In celebration of *Fong Shih*
(American Academy of Arts and Sciences, Cambridge, MA, 12 May 2016)

Mechanics on our Planet: Great Ice Sheets, Rapid Earthquakes

James R. Rice (Harvard) and Ares J. Rosakis (Caltech)

April 2016

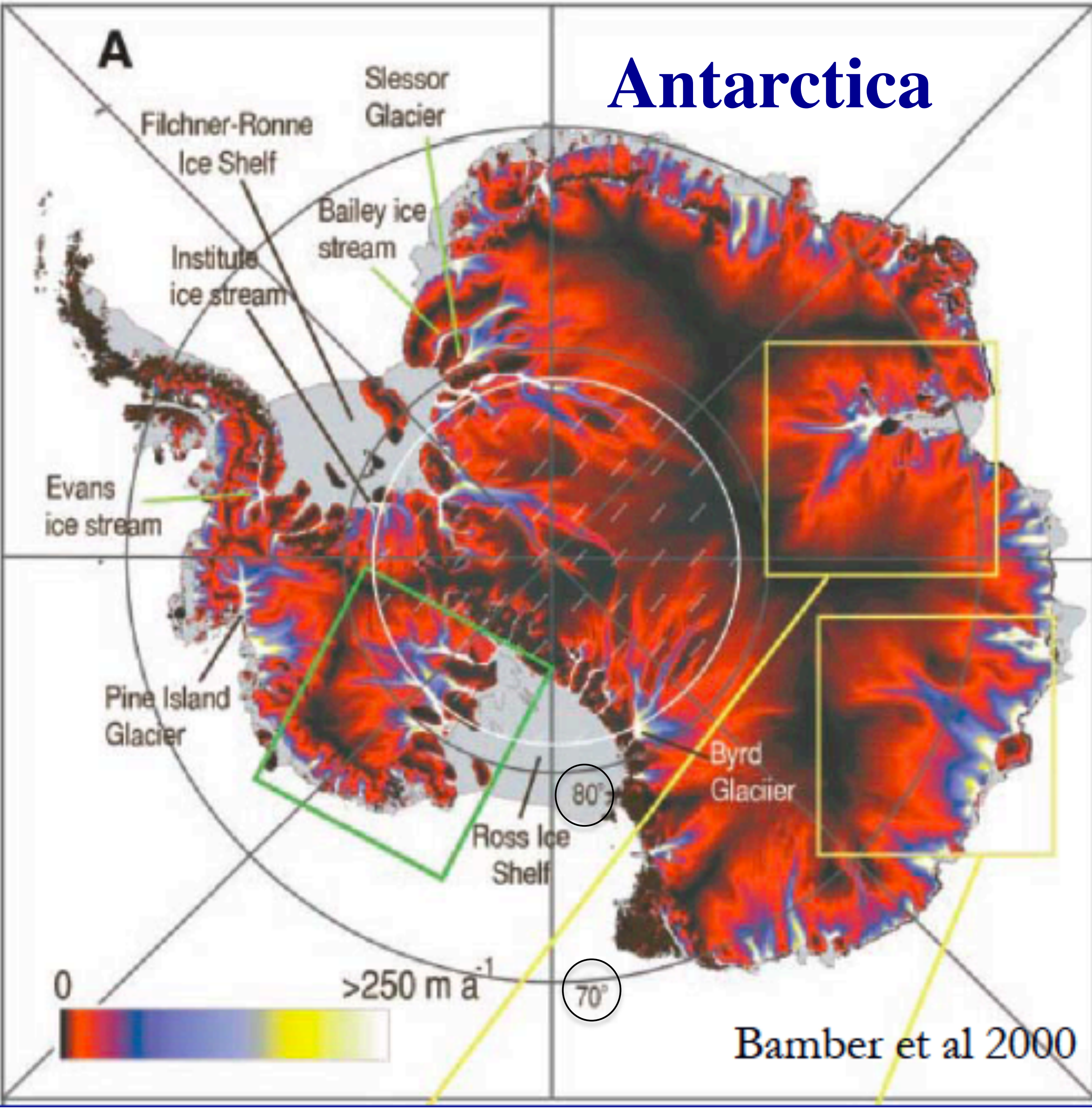
What's a glacier?

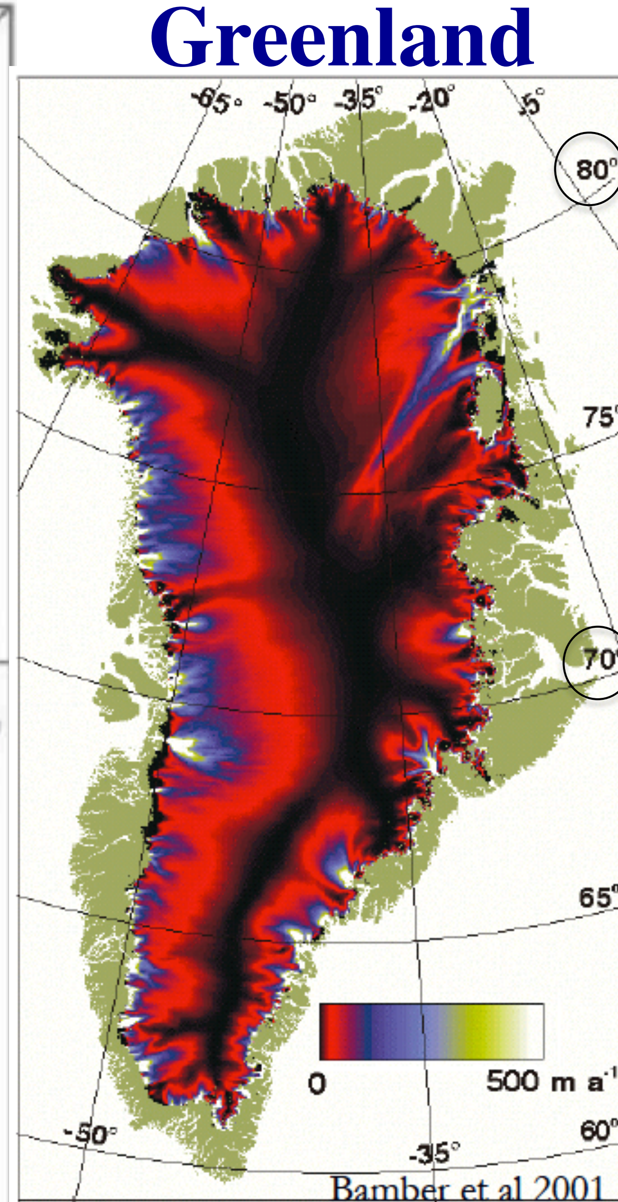
- "A river of ice" that flows under its own weight
- Different types:
  - Mountain (valley), Tidewater, Ice sheet

Described in 11^{th} century

Motion not realized until 16^{th} century!!

The major ice sheets - *not to scale (reduce Greenland ~50%)*





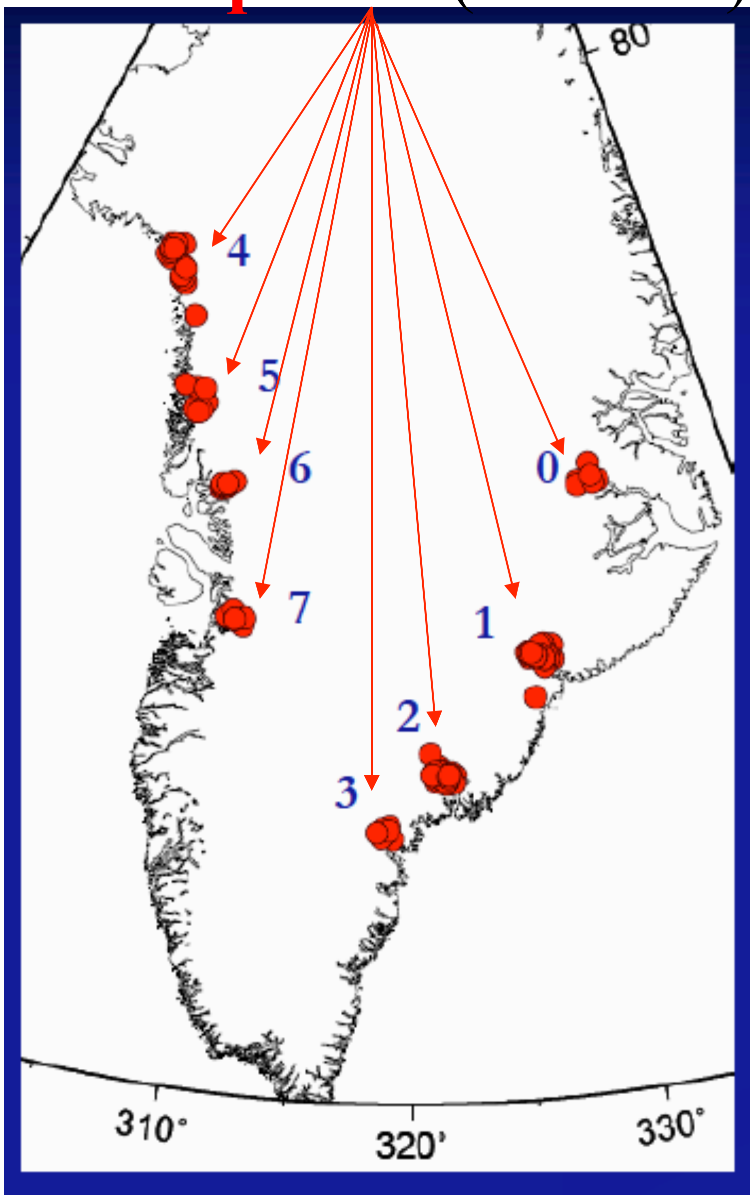
On the Greenland Ice Sheet:

Glacial Earthquakes (discovered and located by Göran Ekström), and their unexpected mechanism

with

Victor C. Tsai (Caltech) and, later, Mark Fahnestock (Univ. New Hampshire)

Source locations of **glacial earthquakes** (Ekström)



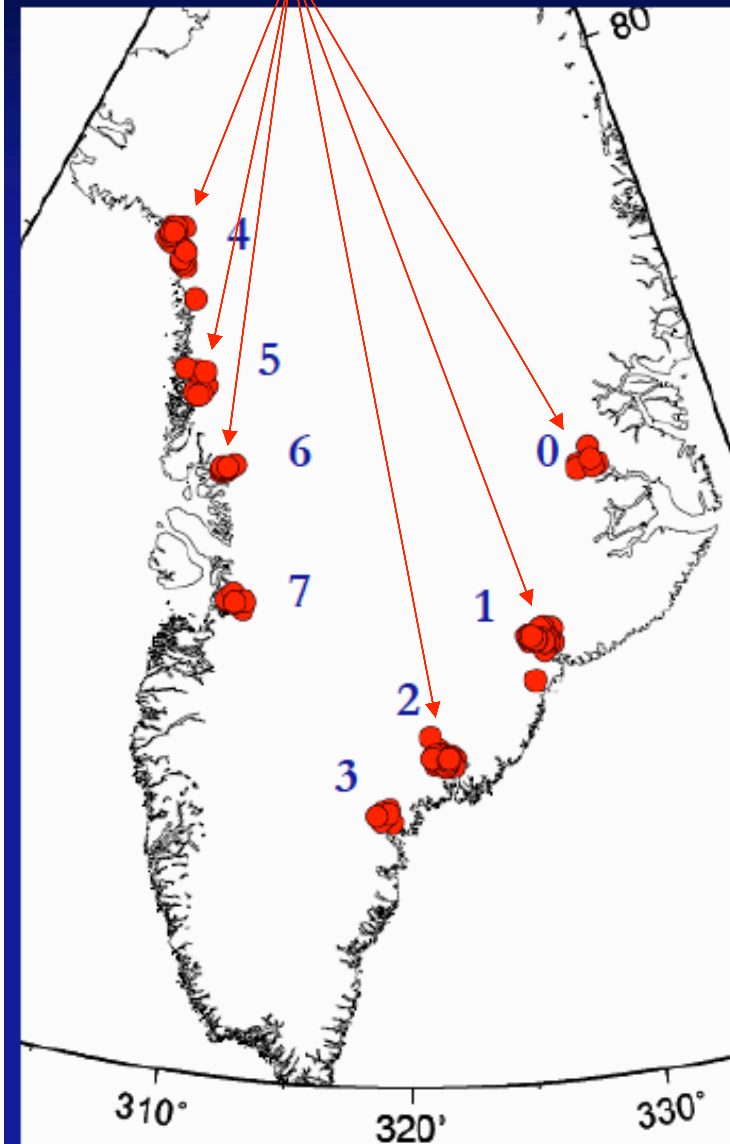
Unusual earthquakes:

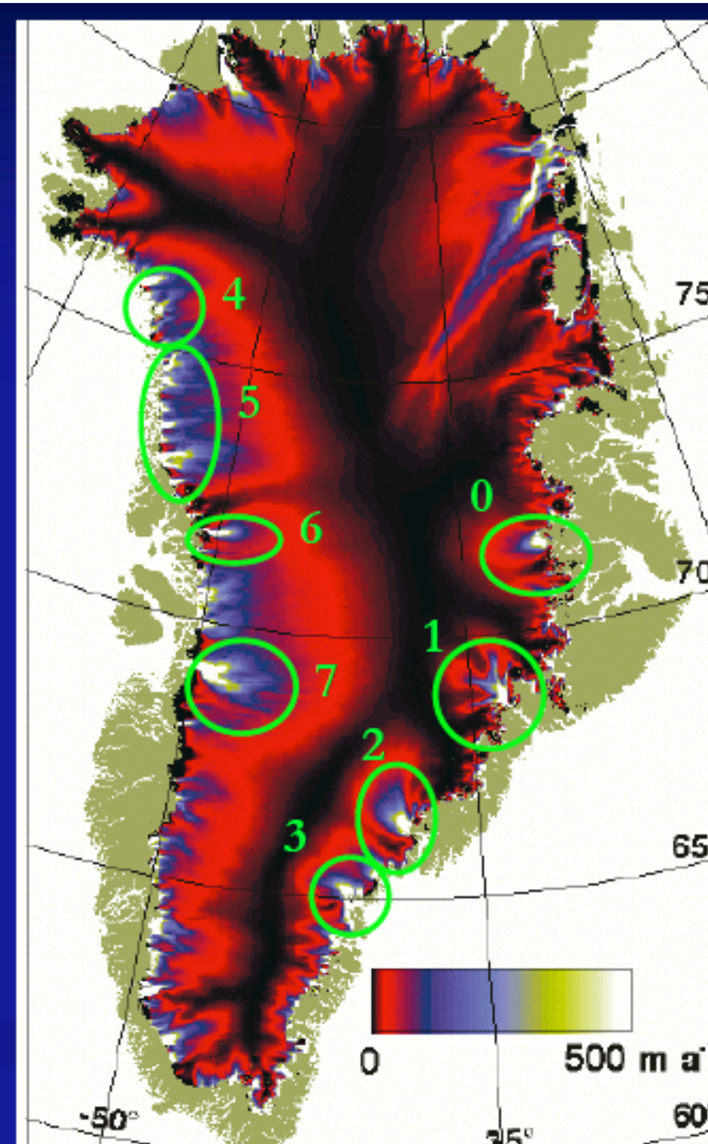
- Magnitude M_{sw} ~ 4.6 to 5.1, measured from signals filtered to 35-150 sec periods.

- Significant energy in periods between 20 and 100 sec (**much longer period than for standard earthquakes of similar M_s** (e.g., typical source duration ~2 sec for M_s ~ 5).

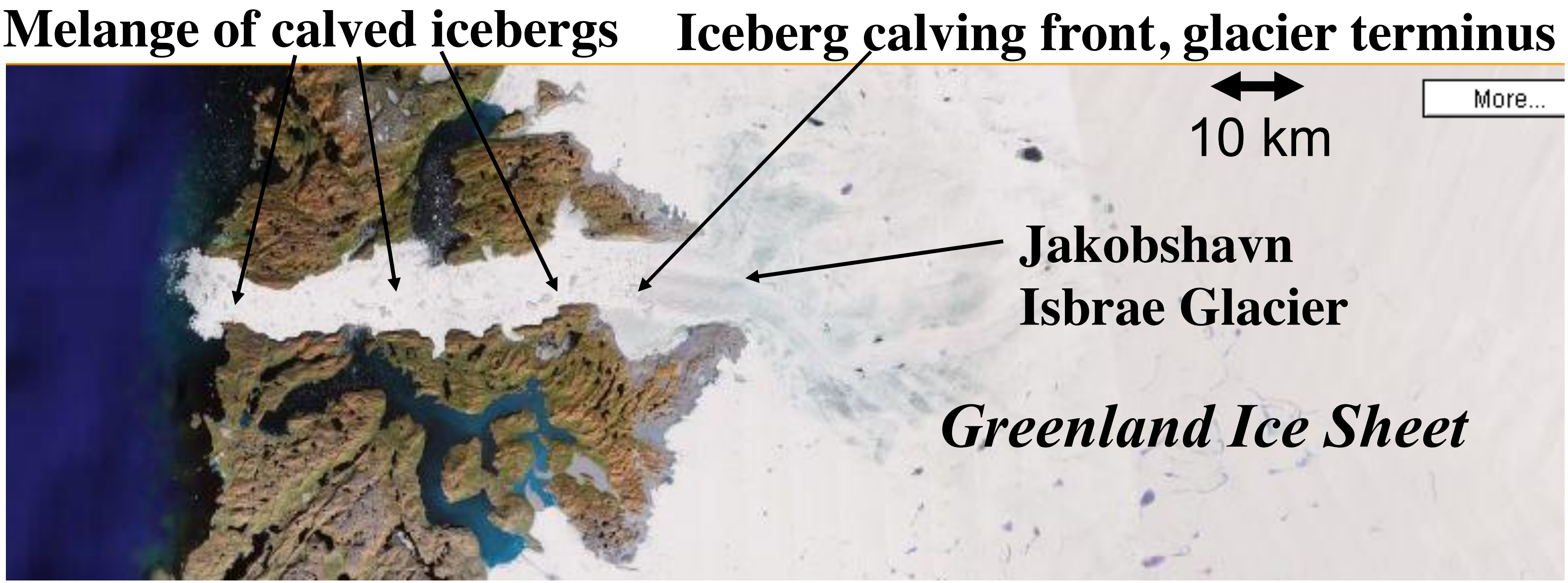
Source locations of **glacial earthquakes** (G. Ekström)

Correlation with areas of high ice flow rates -- at major fjords





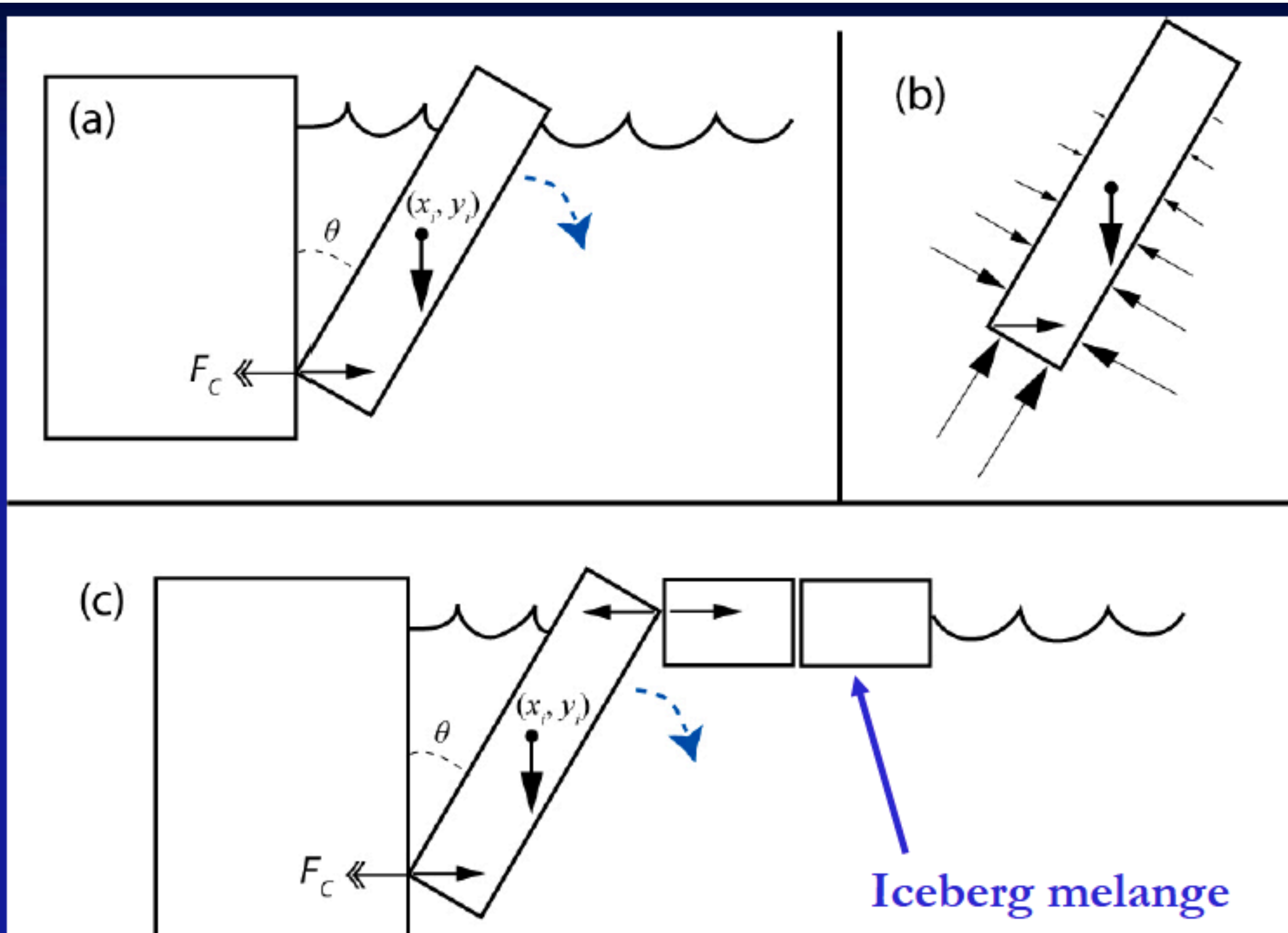
Bamber et al., 2001


Melange of calved icebergs
Iceberg calving front, glacier terminus
10 km
More...
Jakobshavn
Isbrae Glacier
Greenland Ice Sheet


~10 km

What causes glacial EQs?

- ***Fast sliding along boundary of ice sheet, or along internal fault, analogous to normal EQs? Nettles et. al. GPS [AGU,'07]: No!***
- ***Simple iceberg calving models work best! -- timescale.***



Tsai, Rice, Fahnestock, *JGR*, 2008

http://onlinelibrary.wiley.com/doi/10.1029/2008GL035281/abstract

Video by Jason Amundson (Univ. Alaska, Fairbanks) is sped-up; large overturns take time of order 1-2 minute. See Amundson et al., GRL 2008)

Greenland Ice Sheet:

Rapidly draining of surficial lakes and natural hydraulic fractures

Studies with

Victor C. Tsai **(Caltech)**

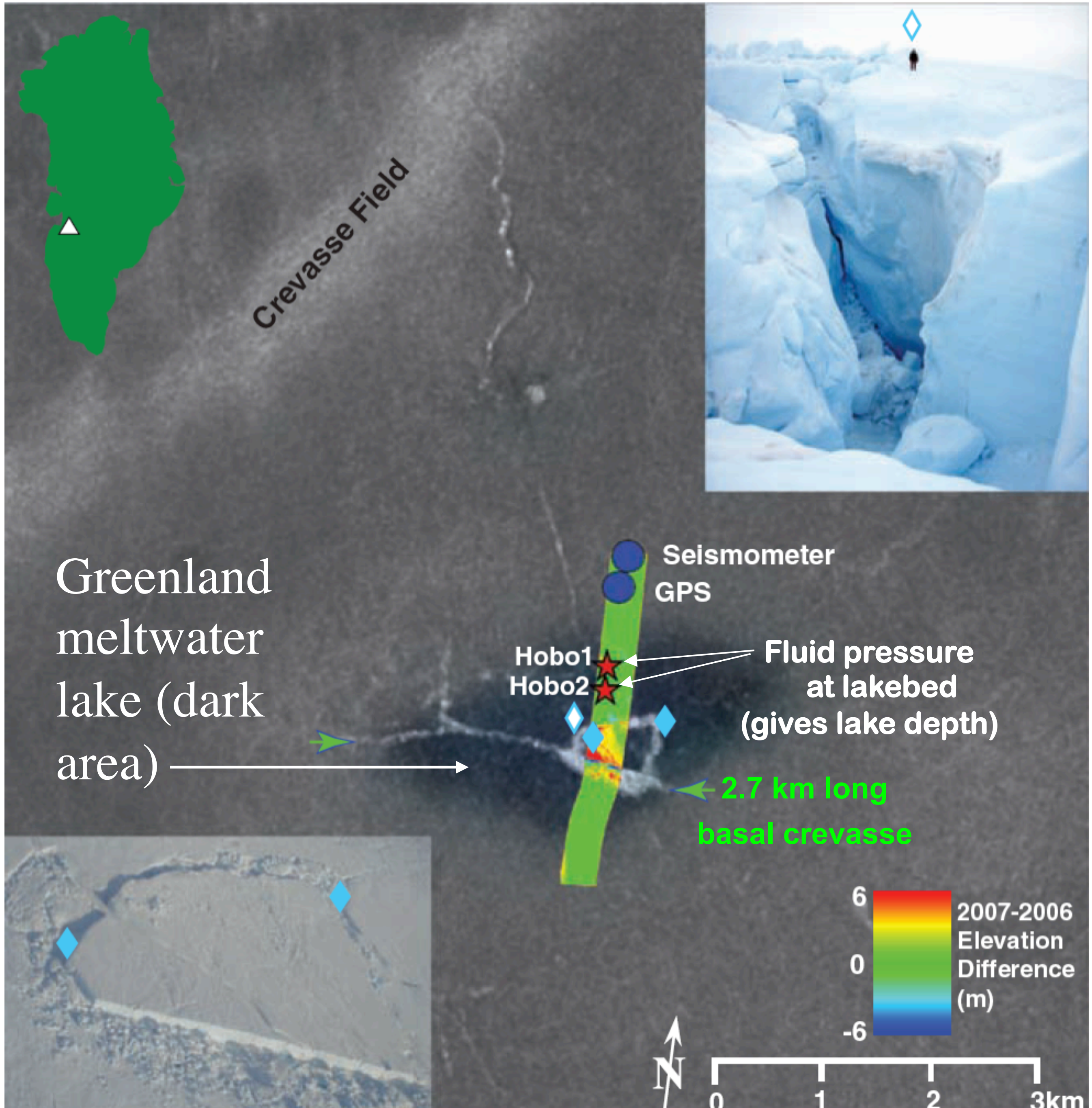
John D. Platt **(Carnegie Inst. of Sci.)**

Matheus C. Fernandes **(Harvard)**

Study motivated by the paper ***Fracture Propagation to the Base of the Greenland Ice Sheet During Supraglacial Lake Drainage***, by Das, Joughin, Behn, Howat, King, Lizarralde & Bhatia, *Science*, May 2008.

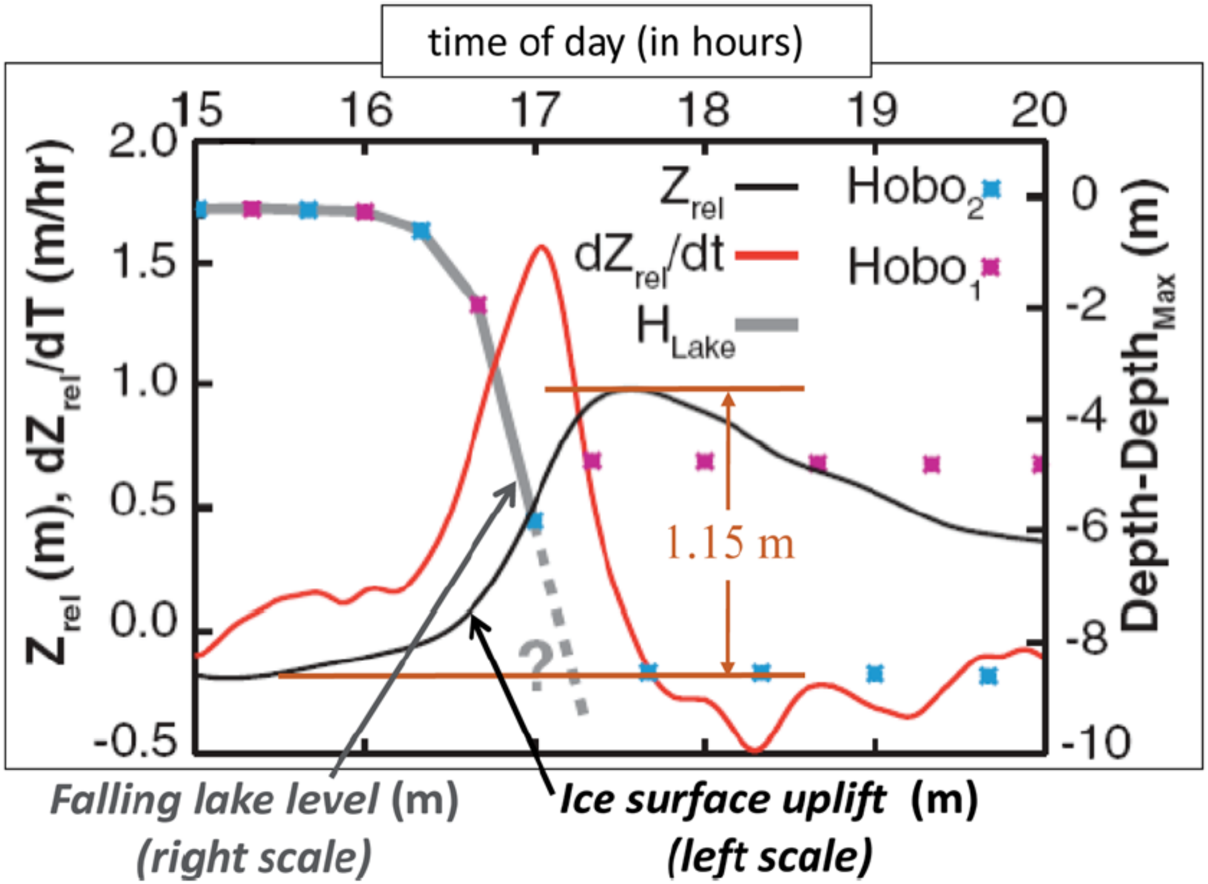
(Das et al., *Sci.*, 2008)

Early October 2006 SAR image (gray-scale background) overlaid with a semi-transparent image recorded by NASA's Moderate Resolution Imaging Spectroradiometer (MODIS) showing the lake extent (blue) on 29 July 2006.



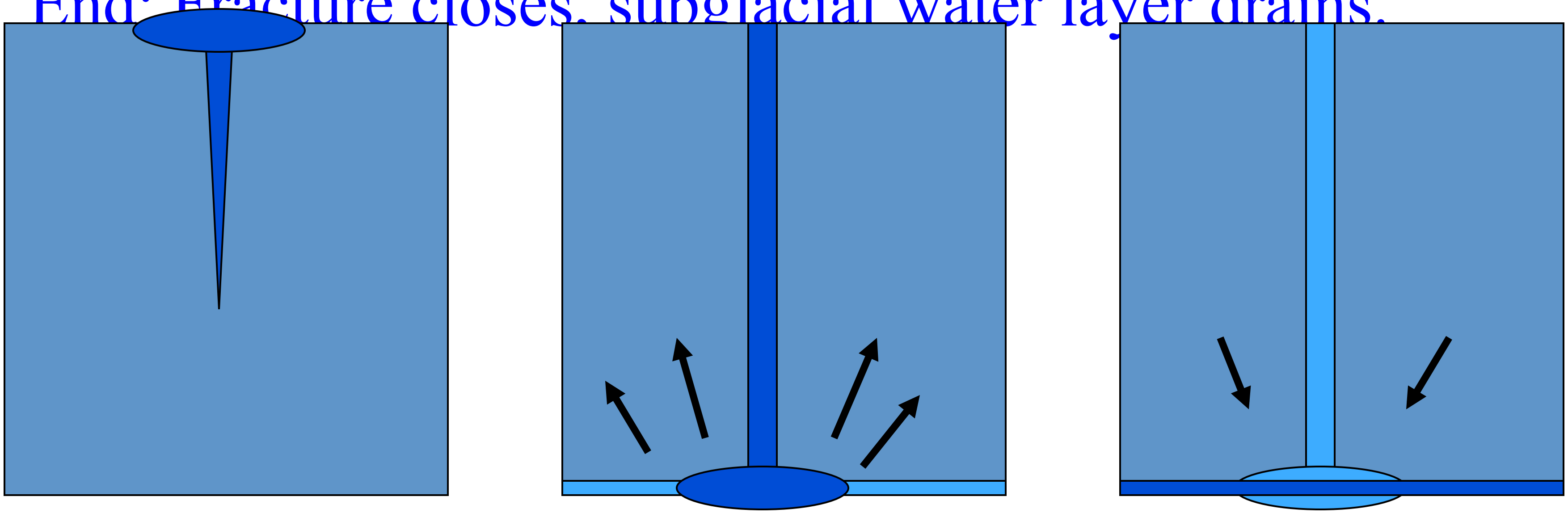
- ***Supraglacial meltwater lake*** began filling July 2006
- Maximum fill at ~0:00 29 July, Vol. = 44 x 10^6 m^3, Surf. = 5.6 km^2
- Level slowly/steadily falls, ~15 mm/hr, for next ~16 hours
- Rapid from 16:00-17:30, max 12 m/hr (Q_{max} > 10,000 m^3/s), avg Q_{avg} ~ 8,700 m^3/s [Compare, Niagra Falls Q_{Niag} ~ 6,000 m^3/s]

[1] Das, Joughin, Behn, Howat, King, Lizarralde & Bhatia, "*Fracture propagation to the base of the Greenland Ice Sheet during supraglacial lake drainage*", Science, v. 320, pp. 778–781, 2008.



Interpretation

- Initially: Crevasse/moulin system gradually propagates to bed by *Weertman* gravitational instability, $\rho_{water} > \rho_{ice}$.
- Middle Stage: Hydraulic cracking and flooding along bed by over-pressure, p > ?$_o$ (?$_o$= ice overburden pressure).
- End: Fracture closes, subglacial water layer drains.



Rubin, ***Propagation of magma-filled cracks*** [Annu. Rev. Earth Planet. Sci., 1995]

$$\rho_{magma} < \rho_{rock}$$

Basaltic dike at tip of Reykjanes Peninsula, southwest Iceland, exposed by glacial erosion (did not make it to surface). Thickness = 40 cm.

Dike (boundaries dotted) terminating in shear zone on Colorado Plateau.

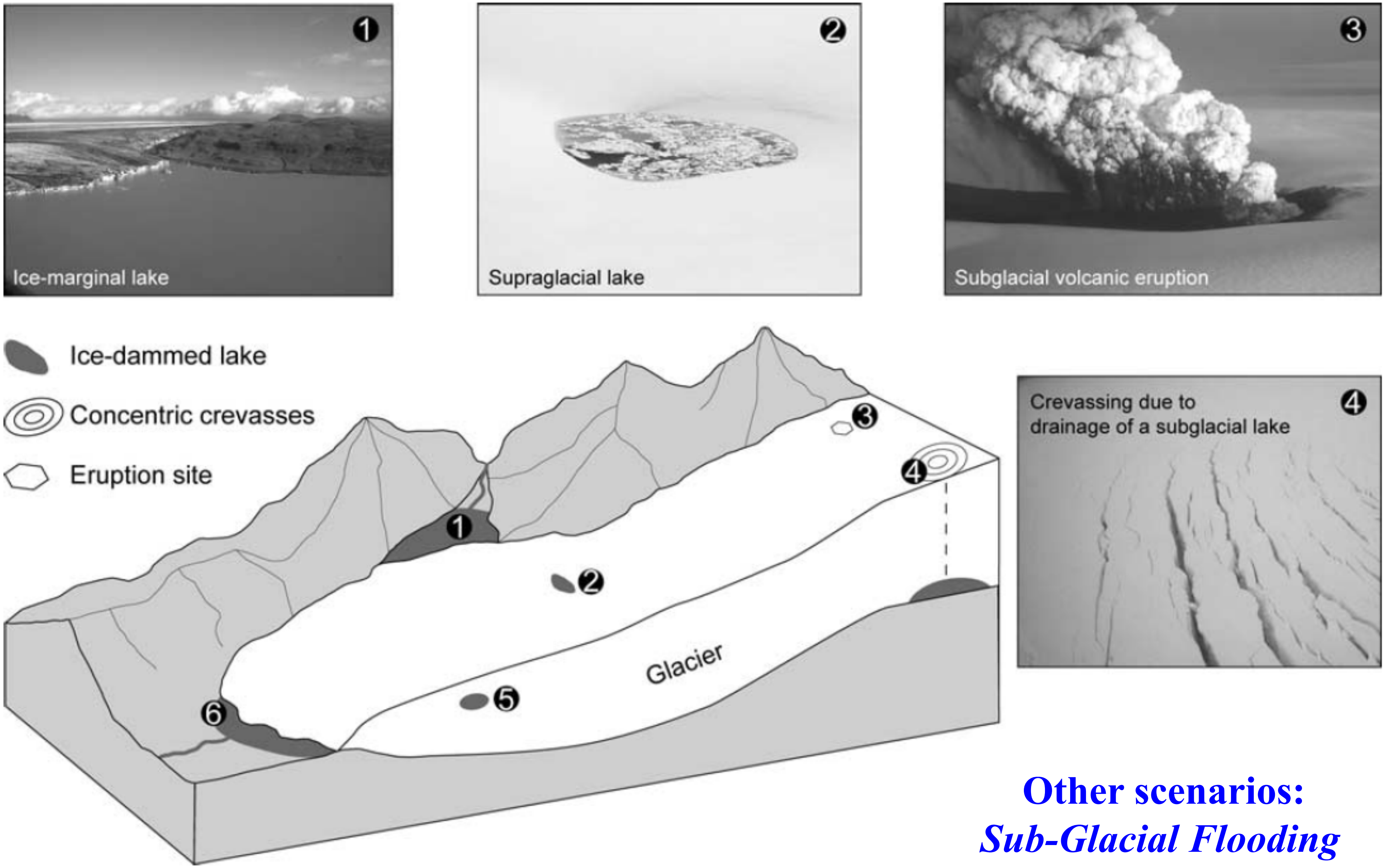


Figure 1. Reservoir sites and meltwater sources for jökulhlaups.

Other scenarios:
Sub-Glacial Flooding
(Jökulhlaup)

from: Roberts, M. J. (2005), Jökulhlaups: A reassessment of floodwater flow through glaciers, *Rev. Geophys.,* 43, RG1002.

Water drainage along glacier beds often creates channels

Natural example, shown in photo by Laura Kehrl: Rothlisberger Channel
(but in a mountain glacier, with channel probably created by outburst underflooding):

Kennicott Glacier, Alaska

Channelized drainage at glacier's bed (Alberta, Canada)

JUNEAU JOURNAL

Alaska Looks for Answers in Glacier's Summer Flood Surges

New York Times
July 22, 2013

First observed in July 2011, also in July 2012 & 2013.

"… in July 2011, … an estimated ten billion gallons gushed out in three days … two smaller bursts this year …"

Mathew Ryan Williams for The New York Times

Visitors leaving caves under the Mendenhall Glacier, near Juneau, Alaska. Unpredictable flood surges have elevated concerns.

By KIRK JOHNSON

JUNEAU, Alaska -- …. unpredictable flood surges at the Mendenhall Glacier, about 14 miles from downtown Juneau, Alaska's capital ….

$$p_{hydrostat} \equiv \rho_{water} g H \geq p_{inlet} \geq \rho_{ice} g H \equiv \sigma_o$$

Rice, Tsai, Fernandes & Platt, *J.Appl.Mech.*, 2015

Average vertical conduit opening:

$\Delta\bar{u} = \Delta\bar{u}^{el} + \Delta\bar{u}^{cr}$ (elastic + prior creep)

$\Delta\bar{u}^{el} \propto p_{inlet} - \sigma_o$

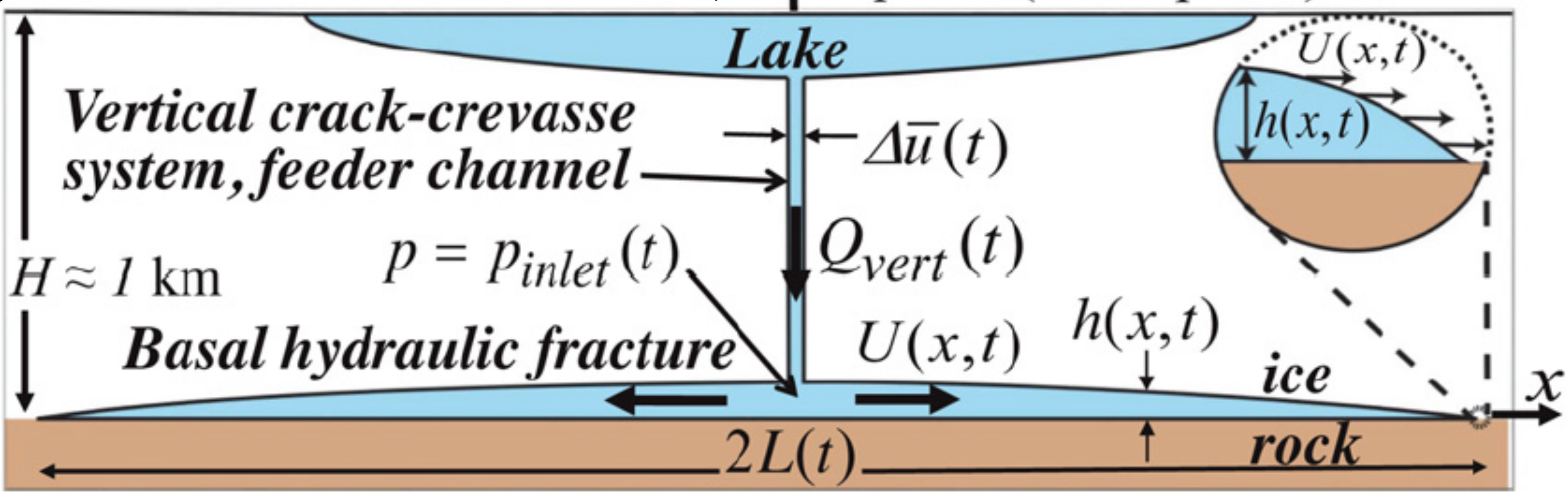
Important

For a given conduit opening $\Delta\bar{u}$, vertical flow rate

$Q_{vert} \propto (p_{hydrostat} - p_{inlet})^{1/2}$

Glen's law, ice deformation: $\dot{\gamma} = A(T)\,\tau^3$

($\dot{\gamma}$ = shear strain rate , τ = shear stress)

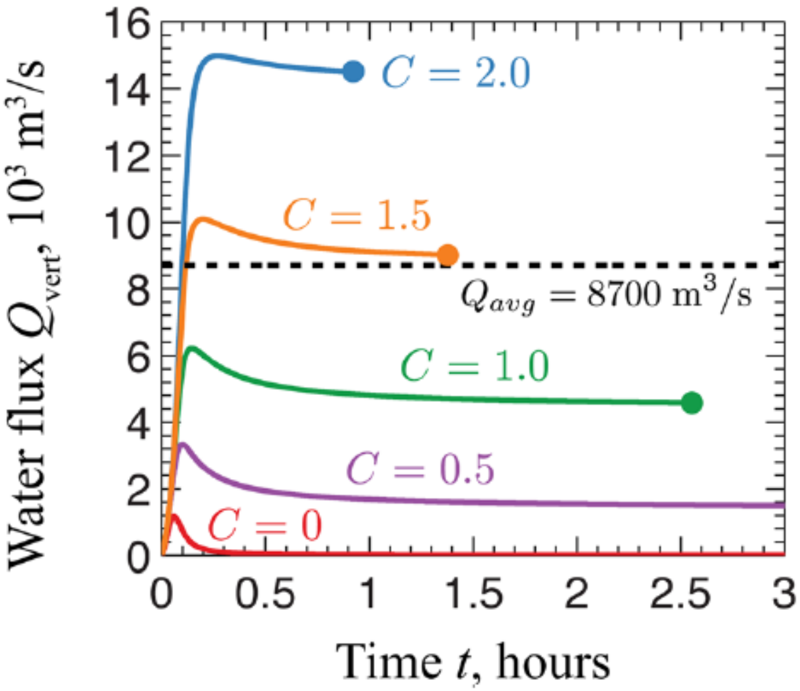


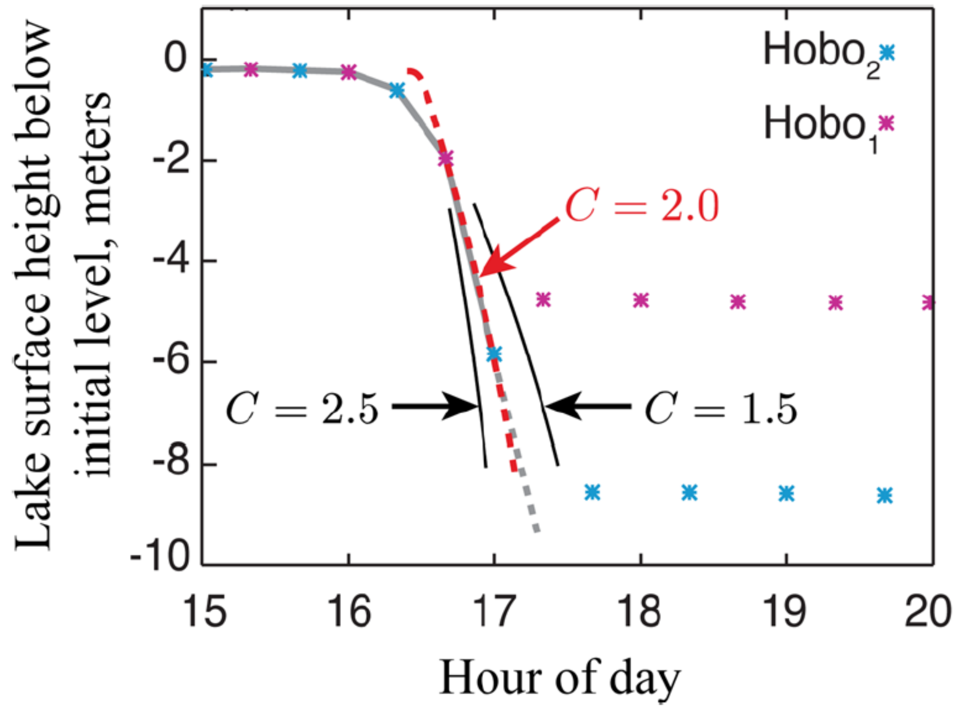
$p_{inlet} - \sigma_o$ ~ controls flow rate Q_{basal} into basal fracture

p_{inlet} ultimately determined by setting $Q_{basal} = Q_{vert}$

Rice et al., *J.Appl.Mech.*, 2015

$$C = \frac{\text{creep opening over 16 hrs of hydrostatic } p}{\text{elastic opening under hydrostatic } p}$$





At $T_{ice} \approx -7^{\circ}$C to -5°C, hydrostatic p over 16 hr of slow leakage $\Rightarrow C \approx 1.42$ to 2.11, in reasonable agreement with the values of $C \approx 1.5$ to 2.0 which would plausibly fit the the model to the observations. (But, at present, we have no temperature measurements below a persistent supraglacial lake to know if -7°C to -5°C is sensible.)

Possibility : High stress levels in ice, due to pressurization of the vertical fissure, cause more rapid ice deformation, at any given T_{ice}, than predicted by Glenn's law, so that a lower T_{ice} would be implied to produce the same drainage time scale.

West Antarctic Ice Sheet:

Rapidly flowing ice streams: What processes control their width?

with

Thibaut Perol **(Harvard),**

John D. Platt **(Carnegie Inst. Of Sci., DC)**

and

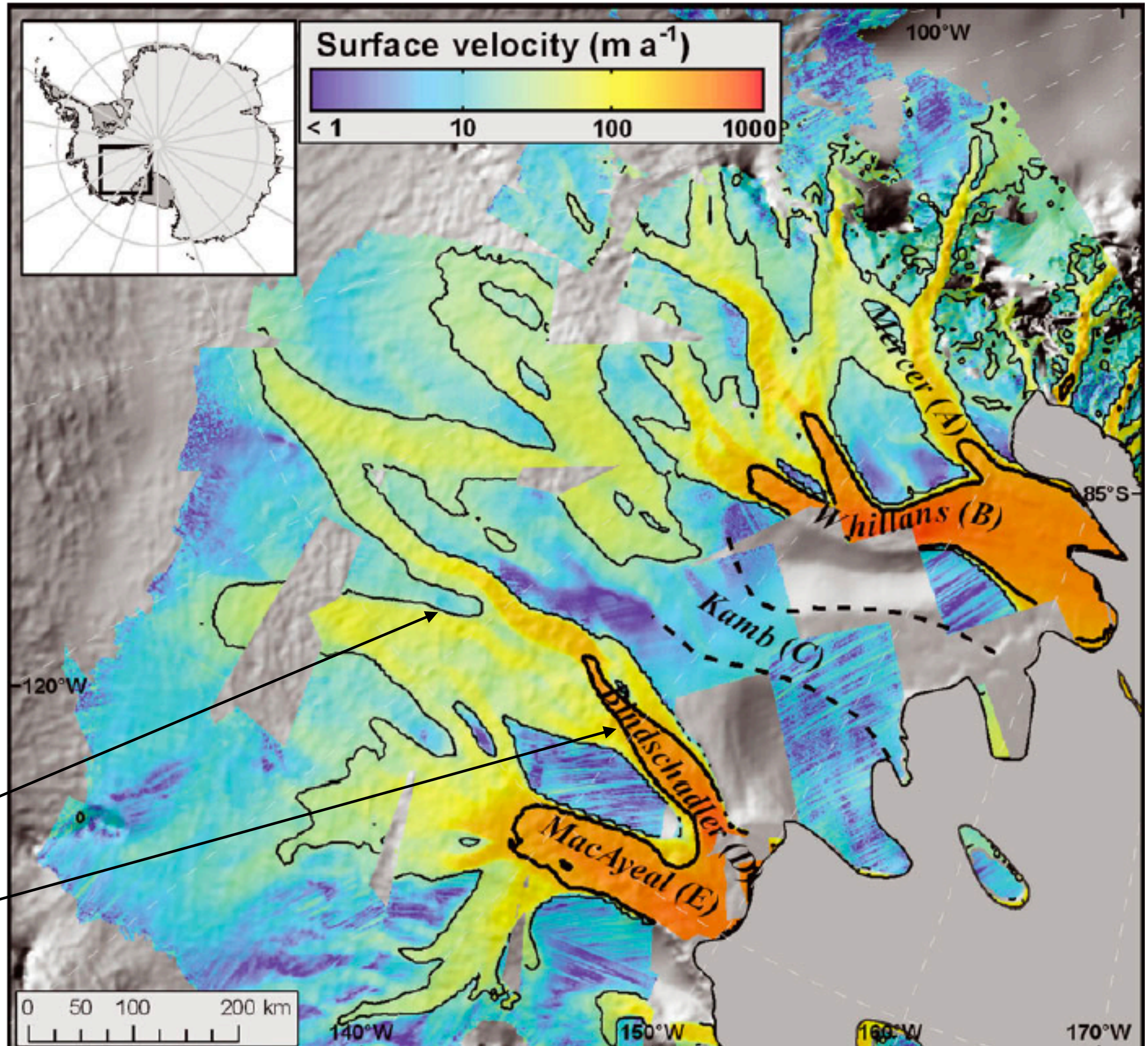
Jenny Suckale **(Stanford)**

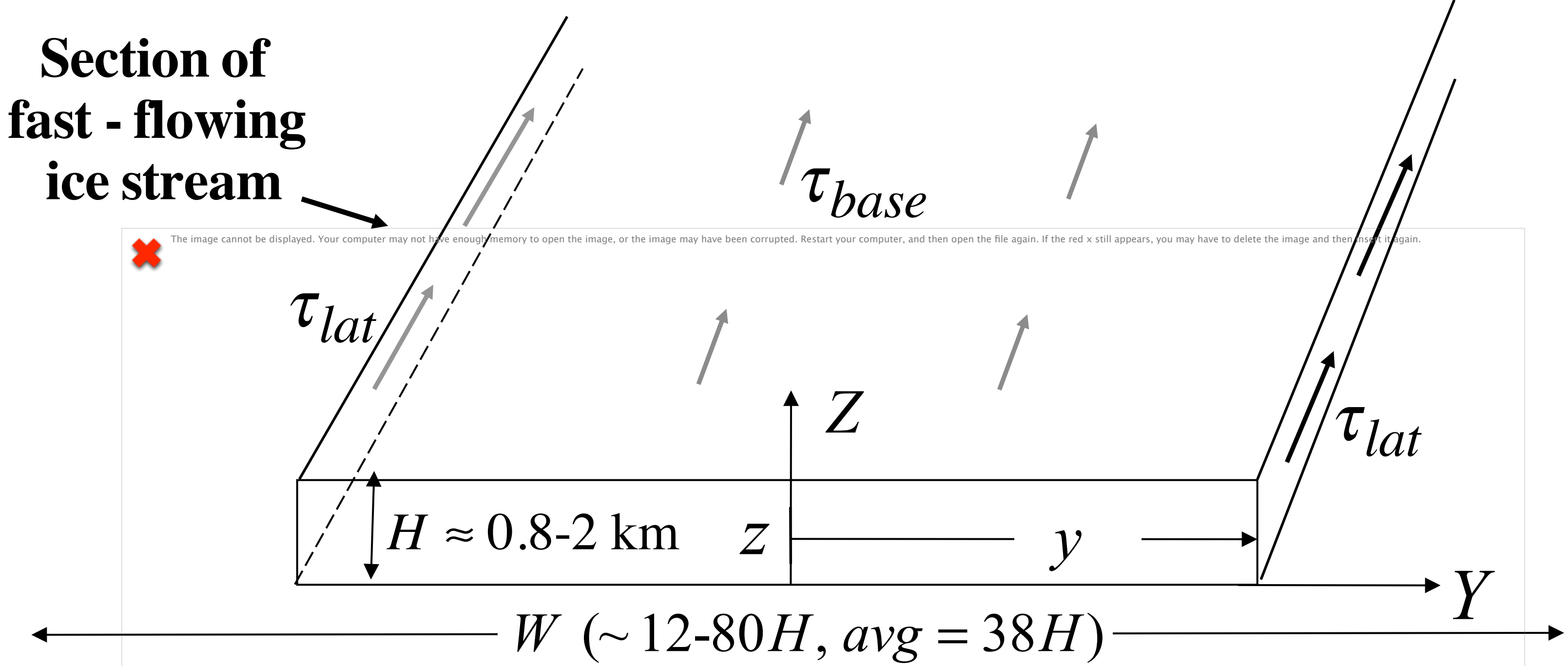
[from Le Brocq, Payne, Siegert & Alley, *J. Glac.*, 2009]

- **Western Antarctica, Siple Coast, *Ice Streams*,** flowing to the Ross Sea ice shelf.

- InSAR velocity (from Joughin et al., *J. Geoph. Res.,* 2002) overlaid on a digital elevation model (Bamber et al., 2009).

- Velocity contours shown are 25 m/yr (thin line) and 250 m/yr (thick line).





$\tau_{grav} = \rho g H S \quad (S = \text{ slope})$

= downslope gravity

force per unit base area

Equilibrium*: $\bar{\tau}_{lat} H = \left(\tau_{grav} - (\tau_{base})_{avg}\right) y$ (low: τ_{base})

$\bar{\tau}_{lat} \rightarrow \left(\tau_{grav} - (\tau_{base})_{avg}\right) W / 2H$ at margins

Increases with stream width W / H ($\sim$ 6-40 at current margins)

$\Rightarrow$ increasing strain rate,

$\dot{\gamma}_{lat} \sim \bar{\tau}_{lat}^3 \sim (W/H)^3$,

$\Rightarrow$ increasing shear heating, $\sim (W/H)^4$,

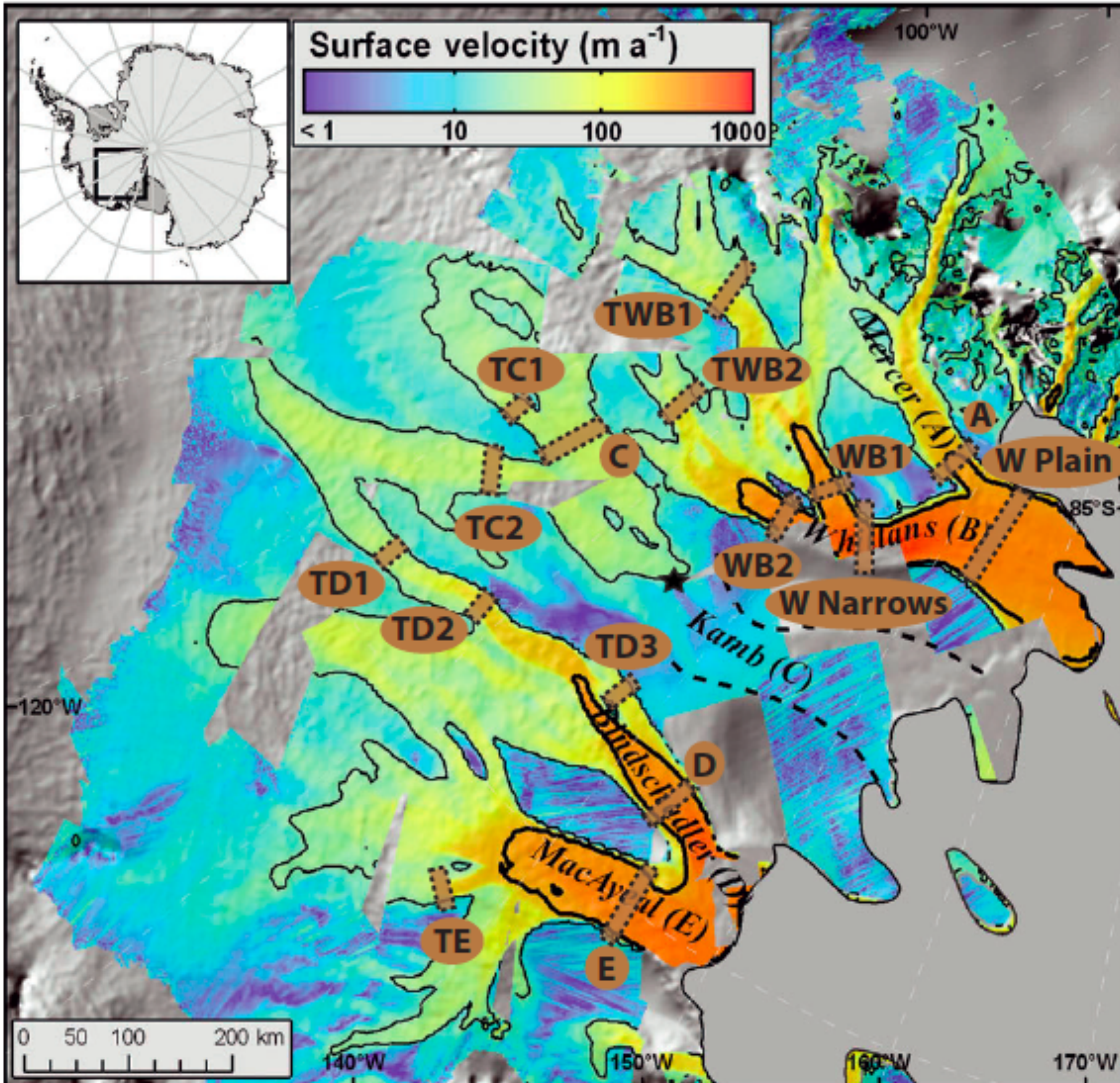
$\Rightarrow$ onset of melting ?

*neglecting any variation in net axial force in sheet, roughly justified [Whillans and van derVeen, *J. Glac.*, 1993]

Our data set, to test concepts:

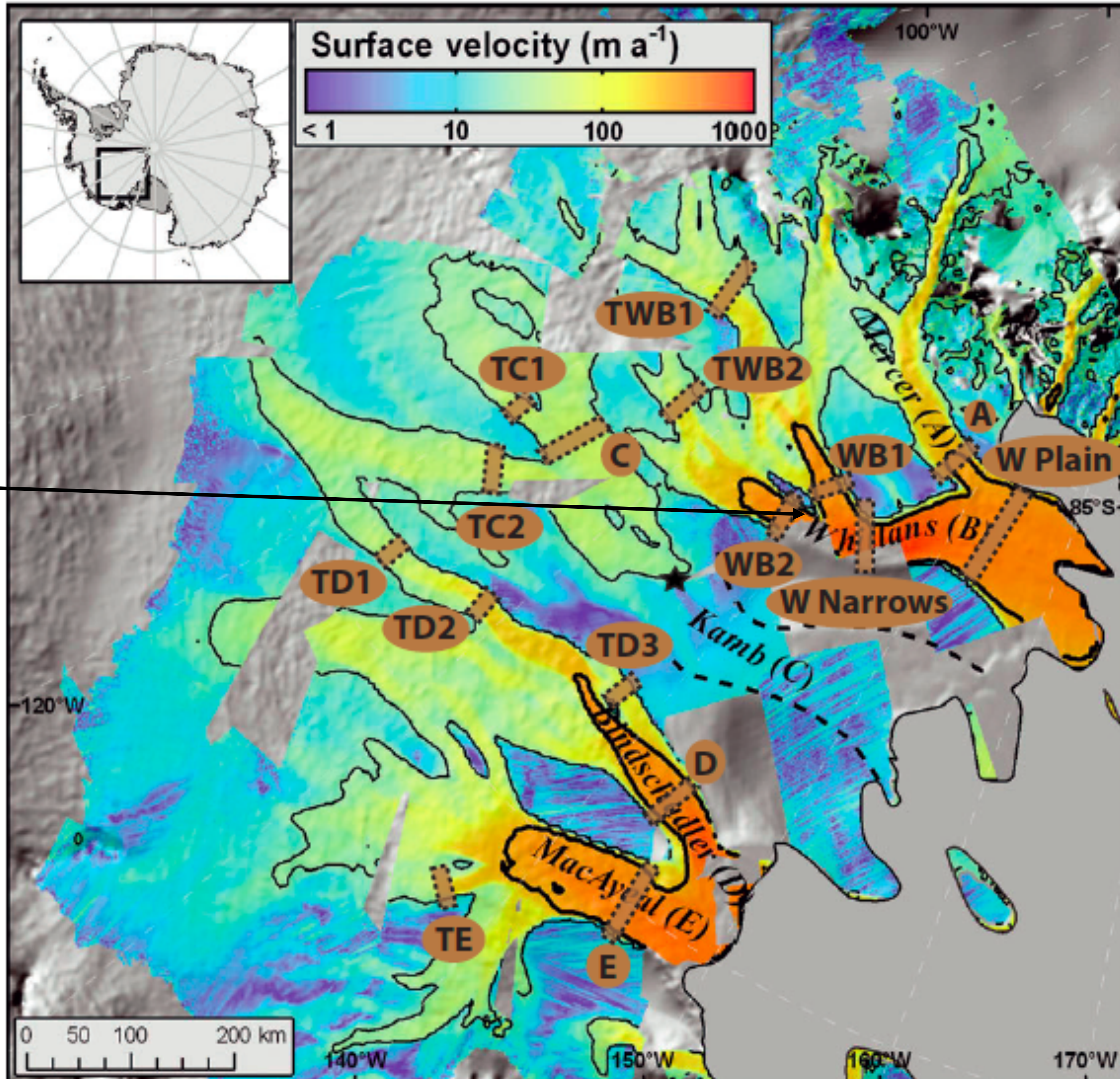
Sixteen ice stream traverses (dotted-lines) for velocity profiles.

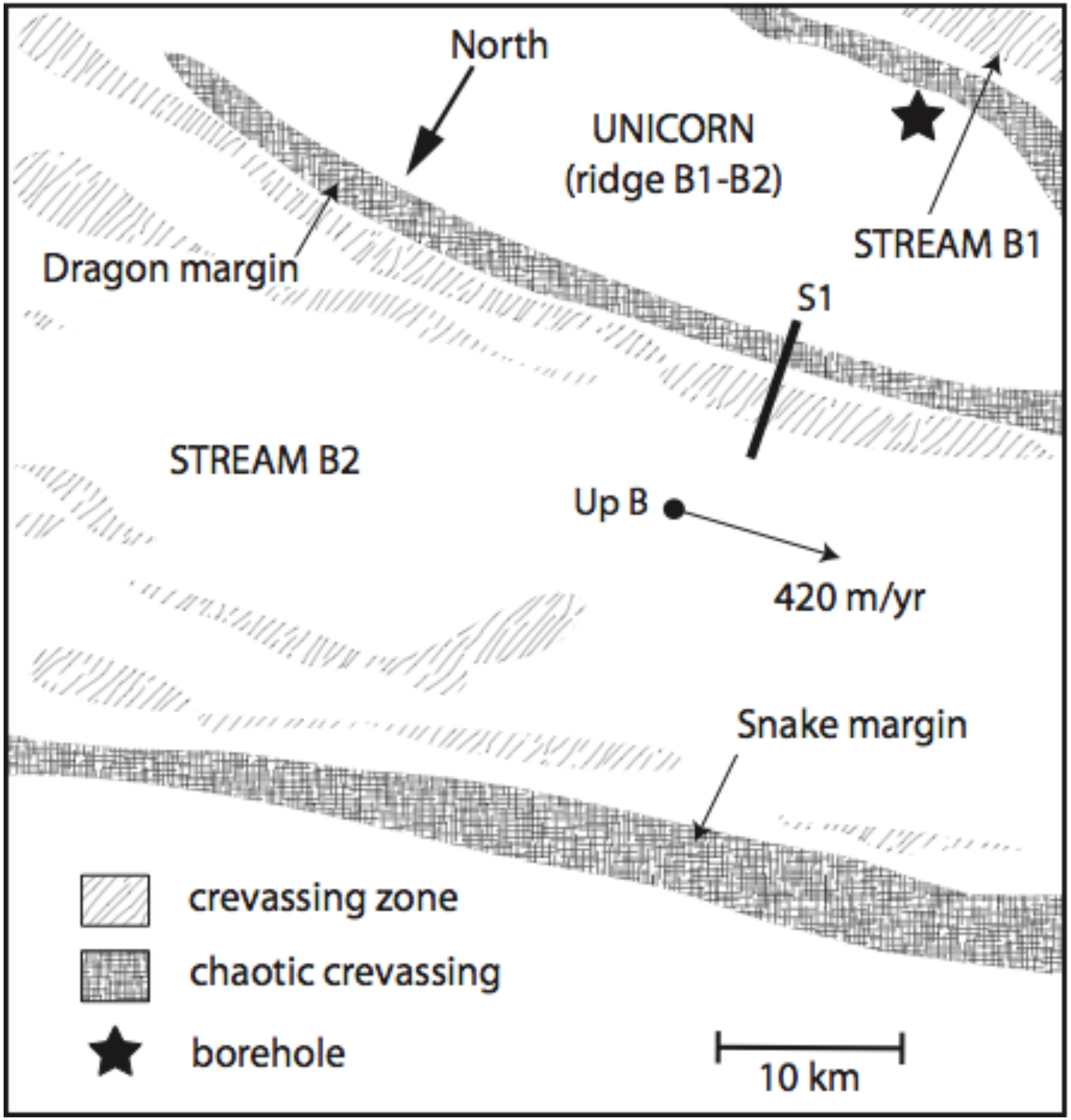
Characterized by Joughin et al. (*J. Geoph. Res.,* 2002)



Focus for most detailed study:

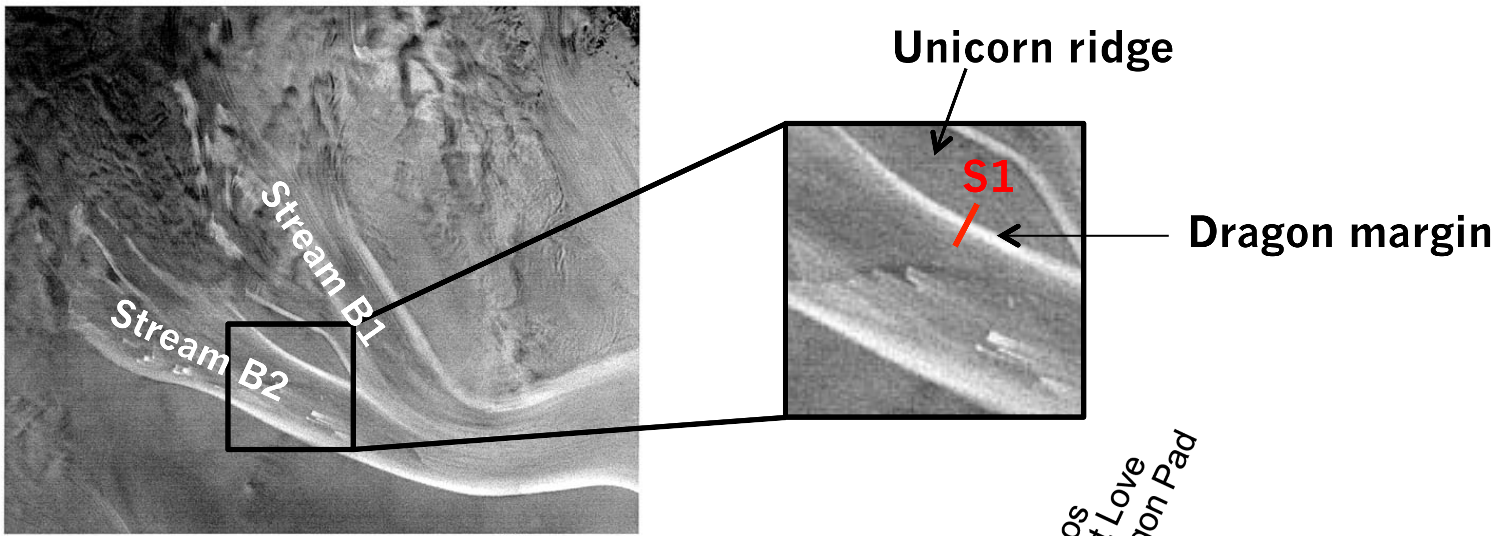
Dragon Margin
(very near to, but not the same as, the WB2 profile)

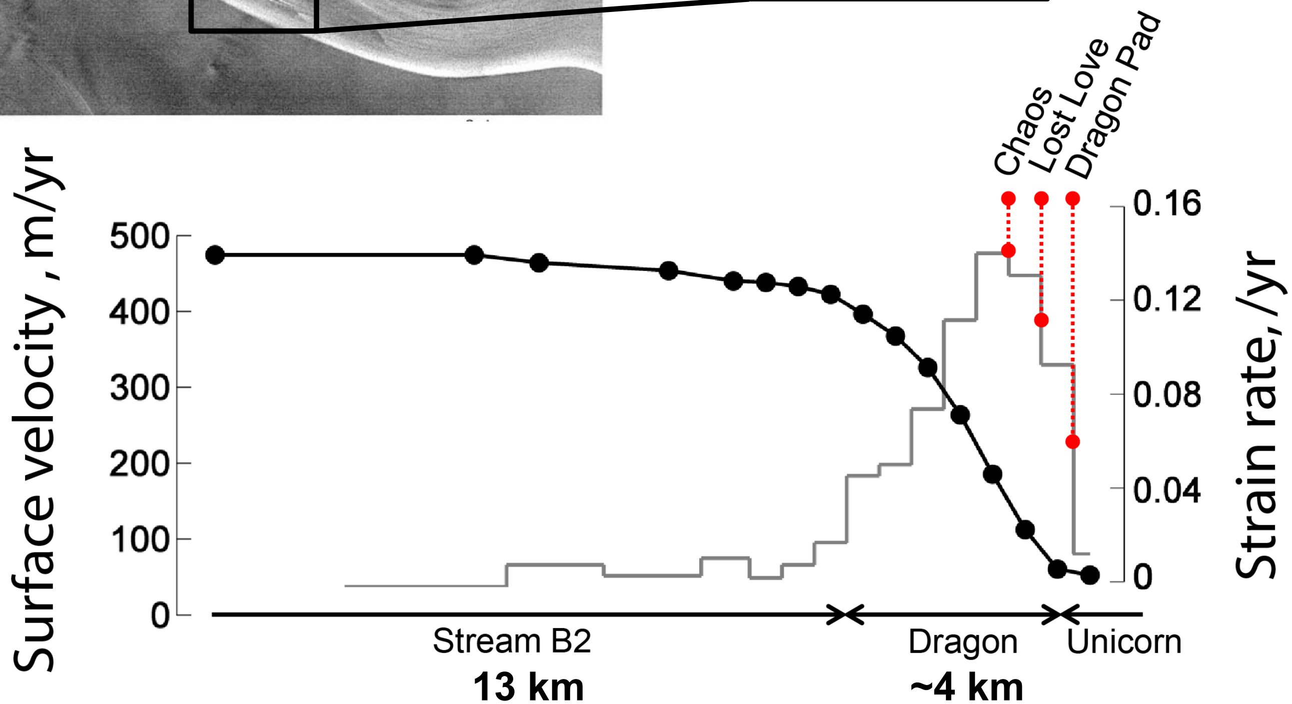




Echelmeyer and Harrison (1999)

Dragon margin





Suckale et al. (2014); data from Harrison et al. (1998), Echelmeyer and Harrison (1999)

Table 1. Parameters taken from *Joughin et al.* [2002] and used for margins of the profiles located in Figure 2. Profiles beginning with the letter T are made at the tributaries of the ice streams.

Ice Stream	Profile	H (m)	W (km)	$\dot{\gamma}_{lat}$ ($10^{-2}.yr^{-1}$)
Mercer	A	1242	39	4.2
Whillans	WB1	1205	35	7.0
	WB2	985	34	9.5
	W Narrows	846	48	13.5
	W Plain	735	121	5.1
	TWB1	2188	25	3.8
	TWB2	1538	25	4.0
Kamb	C	1805	69	1.0
	TC1	1802	17	1.4
	TC2	2196	43	0.9
Bindschadler	D	888	55	5.8
	TD1	1952	24	2.5
	TD2	1412	35	5.4
	TD3	1126	21	2.2
MacAyeal	E	916	78	8.1
	TE	1177	19	5.5

Lateral strain rate $\dot{\gamma}_{lat}$ here is an *average* over ~ 2 km width at Ice Stream margin

Thermo-mechanical properties of ice

Data fits as suggested by Cuffy & Paterson [2010]

Glen's flow law for ice (dislocation creep):

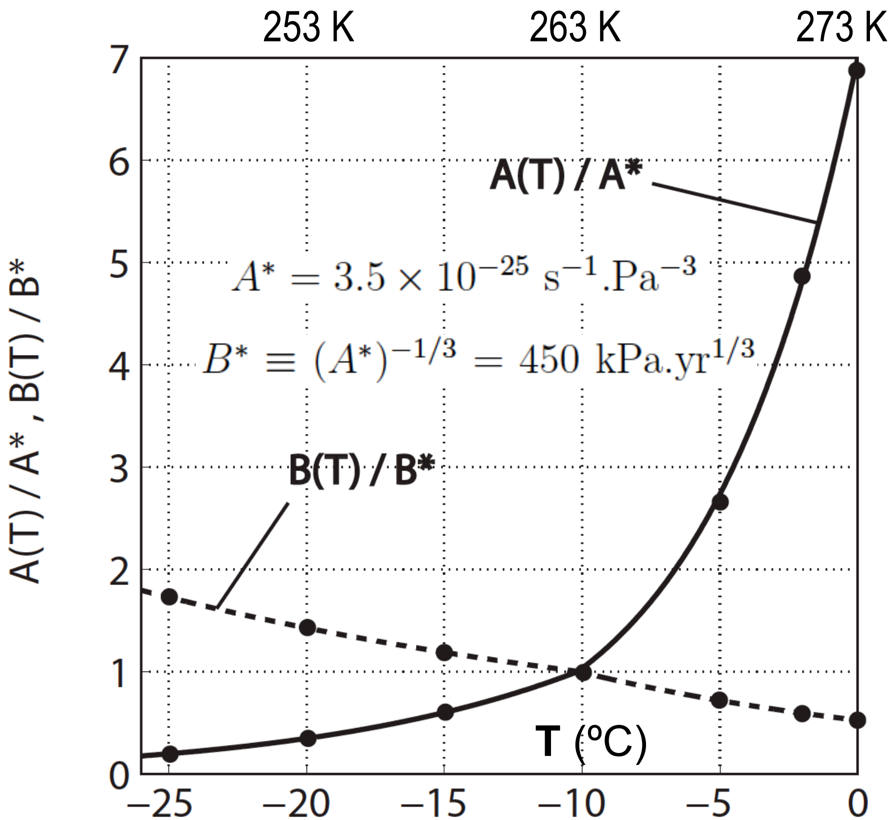
$$\dot{\gamma}/2 = A(T)\tau^3,$$

$$\tau = B(T)(\dot{\gamma}/2)^{1/3}$$

$$\left(B(T) = [1/A(T)]^{1/3}\right)$$

With diffusion creep too,

$$\dot{\gamma}/2 = \tau/2\eta(T, d_g) + A(T)\tau^3$$



Ice is still strong at T_{melt}:

For a given $\dot{\gamma}$:

$$\tau_{T=0°C} \approx 0.5 \times \tau_{T=-13°C}$$

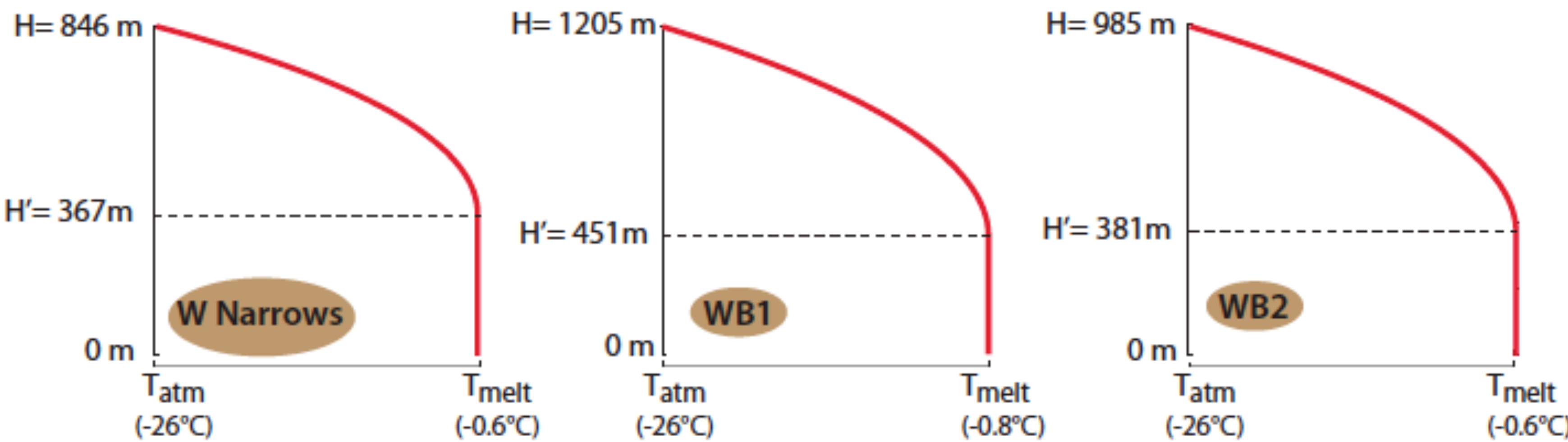
Thermal conductivity:

$$K(T) = 9.828 \frac{\mathrm{J}}{\mathrm{m\,s\,K}} \exp\left(-5.7\times10^{-3}\frac{T}{\mathrm{K}}\right)$$

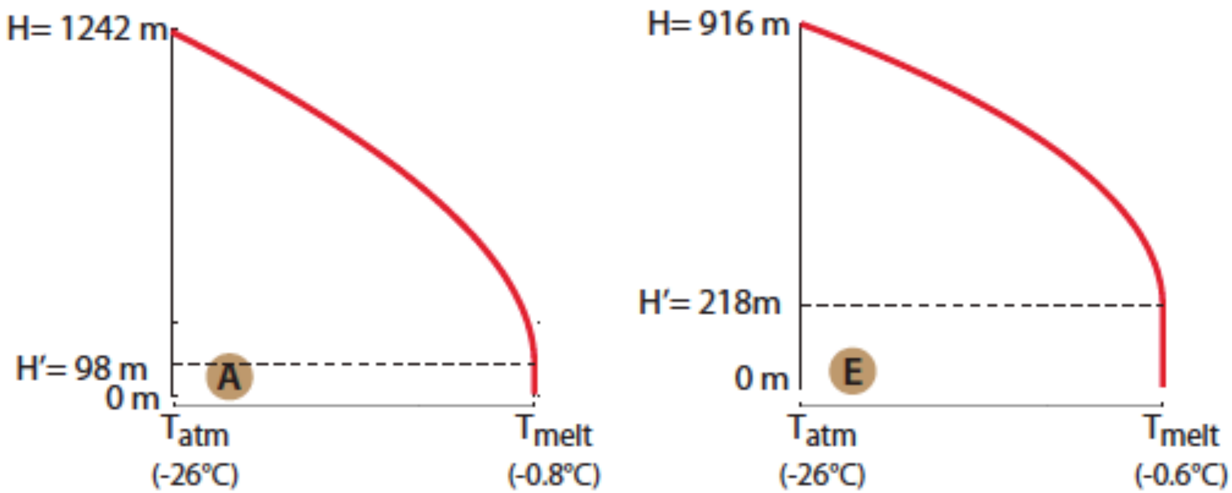
Specific heat:

$$C_i(T) = \left(152.5 + 7.122\frac{T}{\mathrm{K}}\right)\frac{\mathrm{J}}{\mathrm{kg\,K}}$$

$$\frac{d}{dz}\left(K(T)\frac{dT}{dz}\right)+\left(\frac{1}{2A(T)}\right)^{1/3}(\dot{\gamma}_{lat})^{4/3}+\rho C_i(T)(az/H)\frac{dT}{dz}=0 \;\;\&\;\; T\le T_{melt} \;\;\Rightarrow$$



(For all cases,
$a = 0.1$ m/yr)



Typical results, valid for nearly all stream margins:
Substantial thickness of temperate ice (i.e., at $T = T_{melt}$) predicted at base of the ice sheet

2D, antiplane strain analysis : velocity $= u(y,z)\vec{e}_x$, $\dot{\gamma} = \sqrt{\vec{\nabla} u \cdot \vec{\nabla} u}$

Coupled non - linear Poisson equation system
(for velocity u and temperature T) **:**

$$\frac{\partial}{\partial y}\left(\frac{\tau(\dot{\gamma},T)}{\dot{\gamma}}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\tau(\dot{\gamma},T)}{\dot{\gamma}}\frac{\partial u}{\partial z}\right)=-\rho g S \qquad \dot{\gamma}=\max\left(2A(T)\tau^3 \ , \ \tau/\eta(T)\right)$$

$$\frac{\partial}{\partial y}\left(K(T)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K(T)\frac{\partial T}{\partial z}\right)$$
$$=-[1-\hat{H}(T-T_{melt})]\tau(\dot{\gamma},T)\dot{\gamma}+\rho C(T)\left(v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)$$

(here, v & w are regarded as given, e.g., Zotikov's $w=-az/H$, $v=$ const., v_o)

Computational Approach (Suckale, Platt, Perol, Rice [JGR, 2014]) :

- **Multigrid methodology** for iterative solution of coupled nonlinear Poisson systems, embedding constraint $T \leq T_{melt}$.

(Suckale, Platt, Perol, and Rice, JGR 2014)

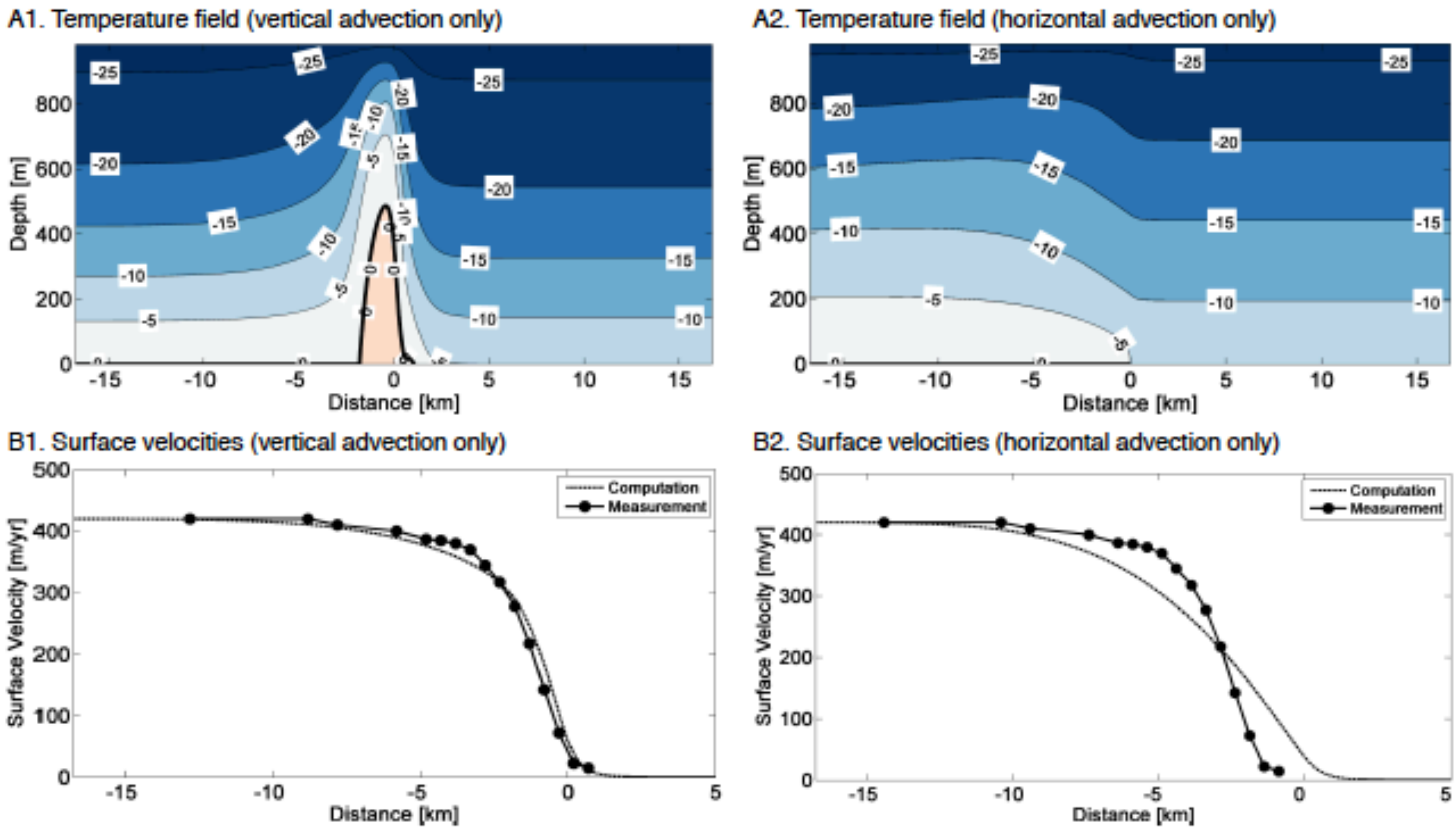


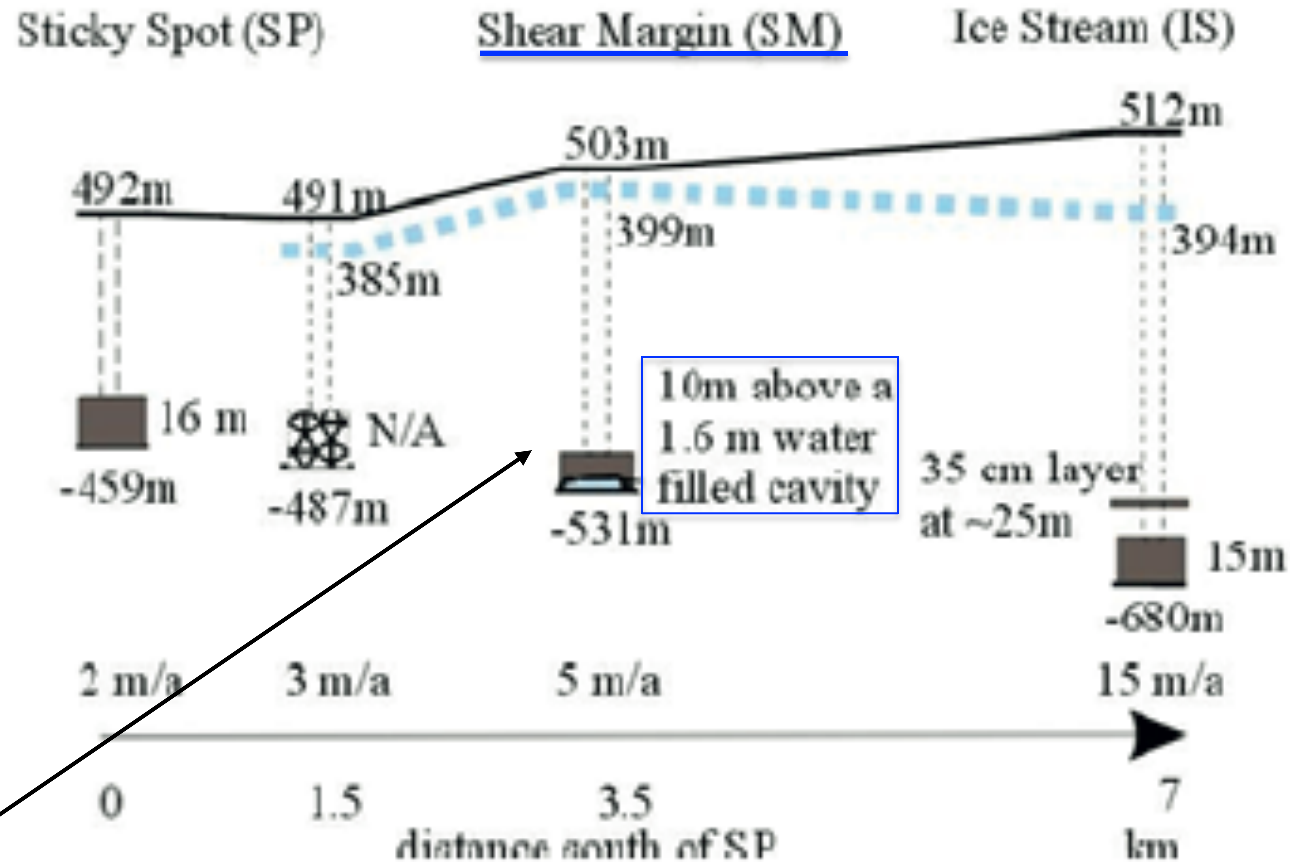
Figure 8. Temperature fields and surface velocities for Dragon margin when including only vertical advection (A1 and B1) with $a = 0.1\,\mathrm{m/yr}$ and only horizontal advection (A2 and B2) with $v = -7.3$ m/yr, respectively. The best fitting basal stresses are $\tau_{base} = 5.31\,\mathrm{kPa}$ (A1 and B1) and $\tau_{base} = 0.94\,\mathrm{kPa}$ (A2 and B2), respectively. Both computations neglect surface crevassing.

Water drainage along glacier beds often creates channels

Natural example, shown in photo by Laura Kehrl: Rothlisberger Channel
(but in a mountain glacier, with channel probably created by outburst underflooding):

Kennicott Glacier, Alaska

Evidence of channel at margins

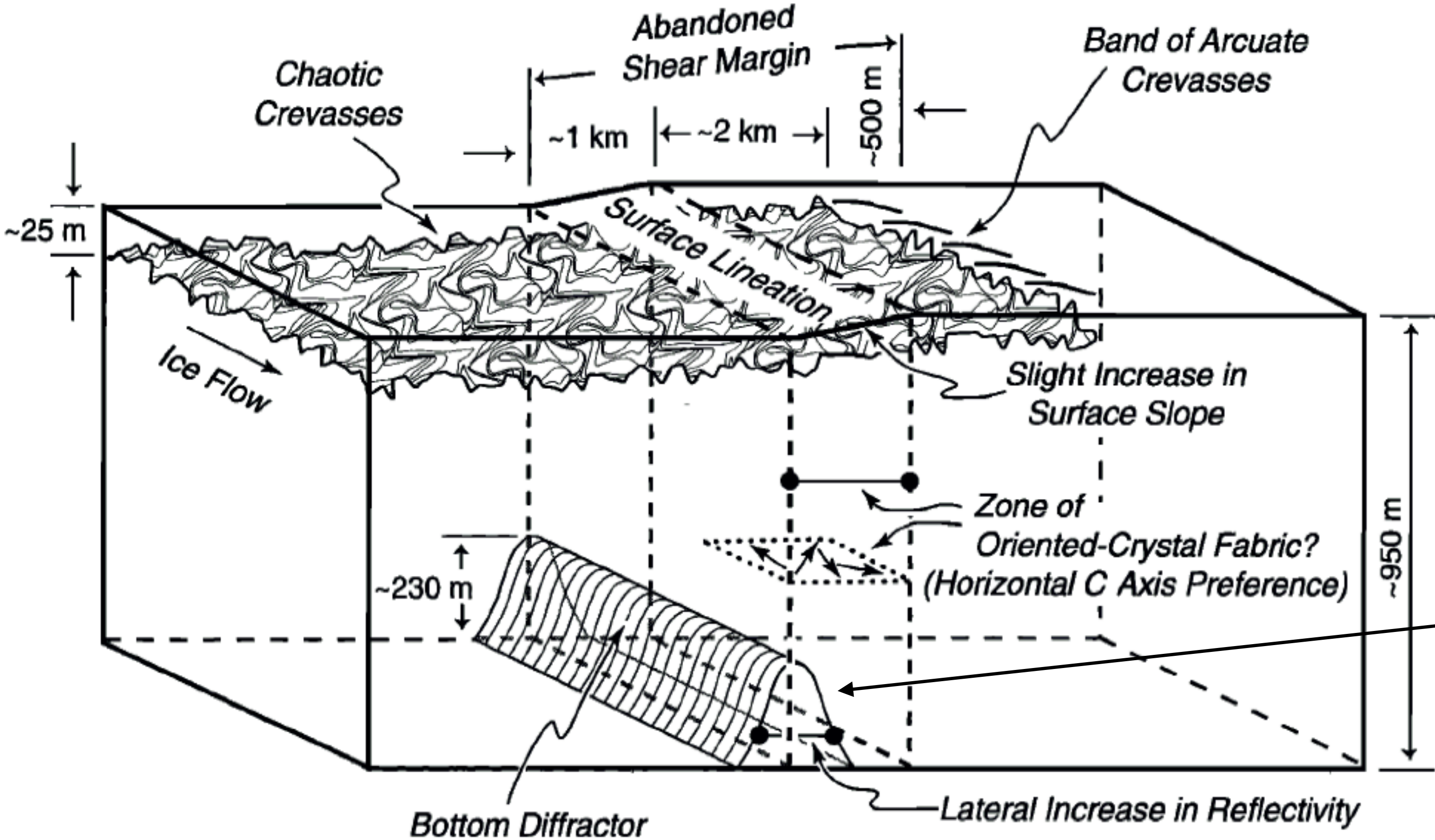


Borehole observation at the presently inactive shear margin of ***Kamb*** **(*C*)** ice stream:

- Found a 1.6 m tall water-filled cavity between 10 m of accreted ice and bed.
- Video of the borehole shows horizontal acceleration of particles sinking into the cavity, indicating flow of water within the cavity -- part of a channel?

[Modified from Vogel PhD (Thesis, 2004) and Vogel et al. (GRL, 2005).]

Possible field evidence of internal melting at margins



Possibly an abandoned position of Dragon Margin (few km away).

• Clarke et al. [2000], in order to explain the bottom diffractors, have invoked **partial melting in temperate ice to a height of 230 m**, due to strain heating, among other possibilities (**entrained sediments, bottom crevasses**).

• Also, Clarke et al. noted a personal communication from H. Engelhardt (Caltech): Abnormal drill resistance encountered from ≈ 56 m above bed. Fresh scratches found on drill tip (assumed to due to **entrained sediments**).

Rothlisberger-Shreve channel analysis

$$\sigma_{hoop} - p_{ch} = \frac{2}{3}(\sigma_o - p_{ch})$$

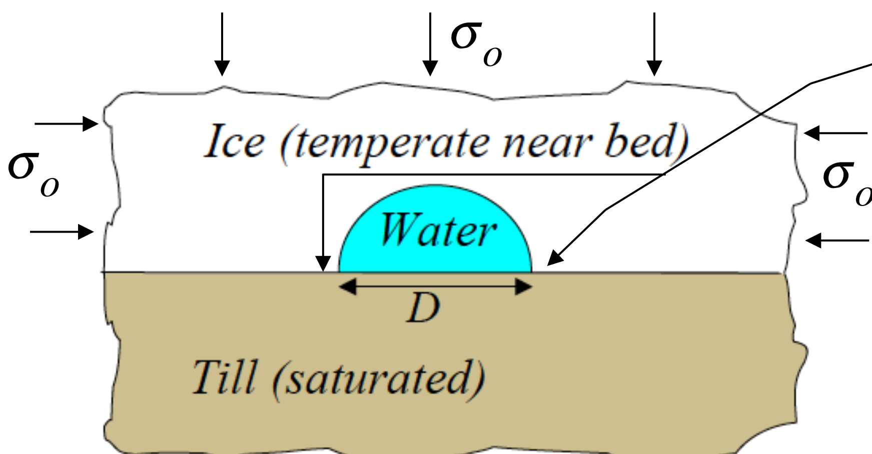
For $Q_w = 100\,\text{km} \times 41\,\text{m}^3/\text{m}\cdot\text{yr}$, and $S = 0.0012$:

Manning Coefficient, n_M $(\text{s}/\text{m}^{1/3})$	0.01	0.02	0.03	0.04
Equivalent Nikuradse Roughness, k (cm)	0.03	1.6	18.0	101.1
Channel Diameter, D (m)	0.9	1.1	1.3	1.5
Effective Normal Stress at Channel Margin, $\sigma_{hoop} - p_{ch}$ (kPa)	369	310	280	261

Clarke sub-glacial flooding range (0.02 to 0.04)

Assumed plausible here (0.02 to 0.03)

Sensitivity : $Q_w \rightarrow 0.25 Q_w \;\Rightarrow\; D \rightarrow 0.59 D\,,\; (\sigma_{hoop} - p_{ch}) \rightarrow 0.89(\sigma_{hoop} - p_{ch})$



Strength $\tau_{ch} = f(\sigma_{hoop} - p_{ch})$

$$\approx 0.5(\sigma_{hoop} - p_{ch}) \approx 150\ \text{kPa}$$

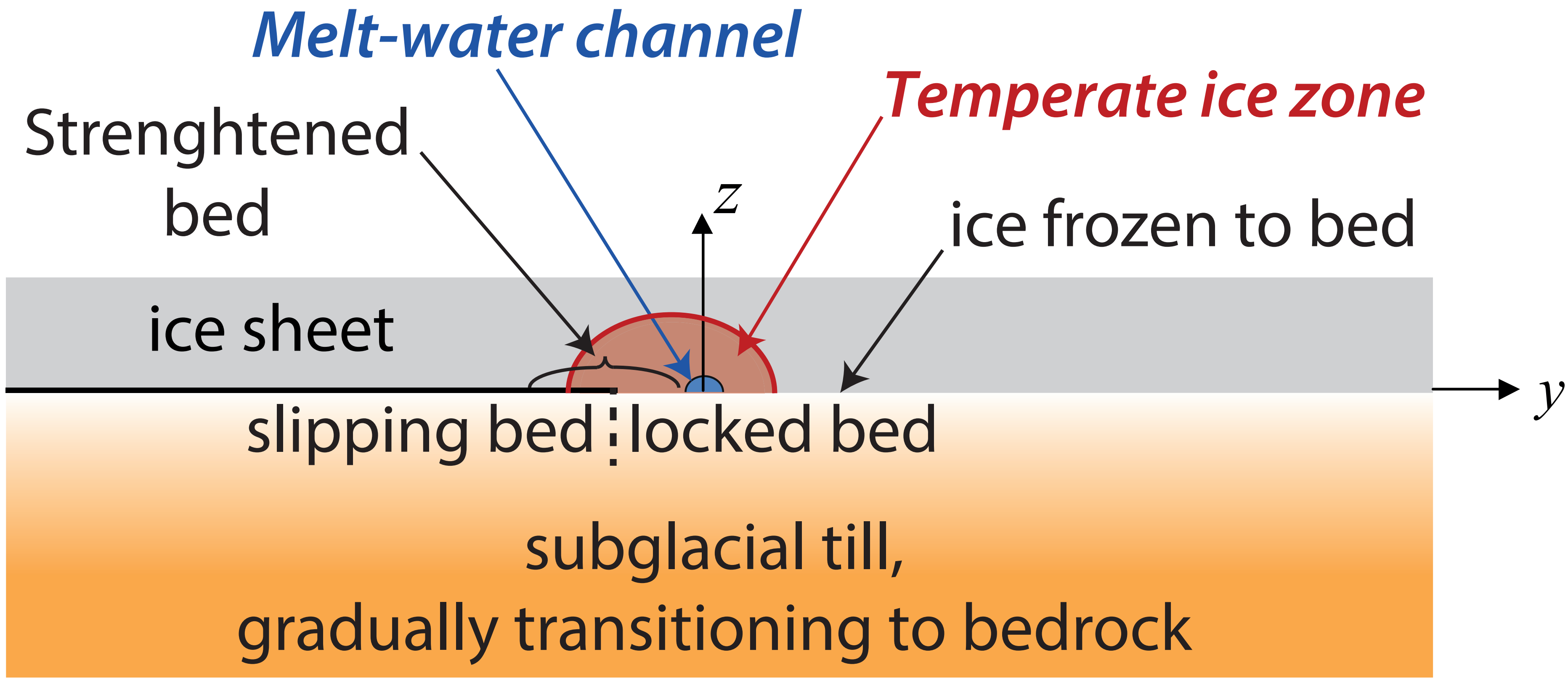
$$\therefore \tau_{ch} / \tau_{base} \approx 20 \text{ to } 45$$

For the 6 major streams, τ_{ch} / τ_{base} average = 32, and range = 12 to 56.

So, a marginal drainage channel could be the source of enhanced basal resistance!

(Perol, Rice, Platt and Suckale, AGU Dec. 2014)

How subglacial hydrology can control the shear margin location of ice streams



(Perol, Rice, Platt and Suckale, AGU Dec. 2014)

Governing equations, mechanical-thermal-hydrologic model of ice stream

- ***Mechanical model of an anti-plane shear flow driven by gravity***

$$\frac{\partial}{\partial y}\left(\frac{\tau(\dot{\gamma},T)}{\dot{\gamma}}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\tau(\dot{\gamma},T)}{\dot{\gamma}}\frac{\partial u}{\partial z}\right)+\rho_{ice}gS=0\ ,\ \tau(\dot{\gamma},T)=\min\left[\left(\frac{\dot{\gamma}}{2A(T)}\right)^{1/3},\ \eta(T)\dot{\gamma}\right]$$

- ***Thermal model***

$$\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)-\rho_{ice}C_{ice}\left(v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)+\left[1-H(T-T_{melt})\right]\tau\dot{\gamma}=0$$

(we take $v=0$ (lateral advection neglected), and $w=-az/H$ (Zotikov))

- ***Subglacial hydrology model (Poiseuille flow in a thin water film)***

$$q_{melt}\equiv\frac{G_{geo}-G_{ice}+\tau_{base}u_b}{\rho_w L}=-\frac{\partial}{\partial y}\left(\frac{h^3}{12\mu_w}\frac{\partial p}{\partial y}\right)\quad \text{(we take thickness } h= \text{ const.)}$$

$\left(\tau_{base}=(\tau_{zx})_{z=0}=f\times(\rho_{ice}gH-p)+c=\text{basal strength},\ u_b=u_{z=0}=\text{basal sliding velocity}\right)$

System solved using Finite Element procedure in COMSOL

Perol, Rice, Platt & Suckale, J. Geophys. Res. - Earth Surface, June 2015

<u>Ice surface deformation data</u> is fit well by a range of models, with the slipping to locked transition occurring at 50 m to 500 m from the R-Channel (at y = 0 here), corresponding to equivalent Poiseuille flow film thicknesses of 0.114 to 0.237 mm:

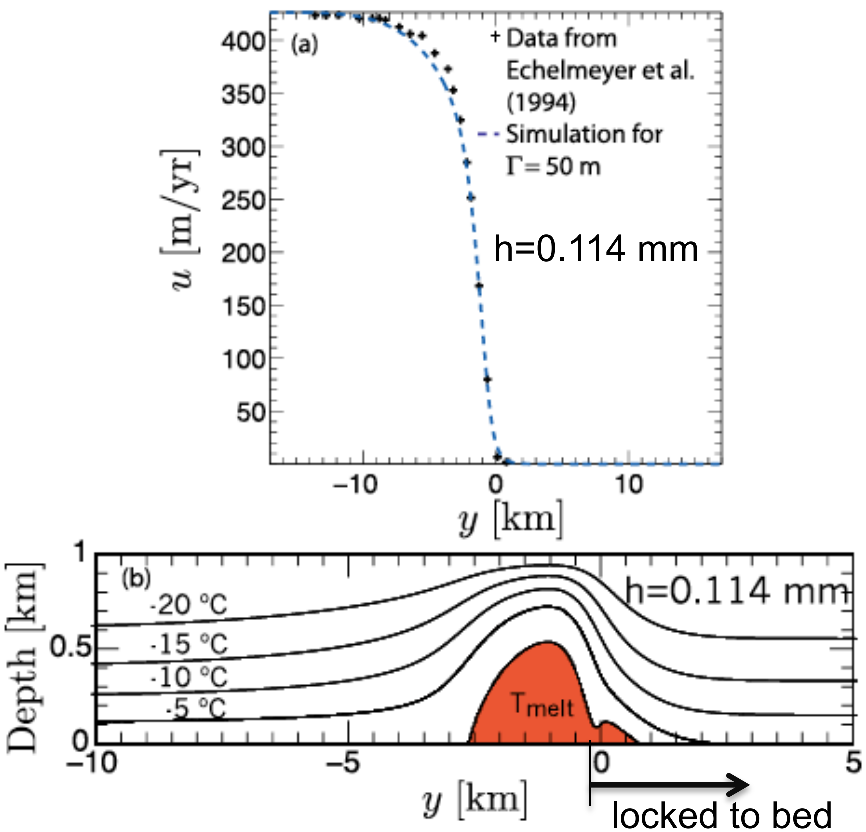


Figure 5. Results for a locking point 50 m away from the channel ($\Gamma = 50$ m). (a) Numerical surface velocities plotted alongside the data from *Echelmeyer et al.* [1994]. (b) Temperature field. In red is the temperate ice zone in which the temperature is capped at the melting point. The basal shear stress is free of singularity for $h = 0.114$ mm.

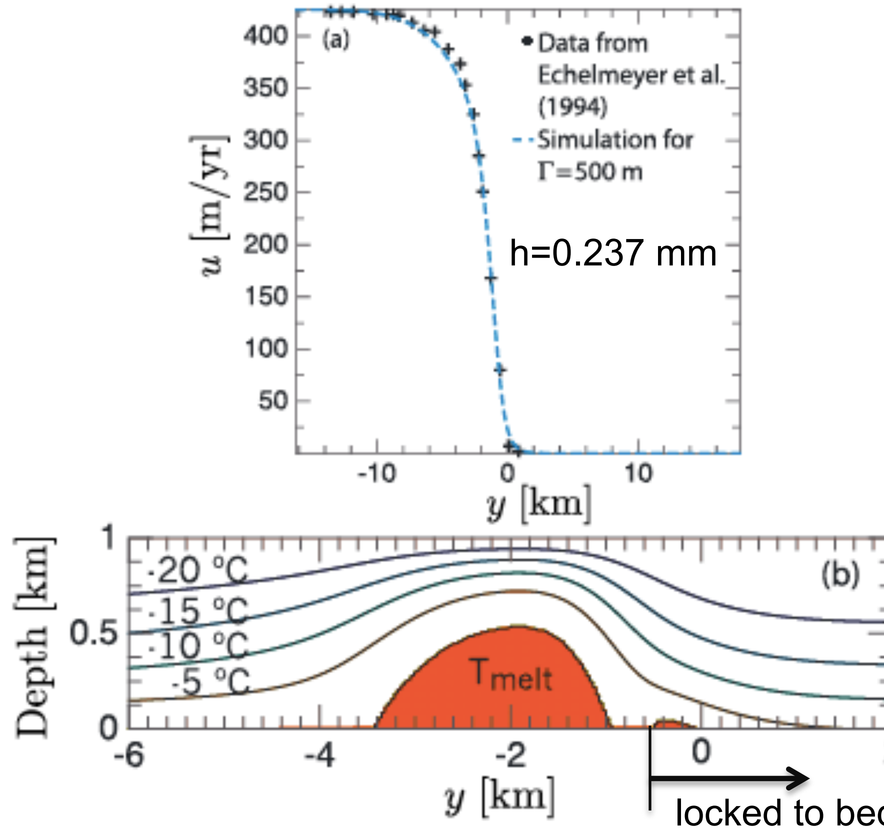


Figure 7. Results for a locking point 500 m away from the channel ($\Gamma = 500$ m). (a) Numerical surface velocities compare with data from *Echelmeyer et al.* [1994]. (b) Temperature field. In red is the temperate ice zone in which the temperature is capped at the melting point. The basal shear stress is free of singularity for $h = 0.237$ mm.