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APPLIED MECHANICS

Mathematically exploring wrinkle evolution

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Wrinkling is one of the most important mechanical deformation modes (for example, buckling and crumpling) that are omnipresent in our daily life: for instance, wrinkled fingers after soaking in water for a prolonged time, the folds within the brain, and metal wrinkles after a car collision, to name a few. These examples already hint at the multi-scale nature of wrinkling, and in reality, the wrinkling process can occur across a wide range of scales from nanometer-scale wrinkled graphene to continental-scale wrinkled geological strata. Wrinkling can sometimes be undesirable as a typical instability failure mode, such as wrinkling-induced device failures of a dielectric elastomer (which is a device converting electrical energy to mechanical work). On the other hand, well-controlled wrinkle patterns can be used to engineer surface properties for novel functional devices. For instance, wrinkled electrodes can provide enhanced power density in batteries due to the increased surface area.

Yet, the modeling of wrinkling processes is still a challenging task mathematically and numerically. The difficulties originate from both the inherited nonlinearity, and more importantly, the coupling with environmental variables and extreme conditions. For example, traditional approaches, such as shell theories, are not generalized in their coordinate system, which restrained their usage to certain simplified geometries with small strains. As a result, this limits the practical use in rational wrinkling design for emerging applications, such as soft polymer thin films in flexible electronics, where arbitrary shapes and large deformations are common. In a recent work, Fan Xu and colleagues developed a new finite strain shell model that can incorporate large deformations and more complex geometries, that is, with variable curvatures.

The authors integrated the general differential geometry and nonlinear elasticity theory to predict the wrinkling morphological evolution in soft shells. To account for the varying curvatures in different geometries, the team utilized a mapping from a parameter space to a physical domain, which allows them to derive two important tensors: metric tensor and curvature tensor. To ensure numerical stability when solving the nonlinear equations with varying curvatures, they developed a numerical algorithm by combining an asymptotic numerical method and the Chebyshev spectral collocation method. It is worth noting that this numerical treatment is more accurate when compared with traditional finite-element-based simulation methods. They showcased two model systems: S-shaped and C-shaped soft thin films. Their simulation revealed a curvature-dependent wrinkling evolution process, for example, the accumulation of wrinkles in the region with low curvature values, which was verified by experimental measurements. More interestingly, they proposed a wrinkling phase diagram as a function of experimentally controllable parameters, such as aspect ratios. To enable rational design of wrinkling patterns and its evolution, advanced mathematical modeling will play more important roles. This work added an important step towards this goal.

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