

# **ON THE SIGN OF SHEAR STRESSES IN THE MOHR'S CIRCLE**

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## **ABSTRACT**

A novel explanation on the location of points in the Mohr's circle is provided, avoiding the ambiguity of the sign interpretation of shear stresses.

**KEY WORDS:** Stress transformation, Mohr's circle.

## 1. INTRODUCTION

The use of Mohr's circle gives a simple way for analysing the plane stress state at a point. Each point gives the values of the normal and shear stresses associated to a plane that passes by the considered point. Each plane can be defined by its outward normal axis. Different sign conventions are mentioned in order to locate this axis in Mohr's circle [Gere and Timoshenko, 1997, Vallabhan, 1997]. The aim of this work is to give a general explanation on the location of axes in the Mohr's circle.

## 2. STRESS TRANSFORMATION AND MOHR'S CIRCLE

The objective is to calculate normal and shear stresses,  $\sigma$  and  $\tau$  respectively, corresponding to any plane that passes through point O, knowing the components  $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$ . Being  $OA = dx$ ,  $OB = dy$  in Figure 1, it is supposed that plane AB passes through O.

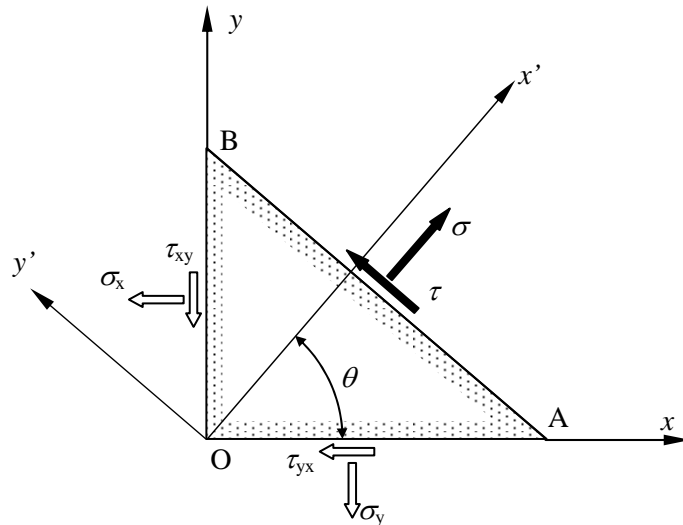


Fig. 1. Stress components in plane-stress.

Applying equilibrium of forces in  $x'$  and  $y'$  directions,  $\sigma$  and  $\tau$  are given by:

$$\begin{aligned}\sigma &= \sigma_x l^2 + 2\tau_{xy} lm + \sigma_y m^2 \\ \tau &= (\sigma_y - \sigma_x) m + \tau_{xy} (l^2 - m^2)\end{aligned}\quad (1)$$

being  $l = \cos\theta$ ,  $m = \sin\theta$ .

Writing Eq.(1) as a function of the double angle it results

$$\begin{aligned}\sigma &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ \tau &= \frac{\sigma_y - \sigma_x}{2} \sin 2\theta + \tau_{xy} \cos 2\theta\end{aligned}\quad (2)$$

Principal directions can be obtained from Eq.(2) by either of the following conditions

$$\left. \begin{aligned} \frac{d\sigma}{d\theta} &= 0 \\ \tau &= 0 \end{aligned} \right\} \Rightarrow \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad (3)$$

In Equation (2), rearranging the first one, squaring and adding them it results

$$\left( \sigma - \frac{\sigma_x + \sigma_y}{2} \right)^2 + \tau^2 = \left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2 \quad (4)$$

Equation (4) corresponds to a circumference in  $\sigma$ ,  $\tau$  axes. The center and the radius are given by

$$\sigma_C = \frac{\sigma_x + \sigma_y}{2} \quad R = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \quad (5)$$

Fig. 2 shows a stress state where it is supposed that  $\sigma_x > \sigma_y > 0$  and  $\tau_{xy} > 0$ . A positive sense is not assigned to  $\tau$  axis, since in other case, there would be positive and negative shear stresses in the Mohr's circle for the same positive  $\tau_{xy}$ . This is the main problem associated to the use of Mohr's circle: which is the criterion for locating axes on the upper or the lower half of the circle?

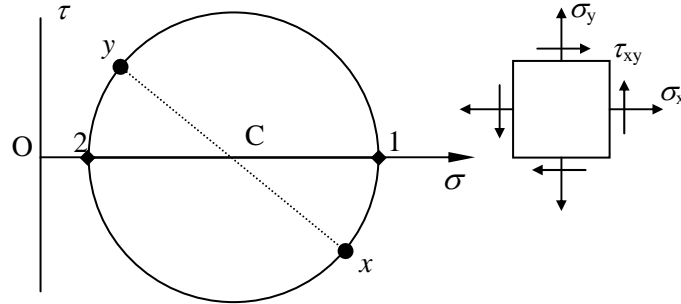


Fig. 2. Mohr's circle for plane stress.

Regarding Equation (3),  $\tan 2\theta_p$  can be positive or negative, depending on the sign of the numerator and denominator. In Figure 3 all possible cases appear, indicating the angle between 0 and  $\pi/2$ . If the tangent is positive the angle is counterclockwise, according to Figure 1 and if the tangent is negative, the angle is clockwise.  $\tau$  axis is not shown, as the location of  $x$  and  $y$  axis in the upper or the lower half of the circle does not depend on the position of the origin O.

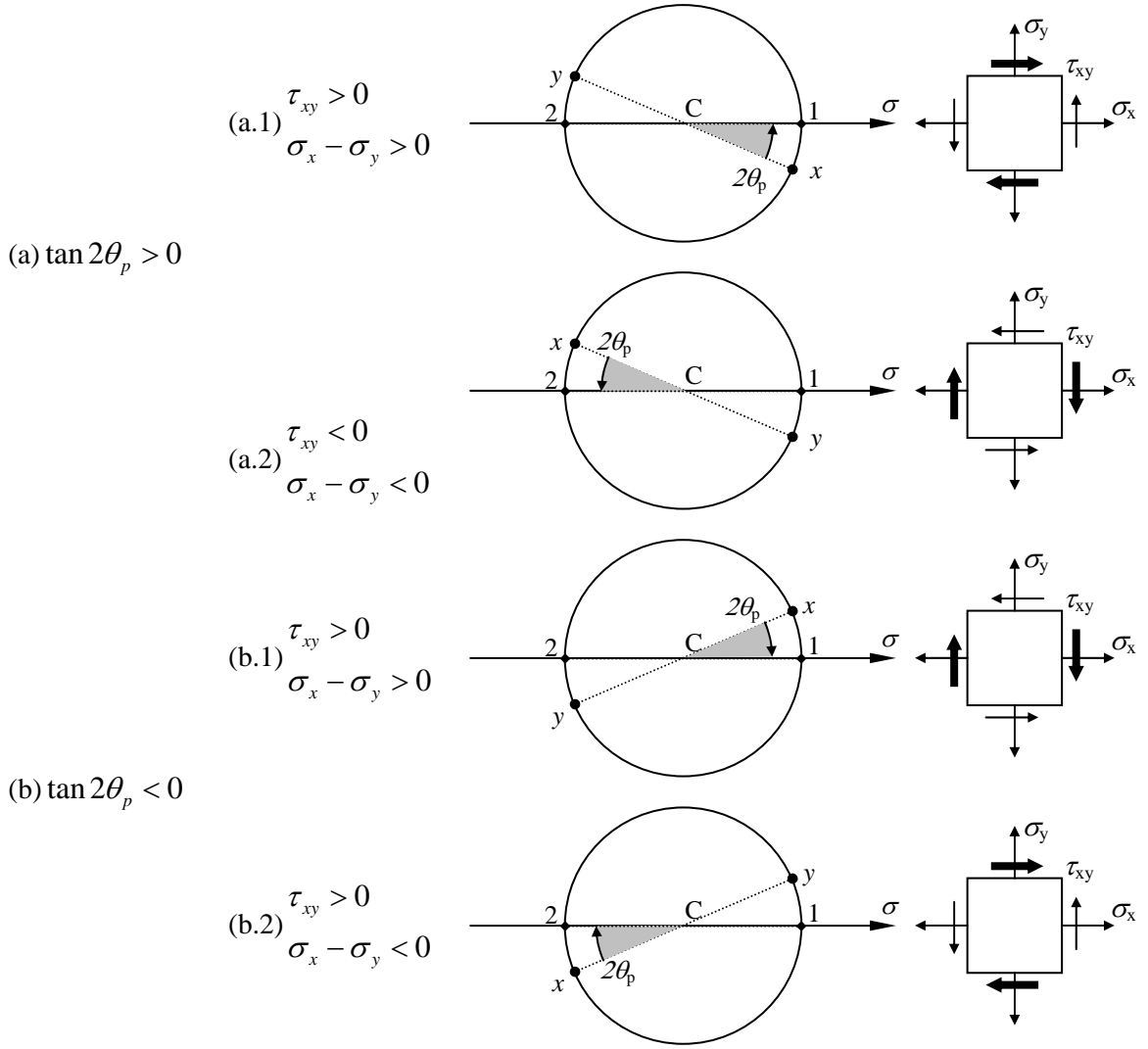


Fig. 3. Location of axes in Mohr's circle: (a) counterclockwise rotation; (b) clockwise rotation.

Corresponding stress states are shown, indicating with thicker arrows shear stresses related to clockwise rotation. It can be seen that in all cases the axis perpendicular to clockwise shear stresses is located in the upper half of the circle.

Therefore, the following criterion appears evident: when the rotation due to shear stresses acting on parallel planes is clockwise, the axis perpendicular to these planes is located at the upper half of the circle. It is worth mentioning that this is not a sign convention, but a consequence of the location of principal directions. Therefore, shear stresses in the Mohr circle's are used in absolute value and their rotation effect in parallel planes gives the position in the upper or the lower half of the circle. In an inverse manner, for a given point in the circle, its position gives the rotation sense of the pair of shear stresses.

### 3. CONCLUSIONS

A general explanation on the location of points in the Mohr's circle has been provided. The contradiction of different signs for the same shear stress that usually appears in Mohr's

circle is avoided, using shear stresses in absolute value. It has been shown that the position of the axis in the upper or the lower half of the circle is related to the rotation associated to the pair of shear stresses perpendicular to the axis. Therefore, the sign of shear stresses in Mohr's circle is not a convention, but a consequence of the position of principal directions.

#### **4. REFERENCES**

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