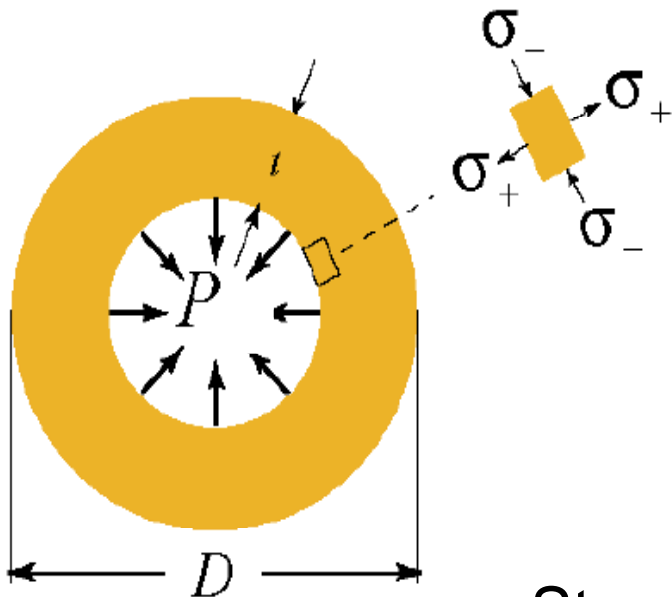
$$\sigma_r \Big|_{r=\frac{D}{2}-t} = P$$

$$\sigma_r \Big|_{r=D/2} = 0$$

$$\sigma_r = -\frac{P(D-2t)^2}{D^2 - (D-2t)^2} + \frac{PD^2(D-2t)^2}{4(D^2 - (D-2t)^2)r^2}$$

$$\sigma_\phi = -\frac{P(D-2t)^2}{D^2 - (D-2t)^2} - \frac{PD^2(D-2t)^2}{4(D^2 - (D-2t)^2)r^2}$$

Strength Failure Mechanism for small cells



Max stresses:

$$\sigma_r \big|_{r=a} = P$$

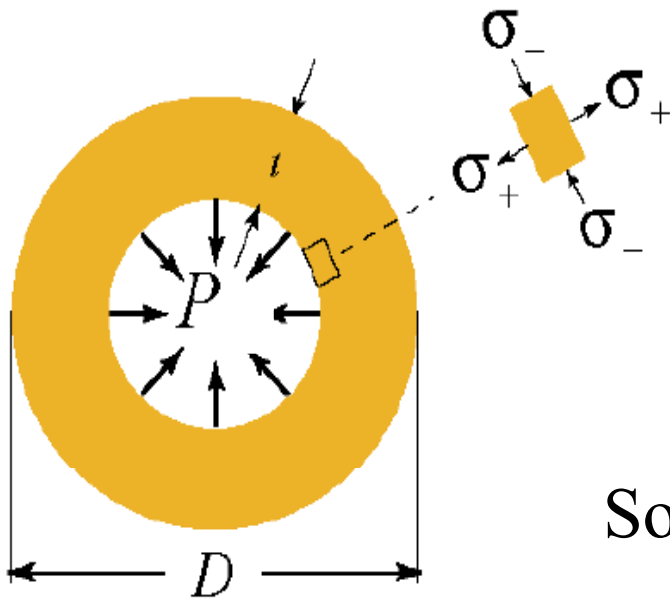
$$\sigma_\phi \big|_{r=a} = -\frac{D^2 + (D - 2t)^2}{D^2 - (D - 2t)^2} P$$

Strength criteria:

$$|\sigma_{r(r=a)}| = P < \sigma_{s+} \quad (= \text{tensile strength})$$

$$|\sigma_{\phi(r=a)}| = \frac{D^2 + (D - 2t)^2}{D^2 - (D - 2t)^2} P < \sigma_{s-} \quad (= \text{compressive strength})$$

Strength Failure Mechanism for small cells




So

$$D < D_{cr} \sim \left[1 + \frac{\sigma_{s-}}{P} + \sqrt{\left(\frac{\sigma_{s-}}{P} \right)^2 - 1} \right] t$$

Detailed parameters...

$$P_{low} = P_0 + (n + 1) \rho g H$$



extra pressure flow resistance index

$$E = 130 \text{ MPa}; \nu = 0.3; t = 1.5 \text{ } \mu\text{m}$$

$$\sigma_{s-} = 2.3 \text{ MPa for rainforest trees}$$

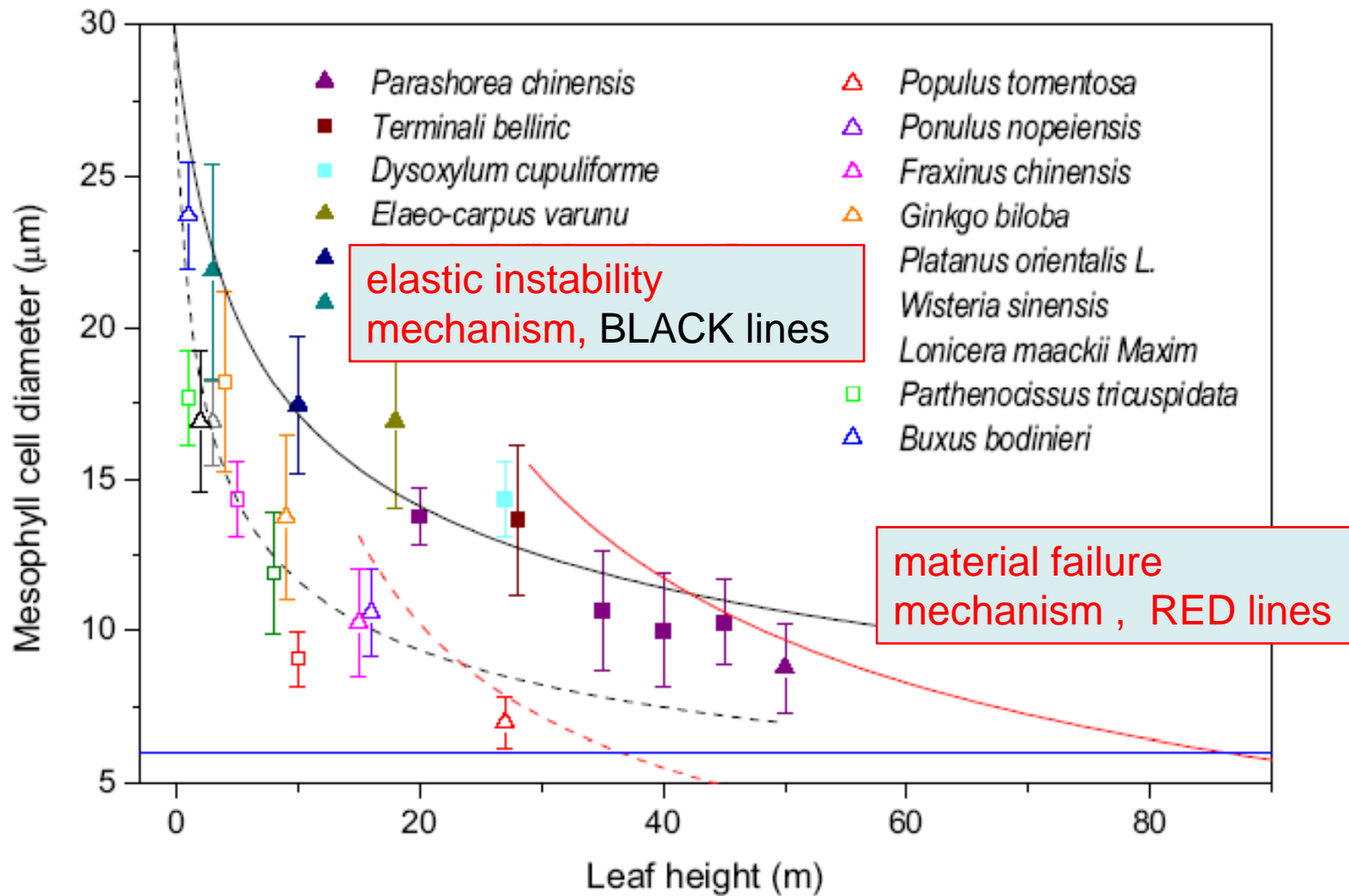
$$\sigma_{s-} = 3.5 \text{ MPa dry-temperate trees}$$

Curve fitting results:

$n = 0.5$ for rainforest zone trees

$n = 4.6$ for dry-temperate zone trees

Model Estimates



In both mechanisms, leaf cells should decrease their sizes to prevent structure and material collapse, with increasing height.